3.4 WS

KEY

Find all the zeros of the polynomial function and write the polynomial as a product of its leading coefficient and its linear factors.

1. $P(x) = x^4 + x^3 - 2x^2 + 4x - 24$

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -2 & 4 & -24 \\ & & 2 & 6 & 8 & 24 \\ \hline & 1 & 3 & 4 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 4 & 12 \\ & & -3 & 0 & -12 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

$x = 2, -3, \pm 2i$

$P(x) = (x-2)(x+3)(x+2i)(x-2i)$

2. $P(x) = x^3 - 3x^2 + 7x - 5$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 7 & -5 \\ & & 1 & -2 & 5 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$x^2 - 2x + 5 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

$x = 1, 1 \pm 2i$

$P(x) = (x-1)(x-1+2i)(x-1-2i)$

3. $P(x) = 2x^4 - x^3 - 4x^2 + 10x - 4$

$$\begin{array}{r|rrrrr} .5 & 2 & -1 & -4 & 10 & -4 \\ & & 1 & 0 & -2 & 4 \\ \hline & 2 & 0 & -4 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -4 & 8 \\ & & -4 & 8 & -8 \\ \hline & 2 & -4 & 4 & 0 \end{array}$$

$2x^2 - 4x + 4 = 0$

$2(x^2 - 2x + 2) = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2} = 1 \pm i$$

$x = \frac{1}{2}, -2, 1 \pm i$

$P(x) = 2\left(x - \frac{1}{2}\right)(x+2)(x-1+i)(x-1-i)$

4. $P(x) = x^5 - 9x^4 + 34x^3 - 58x^2 + 45x - 13$

$p: \pm 1, \pm 13 \quad q: \pm 1 \quad \frac{p}{q}: \pm 1, \pm 13$

$$\begin{array}{r|rrrrrr} 1 & 1 & -9 & 34 & -58 & 45 & -13 \\ & & 1 & -8 & 26 & -32 & 13 \\ \hline & 1 & -8 & 26 & -32 & 13 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 1 & 1 & -8 & 26 & -32 & 13 \\ & & 1 & -7 & 19 & -13 \\ \hline & 1 & -7 & 19 & -13 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 19 & -13 \\ & & 1 & -6 & 13 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

$x = 1 \text{ (mult. 3)}, 3 \pm 2i$

$P(x) = (x-1)^3(x-3+2i)(x-3-2i)$

Use the given zero to find the remaining zeros of each polynomial function.

5. $P(x) = x^4 - 6x^3 + 71x^2 - 146x + 530;\ 2+7i$

$(x-2+7i)(x-2-7i)$

$x^2 - 4x + 4 - 49i^2$

$x^2 - 4x + 53$

$$\begin{array}{r} x^2 - 2x + 10 \\ x^2 - 4x + 53 \overline{)\ x^4 - 6x^3 + 71x^2 - 146x + 530} \\ -x^4 + 4x^3 - 53x^2 \\ \hline -2x^3 + 18x^2 - 146x + 530 \\ +2x^3 - 8x^2 + 106x \\ \hline 10x^2 - 40x + 530 \\ -10x^2 + 40x - 530 \\ \hline 0 \end{array}$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(10)}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$$

$x = 2-7i,\ 1 \pm 3i$

6. $P(x) = 2x^3 - 5x^2 + 6x - 2;\ 1+i$

$(x-1+i)(x-1-i)$

$x^2 - 2x + 1 - i^2$

$x^2 - 2x + 2$

$$\begin{array}{r} 2x - 1 \\ x^2 - 2x + 2 \overline{)\ 2x^3 - 5x^2 + 6x - 2} \\ -2x^3 + 4x^2 - 4x \\ \hline -x^2 + 2x - 2 \\ +x^2 - 2x + 2 \\ \hline 0 \end{array}$$

$2x - 1 = 0$

$x = \frac{1}{2}$

$x = \frac{1}{2},\ 1-i$

Find the polynomial function *P*, with real coefficients, that has the indicated zeros and satisfies the given conditions.

7. Zeros: $2-5i,\ -4$; degree 3

$P(x) = x^3 + 13x + 116$

8. Zeros: $4+3i,\ 5-i$; degree 4

$P(x) = x^4 - 18x^3 + 131x^2 - 458x + 650$

9. Zeros: $3,\ 2i$; degree 3

$P(x) = x^3 - 3x^2 + 4x - 12$

10. Zeros: $\frac{1}{2},\ 4+i$; degree 3

$P(x) = 2x^3 - 17x^2 + 42x - 17$