

3.4 WS 3

KEY

Find all the zeros of the polynomial function and write the polynomial as a product of its leading coefficient and its linear factors.

1. $P(x) = 4x^4 - 4x^3 + 13x^2 - 12x + 3$

$$x = \frac{1}{2} \text{ (mult. 2)}, \pm i\sqrt{3}$$

$$P(x) = 4\left(x - \frac{1}{2}\right)^2 (x + i\sqrt{3})(x - i\sqrt{3})$$

2. $P(x) = 2x^4 - x^3 - 2x^2 + 13x - 6$

$$x = -2, \frac{1}{2}, 1 \pm i\sqrt{2}$$

$$P(x) = 2(x+2)\left(x - \frac{1}{2}\right)(x-1+i\sqrt{2})(x-1-i\sqrt{2})$$

3. $P(x) = 2x^4 - x^3 - 15x^2 + 23x + 15$

$$\begin{array}{r|rrrrr} -3 & 2 & -1 & -15 & 23 & 15 \\ & & -6 & 21 & -18 & -15 \\ \hline & 2 & -7 & 6 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -7 & 6 & 5 \\ & & -1 & 4 & -5 \\ \hline & 2 & -8 & 10 & 0 \end{array}$$

$$2x^2 - 8x + 10 = 0$$

$$2(x^2 - 4x + 5) = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

$$x = \frac{4 \pm 2i}{2} = 2 \pm i$$

$$x = -3, -\frac{1}{2}, 2 \pm i$$

$$P(x) = 2(x+3)\left(x + \frac{1}{2}\right)(x-2+i)(x-2-i)$$

4. $P(x) = 2x^3 - 9x^2 + 18x - 20$

$$x = \frac{5}{2}, 1 \pm i\sqrt{3}$$

$$P(x) = 2\left(x - \frac{5}{2}\right)(x-1+i\sqrt{3})(x-1-i\sqrt{3})$$

Use the given zero to find the remaining zeros of each polynomial function.

5. $P(x) = x^4 - 8x^3 + 18x^2 - 8x + 17$; i

$$x = -i, 4 \pm i$$

6. $P(x) = x^4 - 17x^3 + 112x^2 - 333x + 377$; $5 + 2i$

$$x = 5 - 2i, \frac{7 \pm i\sqrt{3}}{2}$$

Find the polynomial function P , with real coefficients, that has the indicated zeros and satisfies the given conditions.

7. Zeros: $\frac{3}{4}$, $2 + 7i$; degree 3

$$P(x) = 4x^3 - 19x^2 + 224x - 159$$

8. Zeros: i , $3 - 5i$; degree 4

$$P(x) = x^4 - 6x^3 + 35x^2 - 6x + 34$$

9. Zeros: -5 , 3 (multiplicity 2), $2 + i$; degree 5

$$P(x) = x^5 - 5x^4 - 12x^3 + 124x^2 - 285x + 225$$

10. Zeros: $-5, 2, 2 - 3i$; degree 4

$$P(x) = x^4 - x^3 - 9x^2 + 79x - 130$$