3.4 WS 2

KEY

Find all the zeros of the polynomial function and write the polynomial as a product of its leading coefficient and its linear factors.

1. $P(x)=x^4-4x^3+53x^2-196x+196$

$$\begin{array}{c|ccccc} 2 & 1 & -4 & 53 & -196 & 196 \\ & & 2 & -4 & 98 & -196 \\ \hline & 1 & -2 & 49 & -98 & 0 \end{array}$$

$$\begin{array}{c|ccccc} 2 & 1 & -2 & 49 & -98 \\ & & 2 & 0 & 98 \\ \hline & 1 & 0 & 49 & 0 \end{array}$$

$x^2+49=0$

$x=\pm 7i$

$x=2\text{ (mult. 2)}, \pm 7i$

$P(x)=(x-2)^2(x+7i)(x-7i)$

2. $P(x)=x^3-13x^2+65x-125$

$$\begin{array}{c|cccc} 5 & 1 & -13 & 65 & -125 \\ & & 5 & -40 & 125 \\ \hline & 1 & -8 & 25 & 0 \end{array}$$

$x^2-8x+25=0$

$x=\dfrac{8\pm\sqrt{64-4(1)(25)}}{2}=\dfrac{8\pm 6i}{2}=4\pm 3i$

$x=5, 4\pm 3i$

$P(x)=(x-5)(x-4+3i)(x-4-3i)$

3. $P(x)=3x^4-10x^3+15x^2+20x-8$

$$\begin{array}{c|ccccc} -1 & 3 & -10 & 15 & 20 & -8 \\ & & -3 & 13 & -28 & 8 \\ \hline & 3 & -13 & 28 & -8 & 0 \end{array}$$

$$\begin{array}{c|cccc} \frac{1}{3} & 3 & -13 & 28 & -8 \\ & & 1 & -4 & 8 \\ \hline & 3 & -12 & 24 & 0 \end{array}$$

$3x^2-12x+24=0$

$3(x^2-4x+8)=0$

$x=\dfrac{4\pm\sqrt{16-4(1)(8)}}{2}=\dfrac{4\pm 4i}{2}=2\pm 2i$

$x=-1, \frac{1}{3}, 2\pm 2i$

$P(x)=3(x+1)(x-\frac{1}{3})(x-2+2i)(x-2-2i)$

4. $P(x)=3x^5+2x^4+10x^3+6x^2-25x-20$

$$\begin{array}{c|cccccc} -1 & 3 & 2 & 10 & 6 & -25 & -20 \\ & & -3 & 1 & -11 & 5 & 20 \\ \hline & 3 & -1 & 11 & -5 & -20 & 0 \end{array}$$

$$\begin{array}{c|ccccc} -1 & 3 & -1 & 11 & -5 & -20 \\ & & -3 & 4 & -15 & 20 \\ \hline & 3 & -4 & 15 & -20 & 0 \end{array}$$

$$\begin{array}{c|cccc} \frac{4}{3} & 3 & -4 & 15 & -20 \\ & & 4 & 0 & 20 \\ \hline & 3 & 0 & 15 & 0 \end{array}$$

$3x^2+15=0$

$3x^2=-15$

$x^2=-5$

$x=\pm i\sqrt{5}$

$x=-1\text{ (mult. 2)}, \frac{4}{3}, \pm i\sqrt{5}$

$P(x)=3(x+1)^2(x-\frac{4}{3})(x+i\sqrt{5})(x-i\sqrt{5})$

Use the given zero to find the remaining zeros of each polynomial function.

5. $P(x) = x^4 - 4x^3 + 14x^2 - 4x + 13$; $2 - 3i$ & $2+3i$

$(x-2-3i)(x-2+3i)$

$x^2-4x+4-9i^2$

$$\begin{array}{r} x^2+1 \\ x^2-4x+13 \overline{)\,x^4-4x^3+14x^2-4x+13} \\ \underline{-x^4+4x^3-13x^2} \\ x^2-4x+13 \\ \underline{-x^2+4x-13} \\ 0 \end{array}$$

$x^2+1=0$

$x^2=-1$

$x=\pm i$

$x=\pm i, 2+3i$

6. $P(x) = 3x^3 - 29x^2 + 92x + 34$; $5 + 3i$ & $5 - 3i$

$(x-5+3i)(x-5-3i)$

$x^2-10x+25-9i^2$

$$\begin{array}{r} 3x+1 \\ x^2-10x+34 \overline{)\,3x^3-29x^2+92x+34} \\ \underline{-3x^3+30x^2-102x} \\ x^2-10x+34 \\ \underline{-x^2+10x-34} \\ 0 \end{array}$$

$3x+1=0$

$x=\frac{-1}{3}$

$x=\frac{-1}{3}, 5-3i$

Find the polynomial function *P*, with real coefficients, that has the indicated zeros and satisfies the given conditions.

7. Zeros: i, 0; degree 3

$P(x)=x^3+x$

8. Zeros: $3+i$, $2-5i$; degree 4

$P(x)=x^4-10x^3+63x^2-214x+290$

9. Zeros: $3+2i$, 7; degree 3

$P(x)=x^3-13x^2+55x-91$

10. Zeros: $-2, 1, 3, 1+4i$; degree 5

$P(x)=x^5-4x^4+16x^3-18x^2-97x+102$