



## 3.4 Using the Quadratic Formula

**Learning Target** Solve and analyze quadratic equations using the Quadratic Formula and discriminants.

**Success Criteria**

- I can solve quadratic equations using the Quadratic Formula.
- I can find and interpret the discriminant of an equation.
- I can write quadratic equations with different numbers of solutions using the discriminant.

### EXPLORE IT! Analyzing the Quadratic Formula

#### Math Practice

##### View as Components

Can you write the right side of the formula as two fractions? If so, what does each fraction represent?

**Work with a partner.** Recall the *Quadratic Formula*, which can be used to find the solutions of any quadratic equation of the form  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

a. Show how to derive this formula by completing the square for  $ax^2 + bx + c = 0$ . The first step has been done for you.

Step	Justification
$ax^2 + bx + c = 0$	General equation
$ax^2 + bx = -c$	Subtract $c$ from each side.

b. What part of the Quadratic Formula tells whether a quadratic equation has real solutions or imaginary solutions? When does the formula produce real solutions for a quadratic equation? When does it produce imaginary solutions? Explain.

c. Can the Quadratic Formula produce one real solution and one imaginary solution? Explain.

d. Without solving, use your answer to part (b) to determine whether each equation has real solutions or imaginary solutions. What does this tell you about the graph of each equation?

i.  $x^2 - 4x + 3 = 0$

ii.  $x^2 + 4x + 6 = 0$

iii.  $x^2 + 4x + 4 = 0$

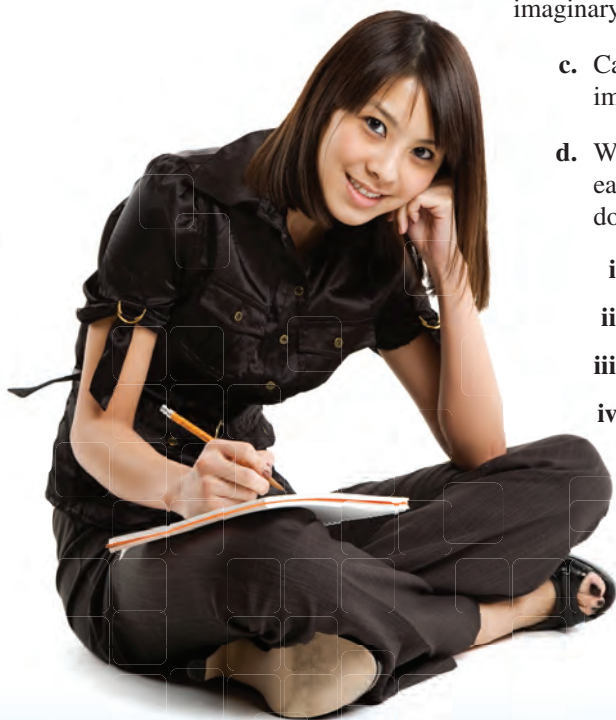
iv.  $x^2 - 6x + 10 = 0$

e. Solve the quadratic equation in as many ways as you can.

i.  $x^2 + 2x - 3 = 0$

ii.  $x^2 - 2x + 2 = 0$

f. Summarize the following methods you have learned for solving quadratic equations: graphing, using square roots, factoring, completing the square, and using the Quadratic Formula. Include when you would use each method.





## Solving Equations Using the Quadratic Formula

### Vocabulary



Quadratic Formula, p. 118  
discriminant, p. 120

You have already learned how to solve quadratic equations by graphing, using square roots, factoring, and completing the square. In the Explore It!, you derived the **Quadratic Formula** by completing the square. You can use the Quadratic Formula to find the solutions of any quadratic equation in standard form.



### KEY IDEA

#### The Quadratic Formula

The solutions of the quadratic equation  $ax^2 + bx + c = 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ .

### EXAMPLE 1 Solving an Equation with Two Real Solutions

Solve  $x^2 + 3x = 5$  using the Quadratic Formula.



#### SOLUTION

Be sure to write the quadratic equation in standard form before applying the Quadratic Formula.

$$x^2 + 3x = 5 \quad \text{Write original equation.}$$

$$x^2 + 3x - 5 = 0 \quad \text{Write in standard form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)} \quad \text{Substitute 1 for } a, 3 \text{ for } b, \text{ and } -5 \text{ for } c.$$

$$x = \frac{-3 \pm \sqrt{29}}{2} \quad \text{Simplify.}$$

► So, the solutions are  $x = \frac{-3 + \sqrt{29}}{2}$  and  $x = \frac{-3 - \sqrt{29}}{2}$ .

### Math Practice

#### Calculate Accurately

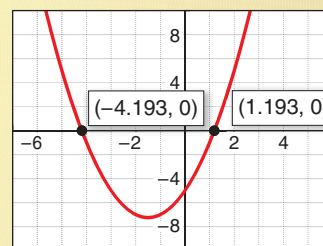
In Example 1, can you say that  $x = -4.193$  is a solution of the equation? Can you say that  $x \approx 1.193$  is a solution? Explain.

#### Check

You can check your solutions by graphing  $y = x^2 + 3x - 5$ .

$$-4.193 \approx \frac{-3 - \sqrt{29}}{2}$$

$$1.193 \approx \frac{-3 + \sqrt{29}}{2}$$



## SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. **MP STRUCTURE** Explain how to identify  $a$ ,  $b$ , and  $c$  when using the Quadratic Formula.

Solve the equation using the Quadratic Formula.

2.  $x^2 - 6x + 4 = 0$

3.  $2x^2 + 4 = -7x$

4.  $5x^2 = x + 8$



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### EXAMPLE 2

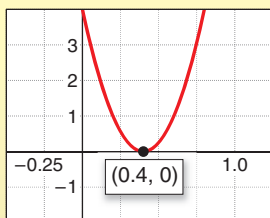
### Solving an Equation with One Real Solution



Solve  $25x^2 - 8x = 12x - 4$  using the Quadratic Formula.

#### Check

Graph  $y = 25x^2 - 20x + 4$ .  
The only  $x$ -intercept is  $\frac{2}{5}$ .



#### SOLUTION

$$25x^2 - 8x = 12x - 4$$

$$25x^2 - 20x + 4 = 0$$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$$

$$x = \frac{20 \pm \sqrt{0}}{50}$$

$$x = \frac{2}{5}$$

Write original equation.

Write in standard form.

$$a = 25, b = -20, c = 4$$

Simplify.

Simplify.

▶ So, the solution is  $x = \frac{2}{5}$ .

### EXAMPLE 3

### Solving an Equation with Imaginary Solutions



Solve  $-x^2 + 4x = 13$  using the Quadratic Formula.

#### SOLUTION

$$-x^2 + 4x = 13$$

$$-x^2 + 4x - 13 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-13)}}{2(-1)}$$

$$x = \frac{-4 \pm \sqrt{-36}}{-2}$$

$$x = \frac{-4 \pm 6i}{-2}$$

$$x = 2 \pm 3i$$

Write original equation.

Write in standard form.

$$a = -1, b = 4, c = -13$$

Simplify.

Write in terms of  $i$ .

Simplify.

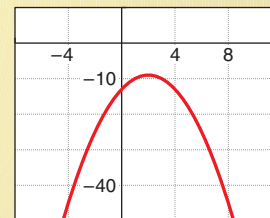
▶ The solutions are  $x = 2 + 3i$  and  $x = 2 - 3i$ .

#### COMMON ERROR

Remember to divide the real part *and* the imaginary part by  $-2$  when simplifying.

**Check** Graph  $y = -x^2 + 4x - 13$ . There are no  $x$ -intercepts. So, the original equation has no real solutions. An algebraic check for one of the imaginary solutions is shown.

$$\begin{aligned} -(2 + 3i)^2 + 4(2 + 3i) &\stackrel{?}{=} 13 \\ -(4 + 12i - 9) + 8 + 12i &\stackrel{?}{=} 13 \\ 5 - 12i + 8 + 12i &\stackrel{?}{=} 13 \\ 13 &= 13 \quad \checkmark \end{aligned}$$



## SELF-ASSESSMENT

1 I do not understand.

2 I can do it with help.

3 I can do it on my own.

4 I can teach someone else.

Solve the equation using the Quadratic Formula.

5.  $x^2 + 41 = -8x$

6.  $-9x^2 = 30x + 25$

7.  $5x - 7x^2 = 3x + 4$

8. **MP REASONING** In Example 2, explain how you can use the solution to factor  $25x^2 - 20x + 4$ .



## Analyzing the Discriminant

In the Quadratic Formula, the expression  $b^2 - 4ac$  is called the **discriminant** of the associated equation  $ax^2 + bx + c = 0$ .

### WORDS AND MATH

*Discriminate* means to mark the distinguishing features of something. The *discriminant* of the Quadratic Formula can be used to discriminate between quadratic equations with different numbers and types of solutions.

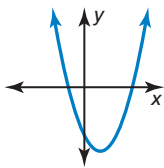
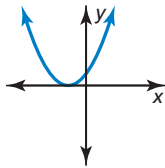
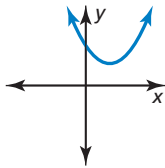
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

You can analyze the discriminant of a quadratic equation to determine the number and type of solutions of the equation.



### KEY IDEA

#### Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two $x$ -intercepts	 One $x$ -intercept	 No $x$ -intercept

### EXAMPLE 4

#### Analyzing the Discriminant



Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a.  $x^2 - 6x + 10 = 0$

b.  $x^2 - 6x + 9 = 0$

c.  $x^2 - 6x + 8 = 0$

### SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
a. $x^2 - 6x + 10 = 0$	$(-6)^2 - 4(1)(10) = -4$	Two imaginary: $3 \pm i$
b. $x^2 - 6x + 9 = 0$	$(-6)^2 - 4(1)(9) = 0$	One real: 3
c. $x^2 - 6x + 8 = 0$	$(-6)^2 - 4(1)(8) = 4$	Two real: 2, 4

## SELF-ASSESSMENT

- 1 I do not understand.    2 I can do it with help.    3 I can do it on my own.    4 I can teach someone else.

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

9.  $4x^2 + 8x + 4 = 0$

10.  $\frac{1}{2}x^2 + x - 1 = 0$

11.  $5x^2 = 8x - 13$

12.  $7x^2 - 3x = 6$

13.  $4x^2 + 6x = -9$

14.  $-5x^2 + 1 = 6 - 10x$

15. **MP REASONING** What is the value of  $b^2$  when a quadratic equation of the form  $ax^2 + bx + c = 0$  has exactly one real solution? Explain your reasoning.

**EXAMPLE 5****Writing Quadratic Equations**

Find a possible pair of integer values for  $a$  and  $c$  so that the equation  $ax^2 - 4x + c = 0$  has the given number and type of solution(s). Then write the equation.

- a. one real solution
- b. two imaginary solutions

**SOLUTION**

- a. For the equation to have one real solution, the discriminant must equal 0.

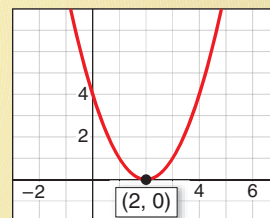
$$\begin{aligned} b^2 - 4ac &= 0 && \text{Write the discriminant.} \\ (-4)^2 - 4ac &= 0 && \text{Substitute } -4 \text{ for } b. \\ 16 - 4ac &= 0 && \text{Evaluate the power.} \\ -4ac &= -16 && \text{Subtract 16 from each side.} \\ ac &= 4 && \text{Divide each side by } -4. \end{aligned}$$

Because  $ac = 4$ , choose two integers whose product is 4, such as  $a = 1$  and  $c = 4$ .

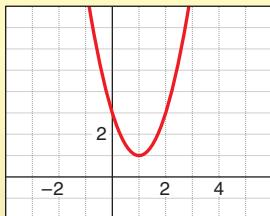
▶ So, one possible equation is  $x^2 - 4x + 4 = 0$ .

**Check** Graph  $y = x^2 - 4x + 4$ . The only  $x$ -intercept is 2. You can also check by factoring.

$$\begin{aligned} x^2 - 4x + 4 &= 0 \\ (x - 2)^2 &= 0 \\ x &= 2 \quad \checkmark \end{aligned}$$

**Check**

The graph of  $y = 2x^2 - 4x + 3$  does not have any  $x$ -intercepts. ✓



- b. For the equation to have two imaginary solutions, the discriminant must be less than zero.

$$\begin{aligned} b^2 - 4ac &< 0 && \text{Write the discriminant.} \\ (-4)^2 - 4ac &< 0 && \text{Substitute } -4 \text{ for } b. \\ 16 - 4ac &< 0 && \text{Evaluate the power.} \\ -4ac &< -16 && \text{Subtract 16 from each side.} \\ ac &> 4 && \text{Divide each side by } -4. \text{ Reverse inequality symbol.} \end{aligned}$$

Because  $ac > 4$ , choose two integers whose product is greater than 4, such as  $a = 2$  and  $c = 3$ .

▶ So, one possible equation is  $2x^2 - 4x + 3 = 0$ .

**SELF-ASSESSMENT**

- 1 I do not understand.    2 I can do it with help.    3 I can do it on my own.    4 I can teach someone else.

16. **OPEN-ENDED** Find another possible equation for Examples 5(a) and 5(b).

Find a possible pair of integer values for  $a$  and  $c$  so that the equation  $ax^2 + 8x + c = 0$  has the given number and type of solution(s). Then write the equation.

17. two real solutions                      18. two imaginary solutions                      19. one real solution



## Solving Real-Life Problems

The function  $h = -16t^2 + s_0$  is used to model the height of a *dropped* object, where  $h$  is the height (in feet),  $t$  is the time in motion (in seconds), and  $s_0$  is the initial height (in feet). For an object that is *launched* or *thrown*, an extra term  $v_0t$  must be added to the model to account for the object's initial vertical velocity  $v_0$  (in feet per second).

$$h = -16t^2 + s_0 \quad \text{Object is dropped.}$$

$$h = -16t^2 + v_0t + s_0 \quad \text{Object is launched or thrown.}$$

### REMEMBER

These models assume that the force of air resistance on the object is negligible. Also, these models apply only to objects on Earth. For planets with stronger or weaker gravitational forces, different models are used.

As shown below, the value of  $v_0$  can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.



$$v_0 > 0$$



$$v_0 < 0$$



$$v_0 = 0$$

### EXAMPLE 6 Modeling Real Life



A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. Does the ball reach a height of 10 feet? 25 feet? Explain your reasoning.

### SOLUTION

Because the ball is *thrown*, use the model  $h = -16t^2 + v_0t + s_0$  to write a function that represents the height of the ball.

$$h = -16t^2 + v_0t + s_0 \quad \text{Write the height model.}$$

$$h = -16t^2 + 30t + 4 \quad \text{Substitute 30 for } v_0 \text{ and 4 for } s_0.$$

To determine whether the ball reaches each height, substitute each height for  $h$  to create two equations. Then solve each equation using the Quadratic Formula.

$$10 = -16t^2 + 30t + 4$$

$$25 = -16t^2 + 30t + 4$$

$$0 = -16t^2 + 30t - 6$$

$$0 = -16t^2 + 30t - 21$$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(-6)}}{2(-16)}$$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(-21)}}{2(-16)}$$

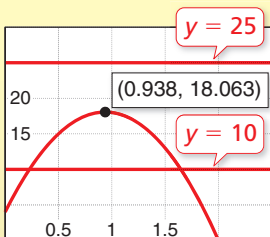
$$t = \frac{-30 \pm \sqrt{516}}{-32}$$

$$t = \frac{-30 \pm \sqrt{-444}}{-32}$$

When  $h = 10$ , the equation has two real solutions,  $t \approx 0.23$  and  $t \approx 1.65$ . When  $h = 25$ , the equation has two imaginary solutions because the discriminant is negative.

▶ So, the ball reaches a height of 10 feet, but it does not reach a height of 25 feet.

**Check** The graph shows that the ball reaches a height of 10 feet but not 25 feet. ✓



## SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

20. **WHAT IF?** The ball leaves the juggler's hand with an initial vertical velocity of 40 feet per second. Does the ball reach a height of 20 feet? 30 feet? Explain.



# 3.4 Practice WITH CalcChat® AND CalcView®



In Exercises 1–14, solve the equation using the Quadratic Formula. Use technology to check your solution(s). ▶ *Examples 1, 2, and 3*

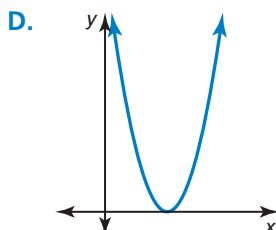
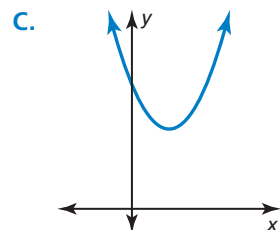
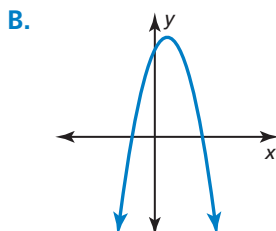
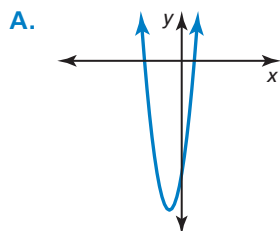
- |                        |                        |
|------------------------|------------------------|
| 1. $x^2 - 4x + 3 = 0$  | 2. $3x^2 + 6x + 3 = 0$ |
| 3. $x^2 + 6x + 15 = 0$ | 4. $6x^2 - 2x + 1 = 0$ |
| 5. $x^2 - 14x = -49$   | 6. $2x^2 + 4x = 30$    |
| 7. $3x^2 + 5 = -2x$    | 8. $-3x = 2x^2 - 4$    |
| 9. $-10x = -25 - x^2$  | 10. $-5x^2 - 6 = -4x$  |
| 11. $-4x^2 + 3x = -5$  | 12. $x^2 + 121 = -22x$ |
| 13. $-z^2 = -12z + 6$  | 14. $-7w + 6 = -4w^2$  |

In Exercises 15–22, find the discriminant of the quadratic equation and describe the number and type of solutions of the equation. ▶ *Example 4*

- |                          |                          |
|--------------------------|--------------------------|
| 15. $x^2 + 12x + 36 = 0$ | 16. $x^2 - x + 6 = 0$    |
| 17. $4n^2 - 4n - 24 = 0$ | 18. $-x^2 + 2x + 12 = 0$ |
| 19. $4x^2 = 5x - 10$     | 20. $-18p = p^2 + 81$    |
| 21. $24x = -48 - 3x^2$   | 22. $-2x^2 - 6 = x$      |

**MP STRUCTURE** In Exercises 23–26, use the discriminant to match each quadratic equation with the graph of its related function. Explain.

- |                         |                            |
|-------------------------|----------------------------|
| 23. $x^2 - 6x + 25 = 0$ | 24. $2x^2 - 20x + 50 = 0$  |
| 25. $3x^2 + 6x - 9 = 0$ | 26. $-5x^2 + 10x + 35 = 0$ |



**ERROR ANALYSIS** In Exercises 27 and 28, describe and correct the error in solving the equation.

27.

**X**

$$x^2 + 10x + 74 = 0$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(74)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{-196}}{2}$$

$$= \frac{-10 \pm 14}{2}$$

$$= 2 \text{ or } -12$$

28.

**X**

$$x^2 + 6x + 8 = 2$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{4}}{2}$$

$$= \frac{-6 \pm 2}{2}$$

$$= -2 \text{ or } -4$$

**OPEN-ENDED** In Exercises 29–34, find a possible pair of integer values for  $a$  and  $c$  so that the quadratic equation has the given number and type of solution(s). Then write the equation. ▶ *Example 5*

29.  $ax^2 + 4x + c = 0$ ; two imaginary solutions
30.  $ax^2 + 6x + c = 0$ ; two real solutions
31.  $ax^2 - 8x + c = 0$ ; two real solutions
32.  $ax^2 - 6x + c = 0$ ; one real solution
33.  $ax^2 + 10x = c$ ; one real solution
34.  $-4x + c = -ax^2$ ; two imaginary solutions
35. **COLLEGE PREP** Determine the number and type of solutions of the equation  $x^2 + 7x = -11$ .
- (A) two real solutions
- (B) one real solution
- (C) two imaginary solutions
- (D) one imaginary solution



36. **COLLEGE PREP** What are the solutions of the equation  $2x^2 + 50 = 16x$ ?

- (A)  $x = 4 \pm 3i$       (B)  $x = -4 \pm 3i$   
 (C)  $x = 1, x = 7$       (D)  $x = -7, x = -1$

**MODELING REAL LIFE** In Exercises 37 and 38, write a function that represents the situation.

37. In a volleyball game, a player on one team spikes the ball over the net when the ball is 10 feet above the court. The spike drives the ball downward with an initial vertical velocity of 55 feet per second.
38. An archer is shooting at targets. The height of the arrow is 5 feet above the ground. Due to safety rules, the archer must aim the arrow parallel to the ground.

39. **MODELING REAL LIFE** A lacrosse player throws a ball in the air from an initial height of 7 feet. The ball has an initial vertical velocity of 35 feet per second. Does the ball reach a height of 26 feet? 30 feet? Explain your reasoning. ▶ *Example 6*



40. **MODELING REAL LIFE** A rocketry club is launching model rockets. The launching pad is 30 feet above the ground. Your model rocket has an initial vertical velocity of 105 feet per second. Your friend's model rocket has an initial vertical velocity of 100 feet per second.
- Does your rocket reach a height of 200 feet? Does your friend's rocket? Explain your reasoning.
  - Which rocket is in the air longer? How much longer?

**MP STRUCTURE** In Exercises 41–46, use the Quadratic Formula to write a quadratic equation that has the given solutions.

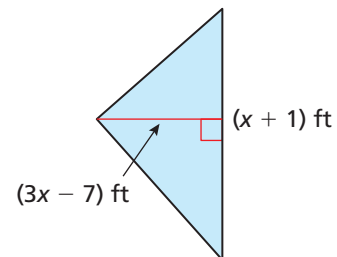
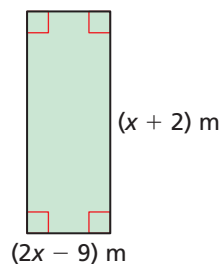
41.  $x = \frac{-8 \pm \sqrt{-176}}{-10}$       42.  $x = \frac{15 \pm \sqrt{-215}}{22}$   
 43.  $x = \frac{-4 \pm \sqrt{-124}}{-14}$       44.  $x = \frac{-9 \pm \sqrt{137}}{4}$   
 45.  $x = \frac{-4 \pm 2}{6}$       46.  $x = \frac{2 \pm 4}{-2}$

**COMPARING METHODS** In Exercises 47–54, solve the quadratic equation using the Quadratic Formula. Then solve the equation using another method. Which method do you prefer? Explain.

47.  $3x^2 - 21 = 3$       48.  $5x^2 + 38 = 3$   
 49.  $2x^2 - 54 = 12x$       50.  $x^2 = 3x + 15$   
 51.  $x^2 + 8x - 13 = 0$       52.  $x^2 - 7x + 12 = 0$   
 53.  $8x^2 + 4x + 5 = 0$       54.  $5x^2 - 50x = -135$

**CONNECTING CONCEPTS** In Exercises 55 and 56, find the value of  $x$ .

55. Area = 24 m<sup>2</sup>      56. Area = 8 ft<sup>2</sup>



**OPEN-ENDED** In Exercises 57–60, find a possible pair of real number values for  $a$  and  $c$  so that the quadratic equation has one real solution. Then write the equation.

57.  $ax^2 - 13x + c = 0$       58.  $ax^2 - \sqrt{2}x + c = 0$   
 59.  $\frac{1}{2}x + c = -ax^2$       60.  $ax^2 + \pi x = -c$

61. **MP PROBLEM SOLVING** A gannet is a bird that feeds on fish by diving into the water. A gannet spots a fish on the surface of the water and flies down 100 feet to catch it. The bird plunges toward the water with an initial vertical velocity of  $-88$  feet per second.

- How much time does the fish have to swim away?
- Another gannet spots the fish at the same time, but it is only 84 feet above the water. It has an initial vertical velocity of  $-70$  feet per second. Which bird will reach the water first? Justify your answer.





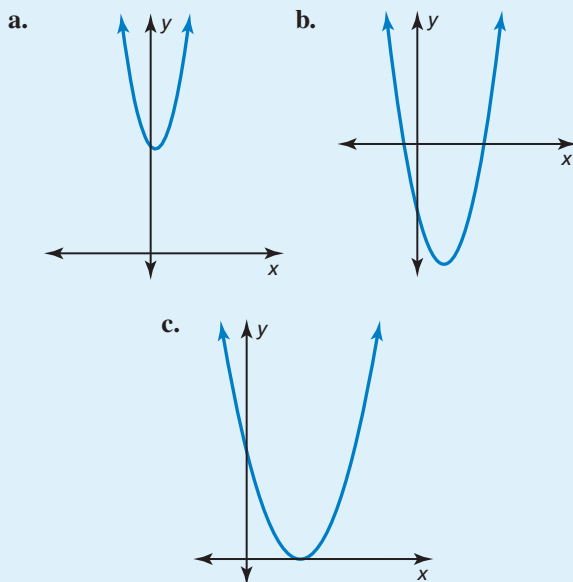


GO DIGITAL

62. **MP PROBLEM SOLVING** The amount  $A$  of nuclear energy (in billions of kilowatt hours) generated for consumer use in the United States can be modeled by the function  $A = -0.6t^2 + 37.2t + 243$ , where  $t$  represents the number of years after 1980.
- In what year did the amount of nuclear energy generated reach 800 billion kilowatt hours?
  - Find the average rate of change from 2000 to 2015 and interpret the meaning in the context of the situation.
  - Do you think this model will be accurate in 2030? Explain your reasoning.
63. **OPEN-ENDED** Describe a real-life situation that can be modeled by  $h = -16t^2 + v_0t + s_0$ . Write the height model for your situation and determine how long your object is in the air.

64. **HOW DO YOU SEE IT?**

The graphs of three quadratic functions are shown. For each graph, determine whether the discriminant of the associated equation is *positive*, *negative*, or *zero*. Then state the number and type of solution(s) of the associated equation. Explain your reasoning.



65. **MAKING AN ARGUMENT** Which method would you use to solve  $4x^2 + 14x + 11 = 0$ , completing the square or using the Quadratic Formula? Explain your reasoning.
66. **MP NUMBER SENSE** The quadratic equation  $ax^2 + 5x + c = 0$  has one real solution. Is it possible for  $a$  and  $c$  to be integers? rational numbers? Explain your reasoning. Then describe the possible values of  $a$  and  $c$ .

67. **ABSTRACT REASONING** For a quadratic equation  $ax^2 + bx + c = 0$  with two real solutions, show that the mean of the solutions is  $-\frac{b}{2a}$ . How is this fact related to the symmetry of the graph of  $y = ax^2 + bx + c$ ?

68. **MODELING REAL LIFE** The Stratosphere Tower in Las Vegas is 921 feet tall and has a “needle” at its top that extends even higher into the air. A thrill ride called Big Shot catapults riders 160 feet up the needle and then lets them fall back to the launching pad.

- The height  $h$  (in feet) of a rider on the Big Shot can be modeled by  $h = -16t^2 + v_0t + 921$ , where  $t$  is the elapsed time (in seconds) after launch and  $v_0$  is the initial vertical velocity (in feet per second). Find  $v_0$  using the fact that the maximum value of  $h$  is  $921 + 160 = 1081$  feet.

- A brochure for the Big Shot states that the ride up the needle takes 2 seconds. Compare this time to the time given by the model  $h = -16t^2 + v_0t + 921$ , where  $v_0$  is the value you found in part (a). Discuss the accuracy of the model.

69. **ABSTRACT REASONING** For what value(s) of  $n$  does the equation  $x^2 - 3x + n = -2$  have one real solution? Explain your reasoning.





GO DIGITAL

70. **CRITICAL THINKING** Solve each absolute value equation.

a.  $|x^2 - 3x - 14| = 4$     b.  $x^2 = |x| + 6$

71. **CRITICAL THINKING** When a quadratic equation with real coefficients has imaginary solutions, why are the solutions complex conjugates? As part of your explanation, show that there is no such equation with solutions of  $3i$  and  $-2i$ .

72. **THOUGHT PROVOKING**

Show how you can use the Quadratic Formula to write the solutions of any quadratic equation of the form  $a(x - h)^2 + k = 0$  in terms of  $a$ ,  $h$ , and  $k$ .

73. **DIG DEEPER** Can you use the Quadratic Formula to find the solutions of  $4x^4 + 35x^2 - 9 = 0$ ? If so, find the solutions. If not, explain why not.



## REVIEW & REFRESH

In Exercises 74–77, solve the system using any method. Explain your choice of method.

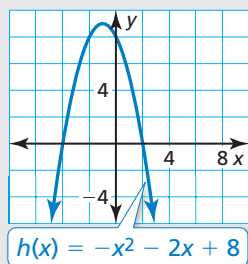
74.  $-x + 2y = 6$   
 $x + 4y = 24$

75.  $y = 2x - 1$   
 $y = x + 1$

76.  $3x + y = 4$   
 $6x + 2y = -4$

77.  $y = -x + 2$   
 $-5x + 5y = 10$

78. Use the graph to solve  $x^2 = -2x + 8$ .



In Exercises 79 and 80, solve the equation by completing the square.

79.  $x^2 - 6x + 10 = 0$     80.  $9x^2 + 36x + 72 = 0$

81. Write  $y = x^2 - 10x + 4$  in vertex form. Then identify the vertex.

82. Find the values of  $x$  and  $y$  that satisfy the equation  $7x - 6i = 14 + yi$ .

In Exercises 83 and 84, multiply the complex number by its complex conjugate.

83.  $6 - 8i$

84.  $-10 + 2i$

In Exercises 85 and 86, solve the equation by graphing.

85.  $-2x + 5 = \frac{1}{3}x - 2$

86.  $|4 - x| = |3x - 8|$

In Exercises 87 and 88, solve the equation using the Quadratic Formula.

87.  $-3 = 4x^2 + 9x$

88.  $x^2 = 1 - x$

89. Write an inequality that represents the graph.

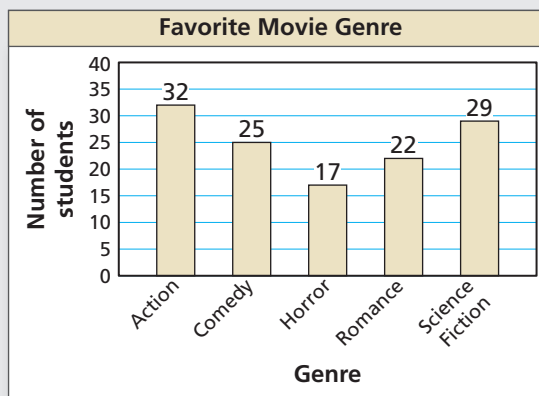


In Exercises 90 and 91, write a function  $g$  whose graph represents the indicated transformations of the graph of  $f$ .

90.  $f(x) = x$ ; translation 9 units down followed by a reflection in the  $y$ -axis

91.  $f(x) = |x|$ ; translation 1 unit down and 2 units left followed by a vertical shrink by a factor of  $\frac{1}{2}$

92. **MODELING REAL LIFE** The bar graph shows the results of a survey that asks a group of students their favorite movie genre. What percent of the students surveyed chose comedy?



In Exercises 93–96, graph the function. Label the vertex and axis of symmetry.

93.  $y = -(x - 1)^2 + 4$     94.  $f(x) = (x + 1)(2x - 3)$

95.  $y = 0.5x^2 + 2x + 5$     96.  $g(x) = -3x^2 - 2$