$$P(x) = (x - 1.5)[x - (2 + i)][x - (2 - i)]$$
$$= (x - 1.5)(x^2 - 4x + 5)$$
$$= x^3 - 5.5x^2 + 11x - 7.5$$

P has the given zeros, but some of its coefficients are not integers. If all of the coefficients were doubled, then the new polynomial function $S = 2P$ would still have the given zeros and all of its coefficients would be integers. Thus,

$$S(x) = 2x^3 - 11x^2 + 22x - 15$$

is a polynomial function that satisfies all the given requirements.

Answers to Exercises 11–13 are on page AA12.

EXERCISE SET 3.4

Concept Check

In Exercises 1 to 4, determine the number of complex zeros for each polynomial function.

1. $P(x) = x^3 - 5x^2 + 6x - 3$ 3
2. $P(x) = 2x^5 - 2x^3 + 7x - 1$ 5
3. $P(x) = 5x^2 + 6x - 3x^3 + 2$ 3
4. $P(x) = 1 - \frac{1}{2}x^4$ 4

In Exercises 5 to 20, find all the zeros of the polynomial function and write the polynomial as a product of its leading coefficient and its linear factors. (*Hint:* First determine the rational zeros.)

5. $P(x) = x^4 + x^3 - 2x^2 + 4x - 24$
   $2, -3, 2i, -2i$; $P(x) = (x - 2)(x + 3)(x - 2i)(x + 2i)$
6. $P(x) = x^3 - 3x^2 + 7x - 5$
   $1, 1 - 2i, 1 + 2i$; $P(x) = (x - 1)(x - 1 + 2i)(x - 1 - 2i)$
7. $P(x) = 2x^4 - x^3 - 4x^2 + 10x - 4$
   $\frac{1}{2}, -2, 1 + i, 1 - i$; $P(x) = 2\left(x - \frac{1}{2}\right)(x + 2)(x - 1 - i)(x - 1 + i)$
8. $P(x) = x^3 - 13x^2 + 65x - 125$
   $5, 4 + 3i, 4 - 3i$; $P(x) = (x - 5)(x - 4 - 3i)(x - 4 + 3i)$
9. $P(x) = x^5 - 9x^4 + 34x^3 - 58x^2 + 45x - 13$
   1 (multiplicity 3), $3 + 2i, 3 - 2i$; $P(x) = (x - 1)^3(x - 3 - 2i)(x - 3 + 2i)$
10. $P(x) = x^4 - 4x^3 + 53x^2 - 196x + 196$
    2 (multiplicity 2), $7i, -7i$; $P(x) = (x - 2)^2(x - 7i)(x + 7i)$
11. $P(x) = 2x^4 - x^3 - 15x^2 + 23x + 15$
12. $P(x) = 3x^4 - 17x^3 - 39x^2 + 337x + 116$
13. $P(x) = 2x^4 - 14x^3 + 33x^2 - 46x + 40$
14. $P(x) = 3x^4 - 10x^3 + 15x^2 + 20x - 8$
    $-1, \frac{1}{3}, 2 + 2i, 2 - 2i$; $P(x) = 3(x + 1)\left(x - \frac{1}{3}\right)(x - 2 - 2i)(x - 2 + 2i)$
15. $P(x) = 2x^3 - 9x^2 + 18x - 20$
    $\frac{5}{2}, 1 + i\sqrt{3}, 1 - i\sqrt{3}$; $P(x) = 2\left(x - \frac{5}{2}\right)(x - 1 - i\sqrt{3})(x - 1 + i\sqrt{3})$
16. $P(x) = 3x^4 - 19x^3 + 59x^2 - 79x + 36$
    $1, \frac{4}{3}, 2 + i\sqrt{5}, 2 - i\sqrt{5}$; $P(x) = 3(x - 1)\left(x - \frac{4}{3}\right)(x - 2 - i\sqrt{5})(x - 2 + i\sqrt{5})$
17. $P(x) = 2x^4 - x^3 - 2x^2 + 13x - 6$
    $-2, \frac{1}{2}, 1 + i\sqrt{2}, 1 - i\sqrt{2}$; $P(x) = 2(x + 2)\left(x - \frac{1}{2}\right)(x - 1 - i\sqrt{2})(x - 1 + i\sqrt{2})$
18. $P(x) = 4x^4 - 4x^3 + 13x^2 - 12x + 3$
    $\frac{1}{2}$ (multiplicity 2), $i\sqrt{3}, -i\sqrt{3}$; $P(x) = 4\left(x - \frac{1}{2}\right)^2(x - i\sqrt{3})(x + i\sqrt{3})$
19. $P(x) = 3x^5 + 2x^4 + 10x^3 + 6x^2 - 25x - 20$
    -1 (multiplicity 2), $\frac{4}{3}, i\sqrt{5}, -i\sqrt{5}$; $P(x) = 3(x + 1)^2\left(x - \frac{4}{3}\right)(x - i\sqrt{5})(x + i\sqrt{5})$
20. $P(x) = 2x^6 - 11x^5 + 5x^4 + 60x^3 - 62x^2 - 64x + 40$
    $-2, -1, \frac{1}{2}, 2, 3 + i, 3 - i$; $P(x) = 2(x + 2)(x + 1)\left(x - \frac{1}{2}\right)(x - 2)(x - 3 - i)(x - 3 + i)$

In Exercises 21 to 32, use the given zero to find the remaining zeros of each polynomial function.

21. $P(x) = 2x^3 - 5x^2 + 6x - 2$; $1 + i$ $1 - i, \frac{1}{2}$
22. $P(x) = 3x^3 - 29x^2 + 92x + 34$; $5 + 3i$ $5 - 3i, -\frac{1}{3}$
23. $P(x) = x^3 + 3x^2 + x + 3$; $-i$ $i, -3$
24. $P(x) = x^4 - 6x^3 + 71x^2 - 146x + 530$; $2 + 7i$
    $2 - 7i, 1 + 3i, 1 - 3i$
25. $P(x) = x^4 - 4x^3 + 14x^2 - 4x + 13$; $2 - 3i$
    $2 + 3i, i, -i$
26. $P(x) = x^5 - 6x^4 + 22x^3 - 64x^2 + 117x - 90$; $3i$
    $-3i, 2, 2 + i, 2 - i$
27. $P(x) = x^4 - 4x^3 + 19x^2 - 30x + 50$; $1 + 3i$
    $1 - 3i, 1 + 2i, 1 - 2i$
28. $P(x) = x^5 - x^4 - 4x^3 - 4x^2 - 5x - 3$; i
    $-i, 3, -1$ (multiplicity 2)
29. $P(x) = x^5 - 3x^4 + 7x^3 - 13x^2 + 12x - 4$; $-2i$
    $2i, 1$ (multiplicity 3)
30. $P(x) = x^4 - 8x^3 + 18x^2 - 8x + 17$; i $-i, 4 + i, 4 - i$
31. $P(x) = x^4 - 17x^3 + 112x^2 - 333x + 377$; $5 + 2i$
32. $P(x) = 2x^5 - 8x^4 + 61x^3 - 99x^2 + 12x + 182$; $1 - 5i$

31. $5 - 2i, \frac{7}{2} + \frac{\sqrt{3}}{2}i, \frac{7}{2} - \frac{\sqrt{3}}{2}i$ 32. $1 + 5i, -1, \frac{3}{2} + \frac{\sqrt{5}}{2}i, \frac{3}{2} - \frac{\sqrt{5}}{2}i$

Indicates Try It Exercises