3.3 WS

KEY

Use the Rational Zero Theorem to list possible rational zeros for each polynomial function.

1. $P(x)=x^3+3x^2-6x-8$

$\pm1, \pm2, \pm4, \pm8$

2. $P(x)=2x^3+x^2-25x+12$

$p: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$q: \pm1, \pm2$

$\frac{p}{q}: \pm1, \pm\frac{1}{2}, \pm2, \pm3, \pm\frac{3}{2}, \pm4, \pm6, \pm12$

Find the smallest positive integer that is the upper bound and the largest negative integer that is a lower bound of the real zeros of each polynomial.

3. $P(x)=x^3+3x^2-6x-6$

Upper Bound

1	1	3	-6	-6
		1	4	-2
	1	4	-2	-8
(2)	1	3	-6	-6
		2	10	8
	1	5	4	14

Lower Bound

-1	1	3	-6	-6
		-1	-2	8
	1	2	-8	2
-2	1	3	-6	-6
		-2	-2	16
	1	1	-8	10
-3	1	3	-6	-6
		-3	0	18
	1	0	-6	12
-4	1	3	-6	-6
		-4	4	8
	1	-1	-2	2
(-5)	1	3	-6	-6
		-5	10	-20
	1	-2	4	-14

Upper Bound of 2, Lower bound of -5

4. $P(x)=2x^3+x^2-25x+10$

Upper Bound

1	2	1	-25	10
		2	3	-22
	2	3	-22	-12
2	2	1	-25	10
		4	10	-30
	2	5	-15	-20
3	2	1	-25	10
		6	21	-12
	2	7	-4	-2
(4)	2	1	-25	10
		8	36	44
	2	9	11	54

Lower Bound

-1	2	1	-25	10
		-2	1	24
	2	-1	-24	34
-2	2	1	-25	10
		-4	6	38
	2	-3	-19	48
-3	2	1	-25	10
		-6	15	30
	2	-5	-10	40
(-4)	2	1	-25	10
		-8	28	-12
	2	-7	3	-2

Upper bound of 4, Lower bound of -4

Use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of each polynomial function.

5. $P(x)=x^3+3x^2-6x-8$

$P(-x)=-x^3+3x^2+6x-8$ (1 sign change; 2 sign changes)

1 positive real zero

2 or 0 negative real zeros

6. $P(x)=2x^3+x^2-25x+12$

$P(-x)=-2x^3+x^2+25x+12$ (1 sign change; 2 sign changes)

2 or 0 positive real zeros

1 negative real zero

Find the zeros of each polynomial function. If a zero is a multiple zero, state it a multiplicity.

7. $P(x) = 2x^3 + x^2 - 25x + 12$

① 3 zeros

$P(x) = 2x^3 + x^2 - 25x + 12$ (1, 2)

$P(-x) = -2x^3 + x^2 + 25x + 12$ (1)

2 or 0 positive real zeros
1 negative real zero

② p: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
q: $\pm 1, \pm 2$

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4$
$\pm 6, \pm 12$

③

	2	1	-25	12
3		6	21	-12
	2	7	-4	0

$2x^2 + 7x - 4 = 0$

-8
$8 \quad -1$
$2 \quad 2$
4

$x = 3, -4, \frac{1}{2}$

8. $P(x) = 2x^4 - 19x^3 + 51x^2 - 31x + 5$

① 4 zeros

$P(x) = 2x^4 - 19x^3 + 51x^2 - 31x + 5$ (1, 2, 3, 4)

$P(-x) = 2x^4 + 19x^3 + 51x^2 + 31x + 5$

4, 2, or 0 positive real zeros
0 negative real zeros

② p: $\pm 1, \pm 5$
q: $\pm 1, \pm 2$

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}$

③

	2	-19	51	-31	5
5		10	-45	30	-5
	2	-9	6	-1	0

	2	-9	6	-1
.5 ($\frac{1}{2}$)		1	-4	1
	2	-8	2	0

$2x^2 - 8x + 2 = 0$

$x = \frac{8 \pm \sqrt{64 - 4(2)(2)}}{4} = \frac{8 \pm \sqrt{48}}{4}$

$x = \frac{8 \pm 4\sqrt{3}}{4} = 2 \pm \sqrt{3}$

$x = 5, \frac{1}{2}, 2 \pm \sqrt{3}$