3.3 WS 3

KEY

Use the Rational Zero Theorem to list possible rational zeros for each polynomial function.

1. $P(x) = 2x^3 + 9x^2 - 2x - 9$

$p: \pm 1, \pm 3, \pm 9$

$q: \pm 1, \pm 2$

$\boxed{\frac{p}{q}: \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}}$

2. $P(x) = 4x^4 - 12x^3 - 3x^2 + 12x - 7$

$p: \pm 1, \pm 7$

$q: \pm 1, \pm 2, \pm 4$

$\boxed{\frac{p}{q}: \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 7, \pm \frac{7}{2}, \pm \frac{7}{4}}$

Find the smallest positive integer that is the upper bound and the largest negative integer that is a lower bound of the real zeros of each polynomial.

3. $P(x) = 3x^4 - 9x^3 + 2x^2 - 5x - 8$

Upper bound of 4,
Lower bound of -1

4. $P(x) = -5x^4 + 14x^3 + 2x^2 - 12x + 325$

Upper bound of 4,
Lower bound of -3

Use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of each polynomial function.

5. $P(x) = \underbrace{x^3 - 19x}_{1} - 30$

$P(-x) = \underbrace{-x^3 + 19x}_{1}\underbrace{ - 30}_{2}$

1 positive real zero
2 or 0 negative real zeros

6. $P(x) = \underbrace{x^5 - 2x^4}_{1} - 35x^3 \underbrace{+ 158x^2}_{2}\underbrace{- 242x}_{3}\underbrace{+ 168}_{4}$

$P(-x) = -x^5 \underbrace{- 2x^4 + 35x^3}_{1} + 158x^2 + 242x + 168$

4, 2, or 0 positive real zeros
1 negative real zero

Find the zeros of each polynomial function. If a zero is a multiple zero, state it a multiplicity.

7. $P(x) = 3x^3 - x^2 - 6x + 2$

$P(-x) = -3x^3 - x^2 + 6x + 2$

① 3 zeros

$\boxed{\text{2 or 0 positive real zeros; 1 negative real zero}}$

② $p: \pm 1, \pm 2 \qquad q: \pm 1, \pm 3$

$\boxed{\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}}$

③
$$\begin{array}{c|cccc} \frac{1}{3} & 3 & -1 & -6 & 2 \\ & & 1 & 0 & -2 \\ \hline & 3 & 0 & -6 & 0 \end{array}$$

$3x^2 - 6 = 0$

$3x^2 = 6$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$

$\boxed{x = \frac{1}{3}, \pm\sqrt{2}}$

8. $P(x) = 4x^4 - 35x^3 + 71x^2 - 4x - 6$

$P(-x) = 4x^4 + 35x^3 + 71x^2 + 4x - 6$

① 4 zeros

$\boxed{\text{3 or 1 positive real zeros; 1 negative real zero}}$

② $p: \pm 1, \pm 2, \pm 3, \pm 6 \qquad q: \pm 1, \pm 2, \pm 4$

$\boxed{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 6}$

③
$$\begin{array}{c|ccccc} 3 & 4 & -35 & 71 & -4 & -6 \\ & & 12 & -69 & 6 & 6 \\ \hline & 4 & -23 & 2 & 2 & 0 \end{array}$$

$$\begin{array}{c|cccc} -\frac{1}{4} & 4 & -23 & 2 & 2 \\ & & -1 & 6 & -2 \\ \hline & 4 & -24 & 8 & 0 \end{array}$$

$4x^2 - 24x + 8 = 0$

$4(x^2 - 6x + 2) = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(1)(2)}}{2}$

$x = \dfrac{6 \pm \sqrt{28}}{2} = \dfrac{6 \pm 2\sqrt{7}}{2}$

$x = 3 \pm \sqrt{7}$

$\boxed{x = 3, -\frac{1}{4}, 3 \pm \sqrt{7}}$

9. $P(x) = x^3 - 2x + 1$ $\qquad P(-x) = -x^3 + 2x + 1$

① 3 zeros

$\boxed{\text{2 or 0 positive real zeros; 1 negative real zero}}$

② $p: \pm 1 \qquad q: \pm 1 \qquad \frac{p}{q}: \pm 1$

③
$$\begin{array}{c|cccc} 1 & 1 & 0 & -2 & 1 \\ & & 1 & 1 & -1 \\ \hline & 1 & 1 & -1 & 0 \end{array}$$

$x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$

$\boxed{x = 1, \frac{-1 \pm \sqrt{5}}{2}}$

10. $P(x) = x^3 - 8x^2 + 8x + 24$

$P(-x) = -x^3 + 8x^2 - 8x + 24$

① 3 zeros

$\boxed{\text{2 or 0 positive real zeros; 1 negativer real zero}}$

② $p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$q: \pm 1$

$\boxed{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}$

③
$$\begin{array}{c|cccc} 6 & 1 & -8 & 8 & 24 \\ & & 6 & -12 & -24 \\ \hline & 1 & -2 & -4 & 0 \end{array}$$

$x^2 - 2x - 4 = 0$

$x = \dfrac{2 \pm \sqrt{4 - 4(1)(-4}}{2}$

$x = \dfrac{2 \pm \sqrt{20}}{2} = \dfrac{2 \pm 2\sqrt{5}}{2}$

$\boxed{x = 6, 1 \pm \sqrt{5}}$