3.3 WS 2

KEY

Use the Rational Zero Theorem to list possible rational zeros for each polynomial function.

1. $P(x)=x^3-19x-30$

$\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

2. $P(x)=6x^4+23x^3+19x^2-8x-4$

P: $\pm1, \pm2, \pm4$

q: $\pm1, \pm2, \pm3, \pm6$

$\pm1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm2, \pm\frac{2}{3}, \pm4, \pm\frac{4}{3}$

Find the smallest positive integer that is the upper bound and the largest negative integer that is a lower bound of the real zeros of each polynomial.

3. $P(x)=-3x^3-2x^2+4x-31$

Upper Bound

1	-3	-2	4	-31
		-3	-5	-1
	-3	-5	-1	-32

Lower Bound

-1	-3	-2	4	-31
		3	-1	
	-3	1	3	

-2	-3	-2	4	-31
		6	-8	8
	-3	4	-4	-23

-3	-3	-2	4	-31
		9	-21	51
	-3	7	-17	20

Upper Bound of 1
Lower Bound of -3

4. $P(x)=2x^3+4x^2-8x+7$

Upper Bound

1	2	4	-8	7
		2	6	
	2	6	-2	

2	2	4	-8	7
		4	16	16
	2	8	8	23

Lower Bound

-1	2	4	-8	7
		-2		
	2	2		

-2	2	4	-8	7
		-4	0	
	2	0	-8	

-3	2	4	-8	7
		-6	6	
	2	-2	-2	

-4	2	4	-8	7
		-8	16	-28
	2	-4	8	-21

Upper Bound of 2
Lower Bound of -4

Use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of each polynomial function.

5. $P(x)=3x^3+11x^2-6x-8$

$P(x) = -3x^3 + 11x^2 + 6x - 8$ (1 sign change; 2 sign changes)

1 positive real zero
2 or 0 negative real zeros

6. $P(x)=2x^5+23x^4+90x^3+152x^2+116x+33$

$P(-x) = -2x^5 + 23x^4 - 90x^3 + 152x^2 - 116x + 23$ (1; 2; 3; 4; 5 sign changes)

0 positive real zeros
5, 3, or 1 negative real zeros

Find the zeros of each polynomial function. If a zero is a multiple zero, state it a multiplicity.

7. $P(x)=3x^3+11x^2-6x-8$

① $P(-x)=-3x^3+11x^2+6x-8$ (1, 2)

3 zeros

1 positive real zero

2 or 0 negative real zeros

② p: $\pm1, \pm2, \pm4, \pm8$

q: $\pm1, \pm3$

$\frac{p}{q}$: $\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}, \pm4, \pm\frac{4}{3}, \pm8, \pm\frac{8}{3}$

③ (-4)

	3	11	-6	-8
		-12	4	8
	3	-1	-2	0

$3x^2-x-2=0$ $\quad -6$

$-3, 2$

$\frac{3}{3}, \frac{2}{3}$

-1

$X=-4, 1, -\frac{2}{3}$

8. $P(x)=2x^4-9x^3-2x^2+27x-12$ (1, 2, 3)

$P(-x)=2x^4+9x^3-2x^2-27x-12$ (1)

① 4 zeros

3 or 1 positive real zeros

1 negative real zeros

② p: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

q: $\pm1, \pm2$

$\frac{p}{q}$: $\pm1, \pm\frac{1}{2}, \pm2, \pm3, \pm\frac{3}{2}, \pm4, \pm6, \pm12$

③ ④

	2	-9	-2	27	-12
		8	-4	-24	12
	2	-1	-6	3	0

.5 ($\frac{1}{2}$)

	2	-1	-6	3
		1	0	-3
	2	0	-6	0

$2x^2-6=0$

$2x^2=6$

$\sqrt{x^2}=\sqrt{3}$

$x=\pm\sqrt{3}$

$X=4, \frac{1}{2}, \pm\sqrt{3}$

9. $P(x)=x^3-7x^2-7x+69$ (1, 2)

$P(-x)=-x^3-7x^2+7x+69$ (1)

① 3 zeros

2 or 0 positive real zeros

1 negative real zero

② p: $\pm1, \pm3, \pm23, \pm69$

q: ±1

$\frac{p}{q}$: $\pm1, \pm3, \pm23, \pm69$

③ (-3)

	1	-7	-7	69
		-3	30	-69
	1	-10	23	0

$X=-3, 5\pm\sqrt{2}$

$x^2-10x+23=0$

$x=\frac{10\pm\sqrt{100-4(1)(23)}}{2}=\frac{10\pm\sqrt{8}}{2}=\frac{10\pm2\sqrt{2}}{2}=5\pm\sqrt{2}$

10. $P(x)=6x^4+23x^3+19x^2-8x-4$ (1)

$P(-x)=6x^4-23x^3+19x^2+8x-4$ (1, 2, 3)

① 4 zeros

1 positive real zero

3 or 1 negative real zero

② p: $\pm1, \pm2, \pm4$

q: $\pm1, \pm2, \pm3, \pm6$

$\frac{p}{q}$: $\pm1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm2, \pm\frac{2}{3}, \pm4, \pm\frac{4}{3}$

③ (-2)

	6	23	19	-8	-4
		-12	-22	6	4
	6	11	-3	-2	0

(-2)

	6	11	-3	-2
		-12	2	2
	6	-1	-1	0

$6x^2-x-1=0$ $\quad -6$

$-3, 2$

$\frac{3}{6}, \frac{2}{6}$

$-\frac{1}{2}, \frac{1}{3}$

$X=\frac{1}{2}, \frac{-1}{3}, -2$ (mult. 2)