3.2 Complex Numbers

Learning Target Understand the imaginary unit i and perform operations with complex numbers.

Success Criteria

- I can define the imaginary unit i and use it to rewrite the square root of a negative number.
- I can add, subtract, and multiply complex numbers.
- I can find complex solutions of quadratic equations and complex zeros of quadratic functions.

EXPLORE IT! Using Complex Numbers

Math Practice

Look for Structure
How can you recognize when a quadratic equation of the form $x^2 + c = 0$ will have solutions that are not real numbers?

Work with a partner.

a. A student solves the equations below as shown. Justify each solution step.

i.

$x^2 = 36$	*Original equation*
$x = \pm\sqrt{36}$	________
$x = \pm 6$	________

ii.

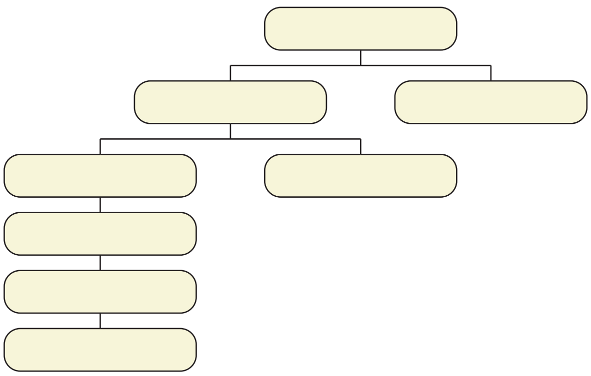
$x^2 = -9$	*Original equation*
$x = \pm\sqrt{-9}$	________
$x = \pm\sqrt{9}\,\sqrt{-1}$	________
$x = \pm 3\sqrt{-1}$	________

b. In your study of mathematics, you have probably worked with only real numbers, which can be represented graphically on the real number line. Describe the solutions of the equation $x^2 = c$ when $c > 0$, when $c = 0$, and when $c < 0$.

c. The solutions of the equation $x^2 = -9$ are *imaginary numbers*, and are typically written as $3i$ and $-3i$. Explain what i represents. What is the value of i^2?

d. In this lesson, the system of numbers is expanded to include imaginary numbers. The real numbers and imaginary numbers compose the set of *complex numbers*. Complete the diagram that shows the relationships among the number systems shown below.

Integers
Natural Numbers
Rational Numbers
Whole Numbers
Real Numbers
Complex Numbers
Irrational Numbers
Imaginary Numbers



e. Determine which subsets of numbers in part (d) contain each number.

i. $\sqrt{9}$ **ii.** $\sqrt{0}$ **iii.** $-\sqrt{4}$ **iv.** $\sqrt{\frac{4}{9}}$ **v.** $\sqrt{2}$ **vi.** $\sqrt{-1}$

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The Imaginary Unit *i*

Vocabulary (VOCAB)

imaginary unit *i*, *p. 100*
complex number, *p. 100*
imaginary number, *p. 100*
pure imaginary number, *p. 100*
complex conjugates, *p. 102*

Not all quadratic equations have real-number solutions. For example, $x^2 = -3$ has no real-number solutions because the square of any real number is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the **imaginary unit *i***, defined as $i = \sqrt{-1}$. Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of *any* negative number.

KEY IDEA

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$.	$\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = i^2 \cdot r = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -1 \cdot 3 = -3$

EXAMPLE 1 Finding Square Roots of Negative Numbers

WATCH

Find the square root of each number.

a. $\sqrt{-25}$ **b.** $\sqrt{-72}$ **c.** $-5\sqrt{-9}$

SOLUTION

a. $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1} = 5i$

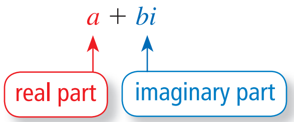
b. $\sqrt{-72} = \sqrt{72} \cdot \sqrt{-1} = \sqrt{36} \cdot \sqrt{2} \cdot i = 6\sqrt{2}\, i = 6i\sqrt{2}$

c. $-5\sqrt{-9} = -5\sqrt{9} \cdot \sqrt{-1} = -5 \cdot 3 \cdot i = -15i$

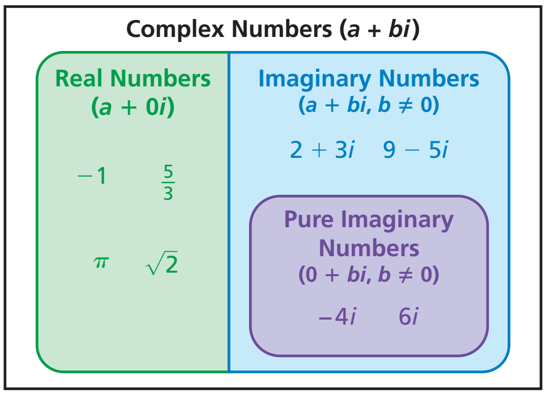
WORDS AND MATH

Numbers of the form $a + bi$ are called *complex* because they have more than one part.

A **complex number** written in *standard form* is a number $a + bi$, where a and b are real numbers. The number a is the *real part*, and the number bi is the *imaginary part*.



If $b \neq 0$, then $a + bi$ is an **imaginary number**. If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number**. The diagram shows how different types of complex numbers are related.



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Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$.

EXAMPLE 2 Equality of Two Complex Numbers

WATCH

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

SOLUTION

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x = 10$ Equate the real parts. $\quad -7i = yi$ Equate the imaginary parts.

$x = 5$ Solve for x. $\quad -7 = y$ Solve for y.

▶ So, $x = 5$ and $y = -7$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. **VOCABULARY** Define the imaginary unit i and explain how you use it.

2. **VOCABULARY** Identify the imaginary part and the real part of the complex number $5 - 2i$.

Find the square root of the number.

3. $\sqrt{-4}$
4. $\sqrt{-12}$
5. $-\sqrt{-36}$
6. $2\sqrt{-54}$

Find the values of *x* and *y* that satisfy the equation.

7. $x + 3i = 9 - yi$
8. $5x + 4i = 20 + 2yi$
9. $9 + 4yi = -2x + 3i$

Operations with Complex Numbers

KEY IDEA

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

EXAMPLE 3 Adding and Subtracting Complex Numbers

WATCH

Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i)$ **b.** $(7 - 6i) - (3 - 6i)$

SOLUTION

a. $(8 - i) + (5 + 4i) = (8 + 5) + (-1 + 4)i$ Definition of complex addition

$= 13 + 3i$ Write in standard form.

b. $(7 - 6i) - (3 - 6i) = (7 - 3) + (-6 + 6)i$ Definition of complex subtraction

$= 4 + 0i$ Simplify.

$= 4$ Write in standard form.

To multiply two complex numbers, use the Distributive Property, or the FOIL Method, just as you do when multiplying real numbers or algebraic expressions.

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EXAMPLE 4 Multiplying Complex Numbers

WATCH

Multiply. Write the answer in standard form.

a. $4i(-6 + i)$

b. $(9 - 2i)(-4 + 7i)$

SOLUTION

a. $4i(-6 + i) = -24i + 4i^2$ — Distributive Property

$= -24i + 4(-1)$ — Use $i^2 = -1$.

$= -4 - 24i$ — Write in standard form.

b. $(9 - 2i)(-4 + 7i) = -36 + 63i + 8i - 14i^2$ — Multiply using FOIL.

$= -36 + 71i - 14(-1)$ — Simplify and use $i^2 = -1$.

$= -36 + 71i + 14$ — Simplify.

$= -22 + 71i$ — Write in standard form.

STUDY TIP

When simplifying an expression that involves complex numbers, be sure to simplify i^2 as -1.

Pairs of complex numbers of the forms $a + bi$ and $a - bi$, where $b \neq 0$, are called **complex conjugates**. Consider the product of complex conjugates below.

$(a + bi)(a - bi) = a^2 - (ab)i + (ab)i - b^2i^2$ — Multiply using FOIL.

$= a^2 - b^2(-1)$ — Simplify and use $i^2 = -1$.

$= a^2 + b^2$ — Simplify.

Because a and b are real numbers, $a^2 + b^2$ is a real number. So, the product of complex conjugates is a real number.

EXAMPLE 5 Multiplying Complex Conjugates

WATCH

Multiply $5 + 2i$ by its complex conjugate.

SOLUTION

The complex conjugate of $5 + 2i$ is $5 - 2i$.

Method 1 Use the FOIL Method.

$(5 + 2i)(5 - 2i) = 25 - 10i + 10i - 4i^2$ — Multiply using FOIL.

$= 25 - 4(-1)$ — Simplify and use $i^2 = -1$.

$= 29$ — Simplify.

Method 2 Use the pattern shown above, $(a + bi)(a - bi) = a^2 + b^2$ where $a = 5$ and $b = 2$.

$(5 + 2i)(5 - 2i) = 5^2 + 2^2$ — $a = 5, b = 2$

$= 25 + 4$ — Evaluate exponents.

$= 29$ — Add.

EXAMPLE 6 Modeling Real Life

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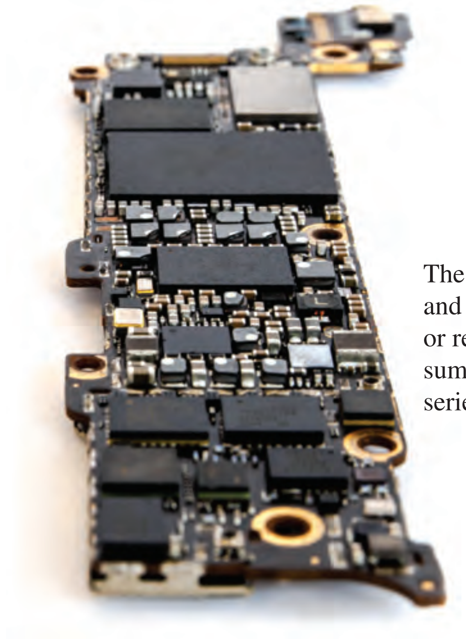
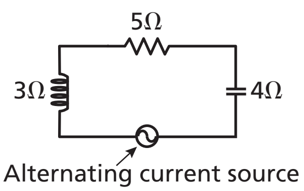
Electrical circuit components, such as resistors, inductors, and capacitors, all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. Each of these quantities is measured in ohms. The symbol used for ohms is Ω, the uppercase Greek letter omega.



Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance (in ohms)	R	L	C
Impedance (in ohms)	R	Li	$-Ci$

The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled. The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the series circuit.



SOLUTION

The resistor has a resistance of 5 ohms, so its impedance is 5 ohms. The inductor has a reactance of 3 ohms, so its impedance is $3i$ ohms. The capacitor has a reactance of 4 ohms, so its impedance is $-4i$ ohms.

$$\text{Impedance of circuit} = 5 + 3i + (-4i)$$
$$= 5 - i$$

▶ The impedance of the circuit is $(5 - i)$ ohms.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Add or subtract. Write the answer in standard form.

10. $(9 - i) + (-6 + 7i)$ **11.** $(3 + 7i) - (8 - 2i)$ **12.** $-4 - (1 + i) - (5 + 9i)$

13. $5 + (-9 + 3i) + 6i$ **14.** $(2 - i) + (1 + i) - 7i$ **15.** $8i - (6 - 3i) + (4 - 4i)$

16. OPEN-ENDED Write two complex numbers with a difference of 9.

Multiply. Write the answer in standard form.

17. $(-3i)(10i)$ **18.** $i(8 - i)$ **19.** $(3 + i)(5 - i)$

Multiply the complex number by its complex conjugate.

20. $1 + i$ **21.** $4 - 7i$ **22.** $-3 - 2i$

23. WHAT IF? In Example 6, what is the impedance of the circuit when the capacitor is replaced with one having a reactance of 7 ohms?

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Complex Solutions and Zeros

EXAMPLE 7 Solving Quadratic Equations

Solve (a) $x^2 + 4 = 0$ and (b) $2x^2 - 11 = -47$.

Math Practice

Look for Structure
How can you use the solutions in Example 7(a) to factor $x^2 + 4$?

SOLUTION

a.

$x^2 + 4 = 0$	Write original equation.
$x^2 = -4$	Subtract 4 from each side.
$x = \pm\sqrt{-4}$	Take square root of each side.
$x = \pm 2i$	Write in terms of *i*.

▶ The solutions are $2i$ and $-2i$.

b.

$2x^2 - 11 = -47$	Write original equation.
$2x^2 = -36$	Add 11 to each side.
$x^2 = -18$	Divide each side by 2.
$x = \pm\sqrt{-18}$	Take square root of each side.
$x = \pm i\sqrt{18}$	Write in terms of *i*.
$x = \pm 3i\sqrt{2}$	Simplify radical.

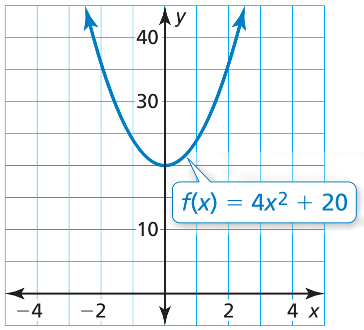
▶ The solutions are $3i\sqrt{2}$ and $-3i\sqrt{2}$.

EXAMPLE 8 Finding Zeros of a Quadratic Function

Math Practice

Use a Graph
A quadratic function without real zeros must have imaginary zeros. How can you use a graph to determine whether a quadratic function has imaginary zeros?

Find the zeros of the function.



SOLUTION

$4x^2 + 20 = 0$	Set $f(x)$ equal to 0.
$4x^2 = -20$	Subtract 20 from each side.
$x^2 = -5$	Divide each side by 4.
$x = \pm\sqrt{-5}$	Take square root of each side.
$x = \pm i\sqrt{5}$	Write in terms of *i*.

▶ So, the zeros of f are $i\sqrt{5}$ and $-i\sqrt{5}$.

Check

$f(i\sqrt{5}) = 4(i\sqrt{5})^2 + 20 = 4 \cdot 5i^2 + 20 = 4(-5) + 20 = 0$ ✓

$f(-i\sqrt{5}) = 4(-i\sqrt{5})^2 + 20 = 4 \cdot 5i^2 + 20 = 4(-5) + 20 = 0$ ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation.

24. $x^2 = -13$ **25.** $x^2 - 8 = -36$ **26.** $3x^2 - 7 = -31$ **27.** $5x^2 + 33 = 3$

Find the zeros of the function.

28. $f(x) = x^2 + 7$ **29.** $f(x) = -x^2 - 4$ **30.** $f(x) = 9x^2 + 1$

3.2 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–8, find the square root of the number. *Example 1*

1. $\sqrt{-36}$
2. $\sqrt{-64}$
3. $\sqrt{-18}$
4. $\sqrt{-24}$
5. $2\sqrt{-16}$
6. $-3\sqrt{-49}$
7. $-4\sqrt{-32}$
8. $6\sqrt{-63}$

In Exercises 9–16, find the values of *x* and *y* that satisfy the equation. *Example 2*

9. $4x + 2i = 8 + yi$
10. $3x + 6i = 27 + yi$
11. $-10x + 12i = 20 + 3yi$
12. $9x - 18i = -36 + 6yi$
13. $2x - yi = 14 + 12i$
14. $-12x + yi = 60 - 13i$
15. $54 - \frac{1}{7}yi = 9x - 4i$
16. $15 - 3yi = \frac{1}{2}x + 2i$

In Exercises 17–26, add or subtract. Write the answer in standard form. *Example 3*

17. $(6 - i) + (7 + 3i)$
18. $(9 + 5i) + (11 + 2i)$
19. $(12 + 4i) - (3 - 7i)$
20. $(2 - 15i) - (4 + 5i)$
21. $(12 - 3i) + (7 + 3i)$
22. $(16 - 9i) - (2 - 9i)$
23. $7 - (3 + 4i) + 6i$
24. $16 - (2 - 3i) - i$
25. $-10 + (6 - 5i) - 9i$
26. $-3 + (8 + 2i) + 7i$

27. **MP STRUCTURE** Write each expression as a complex number in standard form.
    a. $\sqrt{-9} + \sqrt{-4} - \sqrt{16}$
    b. $\sqrt{-16} + \sqrt{8} + \sqrt{-36}$

28. **MP REASONING** The additive inverse of a complex number z is a complex number z_a such that $z + z_a = 0$. Find the additive inverse of each complex number.
    a. $z = 1 + i$
    b. $z = 3 - i$
    c. $z = -2 + 8i$

In Exercises 29–36, multiply. Write the answer in standard form. *Example 4*

29. $3i(-5 + i)$
30. $2i(7 - i)$
31. $(3 - 2i)(4 + i)$
32. $(7 + 5i)(8 - 6i)$
33. $(5 - 2i)(-2 - 3i)$
34. $(-1 + 8i)(9 + 3i)$
35. $(3 - 6i)^2$
36. $(8 + 3i)^2$

ERROR ANALYSIS In Exercises 37 and 38, describe and correct the error in performing the operation and writing the answer in standard form.

37. ✗
$$(3 + 2i)(5 - i) = 15 - 3i + 10i - 2i^2$$
$$= 15 + 7i - 2i^2$$
$$= -2i^2 + 7i + 15$$

38. ✗
$$(4 + 6i)^2 = (4)^2 + (6i)^2$$
$$= 16 + 36i^2$$
$$= 16 + (36)(-1)$$
$$= -20$$

In Exercises 39–46, multiply the complex number by its complex conjugate. *Example 5*

39. $1 - i$
40. $8 + i$
41. $4 + 2i$
42. $5 - 6i$
43. $-2 + 2i$
44. $-1 - 9i$
45. $-3 - 5i$
46. $-7 + 4i$

MP REASONING In Exercises 47 and 48, use the given numbers to complete the equation.

47. $(\_\_\_\_ - \_\_\_\_i) - (\_\_\_\_ - \_\_\_\_i) = 2 - 4i$

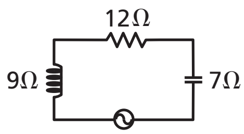
    7 | 4 | 3 | 6

48. $\_\_\_\_i(\_\_\_\_ + \_\_\_\_i) = -18 - 10i$

    −5 | 9 | 2

MODELING REAL LIFE **In Exercises 49–52, find the impedance of the series circuit.** *Example 6*

49.



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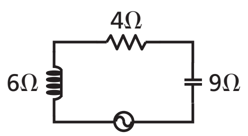

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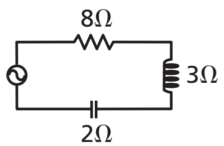

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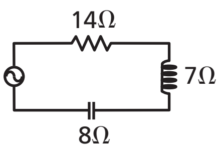

JUSTIFYING STEPS **In Exercises 53 and 54, justify each step in performing the operation.**

53. $11 - (4 + 3i) + 5i$

$= [(11 - 4) - 3i] + 5i$

$= (7 - 3i) + 5i$

$= 7 + (-3 + 5)i$

$= 7 + 2i$

54. $(3 + 2i)(7 - 4i)$

$= 21 - 12i + 14i - 8i^2$

$= 21 + 2i - 8(-1)$

$= 21 + 2i + 8$

$= 29 + 2i$

In Exercises 55–60, solve the equation. *Example 7*

55. $x^2 + 9 = 0$

56. $x^2 + 49 = 0$

57. $x^2 - 4 = -11$

58. $x^2 - 9 = -15$

59. $2x^2 + 6 = -34$

60. $x^2 + 7 = -47$

In Exercises 61–68, find the zeros of the function. *Example 8*

61. $f(x) = 3x^2 + 6$

62. $g(x) = 7x^2 + 21$

63. $h(x) = 2x^2 + 72$

64. $k(x) = -5x^2 - 125$

65. $m(x) = -x^2 - 27$

66. $p(x) = x^2 + 98$

67. $r(x) = -\frac{1}{2}x^2 - 24$

68. $f(x) = -\frac{1}{5}x^2 - 10$

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69. **MP STRUCTURE** Expand $(a - bi)^2$ and write the result in standard form. Use your result to check your answer to Exercise 35.

70. **MP STRUCTURE** Expand $(a + bi)^2$ and write the result in standard form. Use your result to check your answer to Exercise 36.

71. **MP NUMBER SENSE** Write the complex conjugate of $1 - \sqrt{-12}$. Then find the product of the complex conjugates.

72. **MP NUMBER SENSE** Simplify each expression. Then classify your results in the table below.

a. $(-4 + 7i) + (-4 - 7i)$

b. $(2 - 6i) - (-10 + 4i)$

c. $(25 + 15i) - (25 - 6i)$

d. $(5 + i)(8 - i)$

e. $(17 - 3i) + (-17 - 6i)$

f. $(-1 + 2i)(11 - i)$

g. $(7 + 5i) + (7 - 5i)$

h. $(-3 + 6i) - (-3 - 8i)$

Real numbers	Imaginary numbers	Pure imaginary numbers

73. **MP STRUCTURE** The coordinate system shown below is called the *complex plane*. In the complex plane, the point that corresponds to the complex number $a + bi$ is (a, b). Match each complex number with its corresponding point.

a. 2

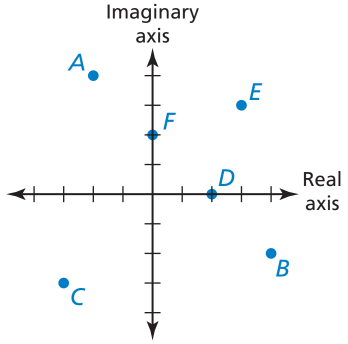
b. $2i$

c. $4 - 2i$

d. $3 + 3i$

e. $-2 + 4i$

f. $-3 - 3i$



74. COMPARING METHODS Describe the methods shown for writing the complex expression in standard form. Which method do you prefer? Explain.

Method 1

$4i(2-3i)+4i(1-2i)=8i-12i^2+4i-8i^2$

$=8i-12(-1)+4i-8(-1)$

$=20+12i$

Method 2

$4i(2-3i)+4i(1-2i)=4i[(2-3i)+(1-2i)]$

$=4i[3-5i]$

$=12i-20i^2$

$=12i-20(-1)$

$=20+12i$

In Exercises 75–80, write the expression as a complex number in standard form.

75. $(3+4i)-(7-5i)+2i(9+12i)$

76. $3i(2+5i)+(6-7i)-(9+i)$

77. $(3+5i)(2-7i^4)$

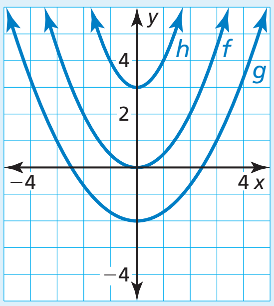
78. $2i^3(5-12i)$

79. $(2+4i^5)+(1-9i^6)-(3+i^7)$

80. $(8-2i^4)+(3-7i^8)-(4+i^9)$

81. MP PATTERNS Make a table that shows the powers of i from i^1 to i^8 in the first row and the simplified forms of these powers in the second row. Describe the pattern you observe in the table. Then use the pattern to evaluate i^{25}, i^{50}, i^{75}, and i^{100}.

82. HOW DO YOU SEE IT?
The graphs of three functions are shown. Which function(s) have real zeros? imaginary zeros? Explain your reasoning.



83. MP NUMBER SENSE Write each product as a complex number in standard form.

a. $(2-3i)^3$

b. $(3+i)^4$

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84. MP REASONING Is it possible for a quadratic equation to have one real solution and one imaginary solution? Explain your reasoning.

85. COLLEGE PREP Which expressions are equivalent to $1+i$? Select all that apply.

Ⓐ $(4-i)-(3+2i)$

Ⓑ $-i(i^2+i)$

Ⓒ $(2-i)(i+1)$

Ⓓ $i^{20}-i^{21}$

86. MP NUMBER SENSE Write a pair of complex numbers whose sum is −4 and whose product is 53.

87. JUSTIFYING STEPS Justify each step in the simplification of i^2.

Algebraic Step	Justification
$i^2=\left(\sqrt{-1}\right)^2$	
$=-1$	

88. MAKING AN ARGUMENT Your friend claims that the conclusion in Exercise 87 is incorrect because $i^2=i\cdot i=\sqrt{-1}\cdot\sqrt{-1}=\sqrt{-1(-1)}=\sqrt{1}=1$. Is your friend correct? Explain.

89. CRITICAL THINKING Rewrite each expression with a real denominator.

a. $\dfrac{1+3i}{2i}$ **b.** $\dfrac{4-2i}{1+i}$

90. MP LOGIC The sum of two complex numbers is $7+8i$. The difference of the numbers is $1-2i$. What is the product of the numbers?

91. CRITICAL THINKING Determine whether each statement is true or false. If it is true, give an example. If it is false, give a counterexample.

a. The sum of two imaginary numbers is always an imaginary number.

b. The product of two pure imaginary numbers is always a real number.

c. A pure imaginary number is an imaginary number.

d. A complex number is a real number.

92. **CRITICAL THINKING** The zeros of a quadratic function are $3 \pm 4i$.

a. What do you know about the vertex of the function? Explain.

b. Write and graph a quadratic function that has these zeros.

93. **DIG DEEPER** Write $\sqrt{i}$ as a complex number in standard form. (*Hint:* Use the equation $\sqrt{i} = a + bi$ to write a system of equations in terms of a and b.)

GO DIGITAL

94. **THOUGHT PROVOKING**
Create a circuit that has an impedance of $14 - 3i$.

REVIEW & REFRESH

WATCH

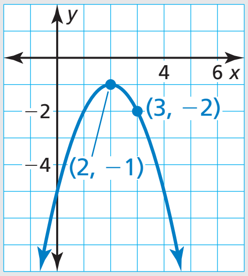
In Exercises 95 and 96, graph the function and its parent function. Then describe the transformation.

95. $f(x) = \frac{1}{4}x^2 + 1$ **96.** $f(x) = -\frac{1}{2}x - 4$

In Exercises 97 and 98, simplify the expression. Write your answer using only positive exponents.

97. $\left(\dfrac{4c^4d^{-5}}{8c^0d^4}\right)^2$ **98.** $\left(\dfrac{3m^{-5}}{m^{-6}n}\right)^3 \bullet \left(\dfrac{-4m^3n^{-1}}{2mn^{-7}}\right)^4$

99. Write a quadratic function in vertex form whose graph is shown.



100. **MP REASONING** Find the zeros of the function $f(x) = 9x^2 + 2$. Does the graph of the function intersect the x-axis? Explain your reasoning.

In Exercises 101 and 102, write an equation of the parabola with the given characteristics.

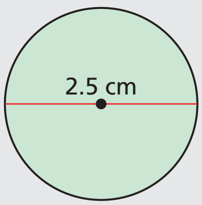
101. focus: $(0, -2)$
directrix: $y = 2$

102. vertex: $(5, -4)$
directrix: $x = 2$

103. Make a scatter plot of the data. Then describe the relationship between the data.

x	4	2	7	5	2	6	3	5
y	37	62	45	79	29	18	55	64

104. Find the circumference of the circle.



105. **MODELING REAL LIFE** A screen printing shop sells long-sleeved shirts and short-sleeved shirts. Order A includes 3 long-sleeved shirts and 12 short-sleeved shirts for a total cost of \$165. Order B includes 8 long-sleeved shirts and 2 short-sleeved shirts for a total cost of \$140. What is the cost of each type of shirt?

In Exercises 106–109, add, subtract, or multiply. Write the answer in standard form.

106. $-3i(9 - 4i)$ **107.** $(7 + 8i) - (-4 + 9i)$

108. $(5 - 2i)(1 - 12i)$ **109.** $(-15 - i) + (6 + 3i)$

In Exercises 110 and 111, write an inequality that represents the graph.

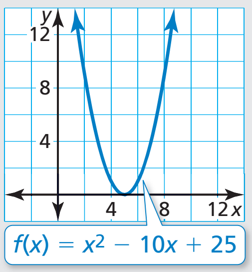
110.

111.

In Exercises 112 and 113, find the inverse of the function. Then graph the function and its inverse.

112. $f(x) = -3x + 6$ **113.** $f(x) = 2x^2 - 18, x \geq 0$

114. Use the graph to solve $x^2 - 10x + 25 = 0$.



In Exercises 115 and 116, graph the system. Identify a solution, if possible.

115. $y > x - 1$
$y \leq -4$

116. $x + y \leq 3$
$y + 2 \geq -4x$