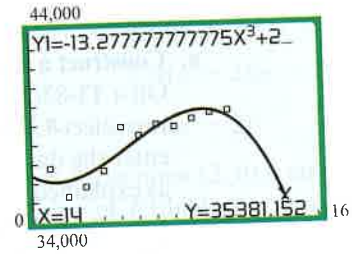
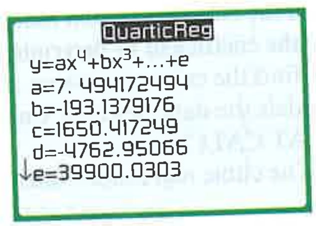
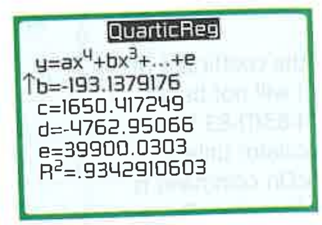


A **quartic regression** function will model the data in an application at least as well as does a cubic regression function. To illustrate this, find the quartic regression function for the movie screen data in Example 7. On a TI-83/TI-83 Plus/TI-84 Plus calculator, this can be accomplished by selecting **7: QuartReg** in the STAT CALC menu.



The coefficients of the quartic regression function

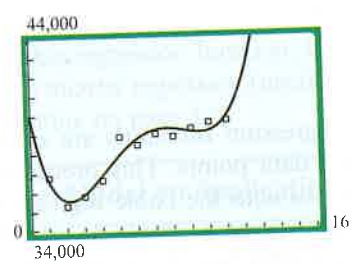


R^2 is the coefficient of determination

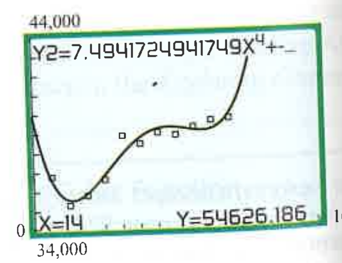
The quartic model is

$$P(x) = 7.494172494x^4 - 193.1379176x^3 + 1650.417249x^2 - 4762.95066x + 39900.0303$$

The coefficient of determination ($R^2 = 0.9342910603$) and the graphs of the quartic regression function, shown below, confirm that the quartic function does provide a better fit to the data than does the cubic regression function.



A graph of the quartic regression function and the data



The quartic regression function estimates about 54,626 movie screens in 2014.

Using extrapolation, we find that the quartic regression function estimates about 54,626 movie screens in 2014. See the graph on the right above. This large increase, from 39,641 screens in 2011, is also unlikely.

Answers to Exercises 15–20, 45–60, 75a, and 80 are on page AA12.

EXERCISE SET 3.2

Concept Check

1. What is the degree of

$$P(x) = 4x^3 - x^2 + 3x - 5 \quad 3$$

2. What is the leading term of

$$P(x) = 5x - 3x^2 + 2x^3 - 1 \quad 2x^3$$

3. What is the largest possible number of turning points for the graph of a polynomial function of degree 4? 3

Indicates Try It Exercises

4. The factored form of $P(x) = x^3 + 2x^2 - 15x$ is
$$P(x) = x(x - 3)(x + 5)$$
What are the zeros of P? 0, 3, and −5

In Exercises 5 to 12, examine the leading term and determine the far-left and far-right behavior of the graph of the polynomial function.

5. $P(x) = 3x^4 - 2x^2 - 7x + 1$
Up to the far left, up to the far right

6. $P(x) = -2x^3 - 6x^2 + 5x - 1$
Up to the far left, down to the far right

7. $P(x) = 5x^5 - 4x^3 - 17x^2 + 2$
Down to the far left, up to the far right

8. $P(x) = -6x^4 - 3x^3 + 5x^2 - 2x + 5$
Down to the far left, down to the far right

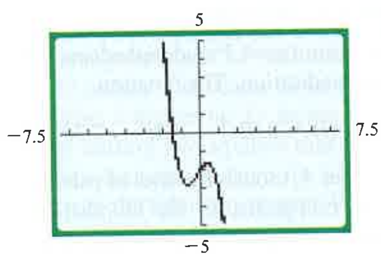
9. $P(x) = -450 + x^6$
Up to the far left, up to the far right

10. $P(x) = 5 - 2x - 4x^2 - 3x^3$
Up to the far left, down to the far right

11. $P(x) = -x^5 + 4x^2 + 9$
Up to the far left, down to the far right

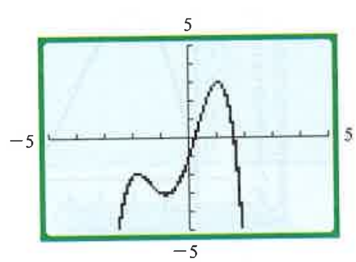
12. $P(x) = \frac{3}{4}x^4 - 5x^3 + 6x^2 - 3x - 7$
Up to the far left, up to the far right

13. The following graph is the graph of a third-degree (cubic) polynomial function. What does the far-left and far-right behavior of the graph say about the leading coefficient a?
$a < 0$



$P(x) = ax^3 + bx^2 + cx + d$

14. The following graph is the graph of a fourth-degree (quartic) polynomial function. What does the far-left and far-right behavior of the graph say about the leading coefficient a?
$a < 0$



$P(x) = ax^4 + bx^3 + cx^2 + dx + e$

In Exercises 15 to 20, use a graphing utility to graph each polynomial. Use the maximum and minimum features of the graphing utility to estimate, to the nearest tenth, the coordinates of the points where $P(x)$ has a relative maximum or a relative minimum. For each point, indicate whether the y value is a relative maximum or a relative minimum. The number in parentheses to the right of the polynomial is the total number of relative maxima and minima.

15. $P(x) = x^3 + x^2 - 9x - 9$ (2) Relative maximum $y \approx 5.0$ at $x \approx -2.1$, relative minimum $y \approx -16.9$ at $x \approx 1.4$

16. $P(x) = x^3 + 4x^2 - 4x - 16$ (2) Relative maximum $y \approx 5.0$ at $x \approx -3.1$, relative minimum $y \approx -16.9$ at $x \approx 0.4$

17. $P(x) = x^3 - 3x^2 - 24x + 3$ (2) Relative maximum $y \approx 31.0$ at $x \approx -2.0$, relative minimum $y \approx -77.0$ at $x \approx 4.0$

18. $P(x) = -2x^3 - 3x^2 + 12x + 1$ (2) Relative maximum $y \approx 8.0$ at $x \approx 1.0$, relative minimum $y \approx -19.0$ at $x \approx -2.0$

19. $P(x) = x^4 - 4x^3 - 2x^2 + 12x - 5$ (3) Relative maximum $y \approx 2.0$ at $x \approx 1.0$, relative minima $y \approx -14.0$ at $x \approx -1.0$ and $y \approx -14.0$ at $x \approx 3.0$

20. $P(x) = x^4 - 10x^2 + 9$ (3) Relative maximum $y \approx 9.0$ at $x \approx 0.0$, relative minima $y \approx -16.0$ at $x \approx -2.2$ and $y \approx -16.0$ at $x \approx 2.2$

In Exercises 21 to 26, find the real zeros of each polynomial function by factoring. The number in parentheses to the right of each polynomial indicates the number of real zeros of the given polynomial function.

21. $P(x) = x^3 - 2x^2 - 15x$ (3) −3, 0, 5

22. $P(x) = x^3 - 6x^2 + 8x$ (3) 0, 2, 4

23. $P(x) = x^4 - 13x^2 + 36$ (4) −3, −2, 2, 3

24. $P(x) = 4x^4 - 37x^2 + 9$ (4) $-3, -\frac{1}{2}, \frac{1}{2}, 3$

25. $P(x) = x^5 - 5x^3 + 4x$ (5) −2, −1, 0, 1, 2

26. $P(x) = x^5 - 25x^3 + 144x$ (5) −4, −3, 0, 3, 4

In Exercises 27 to 36, use the Intermediate Value Theorem to verify that P has a zero between a and b.

27. $P(x) = 2x^3 + 3x^2 - 23x - 42$; $a = 3, b = 4$

28. $P(x) = 4x^3 - x^2 - 6x + 1$; $a = 0, b = 1$

29. $P(x) = 3x^3 + 7x^2 + 3x + 7$; $a = -3, b = -2$

30. $P(x) = 2x^3 - 21x^2 - 2x + 25$; $a = 1, b = 2$

31. $P(x) = 4x^4 + 7x^3 - 11x^2 + 7x - 15$; $a = 1, b = 1\frac{1}{2}$

32. $P(x) = 5x^3 - 16x^2 - 20x + 64$; $a = 3, b = 3\frac{1}{2}$

33. $P(x) = x^4 - x^2 - x - 4$; $a = 1.7, b = 1.8$

34. $P(x) = x^3 - x - 8$; $a = 2.1, b = 2.2$

35. $P(x) = -x^4 + x^3 + 5x - 1$; $a = 0.1, b = 0.2$

36. $P(x) = -x^3 - 2x^2 + x - 3$; $a = -2.8, b = -2.7$

In Exercises 37 to 44, determine the *x*-intercepts of the graph of *P*. For each *x*-intercept, use the Even and Odd Powers of $(x - c)$ Theorem to determine whether the graph of *P* crosses the *x*-axis or intersects but does not cross the *x*-axis.

37. $P(x) = (x - 1)(x + 1)(x - 3)$
Crosses the *x*-axis at (−1, 0), (1, 0), and (3, 0)

38. $P(x) = (x + 2)(x - 6)^2$
Crosses the *x*-axis at (−2, 0); intersects but does not cross at (6, 0)

39. $P(x) = x(x - 5)^2(x - 3)$ Crosses the *x*-axis at (3, 0) and at (0, 0); intersects but does not cross at (5, 0)

40. $P(x) = -(2x - 8)(x - 7)^2$
Crosses the *x*-axis at (4, 0); intersects but does not cross at (7, 0)

41. $P(x) = x^2(x - 15)(2x - 7)^2$ Crosses the *x*-axis at (15, 0); intersects but does not cross at (0, 0) and at $\left(\frac{7}{2}, 0\right)$

42. $P(x) = x(x + 4)(x - 5)^2$ Crosses the *x*-axis at (0, 0) and (−4, 0); intersects but does not cross at (5, 0)

43. $P(x) = x^3 - 6x^2 + 9x$
Crosses the *x*-axis at (0, 0); intersects but does not cross at (3, 0)

44. $P(x) = x^4 + 3x^3 + 4x^2$
Intersects but does not cross the *x*-axis at (0, 0)

In Exercises 45 to 60, sketch the graph of the polynomial function. Do not use a graphing utility.

45. $P(x) = x^3 - x^2 - 2x$

46. $P(x) = x^3 + 2x^2 - 3x$

47. $P(x) = -x^3 - 2x^2 + 5x + 6$
(In factored form, $P(x) = -(x + 3)(x + 1)(x - 2)$.)

48. $P(x) = -x^3 - 3x^2 + x + 3$
(In factored form, $P(x) = -(x + 3)(x + 1)(x - 1)$.)

49. $P(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$
(In factored form, $P(x) = (x + 1)(x - 1)^2(x - 3)$.)

50. $P(x) = x^4 - 6x^3 + 8x^2$

51. $P(x) = x^3 + 6x^2 + 5x - 12$
(In factored form, $P(x) = (x - 1)(x + 3)(x + 4)$.)

52. $P(x) = -x^3 + 4x^2 + x - 4$

53. $P(x) = -x^3 + 7x - 6$

54. $P(x) = x^3 - 6x^2 + 9x$
(In factored form, $P(x) = x(x - 3)^2$.)

55. $P(x) = -x^3 + 4x^2 - 4x$
(In factored form, $P(x) = -x(x - 2)^2$.)

56. $P(x) = -x^4 + 2x^3 + 3x^2 - 4x - 4$
(In factored form, $P(x) = -(x - 2)^2(x + 1)^2$.)

57. $P(x) = -x^4 + 3x^3 + x^2 - 3x$

58. $P(x) = \frac{1}{2}x^4 + x^3 - 2x^2 - x + \frac{3}{2}$
(In factored form, $P(x) = \frac{1}{2}(x - 1)^2(x + 1)(x + 3)$.)

59. $P(x) = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$
(In factored form, $P(x) = (x + 1)^3(x - 2)^2$.)

60. $P(x) = 2x^5 - 3x^4 - 4x^3 + 3x^2 + 2x$

In Exercises 61 to 66, use translation, reflection, or both concepts to explain how the graph of *P* can be used to produce the graph of *Q*.

61. $P(x) = x^3 + x$; $Q(x) = x^3 + x + 2$
Shift the graph of *P* vertically upward 2 units.

62. $P(x) = x^4$; $Q(x) = x^4 - 3$
Shift the graph of *P* vertically downward 3 units.

63. $P(x) = x^4$; $Q(x) = (x - 1)^4$
Shift the graph of *P* horizontally 1 unit to the right.

64. $P(x) = x^3$; $Q(x) = (x + 3)^3$
Shift the graph of *P* horizontally 3 units to the left.

65. $P(x) = x^5$; $Q(x) = -(x - 2)^5 + 3$ Shift the graph of *P* horizontally 2 units to the right and reflect this graph about the *x*-axis. Then shift the resulting graph vertically upward 3 units.

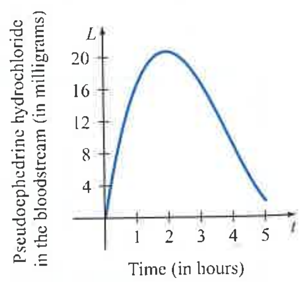
66. $P(x) = x^6$; $Q(x) = (x + 4)^6 - 5$
Shift the graph of *P* horizontally 4 units to the left and vertically downward 5 units.

67. **Medication Level** Pseudoephedrine hydrochloride is an allergy medication. The function

$$L(t) = 0.03t^4 + 0.4t^3 - 7.3t^2 + 23.1t$$

where $0 \le t \le 5$, models the level of pseudoephedrine hydrochloride, in milligrams, in the bloodstream of a patient *t* hours after 30 milligrams of the medication have been taken.

a. Use a graphing utility and the function $L(t)$ to determine the maximum level of pseudoephedrine hydrochloride in the patient's bloodstream. Round your result to the nearest 0.01 milligram. 20.69 mg



b. At what time *t*, to the nearest minute, is this maximum level of pseudoephedrine hydrochloride reached? 118 min

68. **Profit** A software company produces a computer game. The company has determined that its profit *P*, in dollars, from the manufacture and sale of *x* games is given by

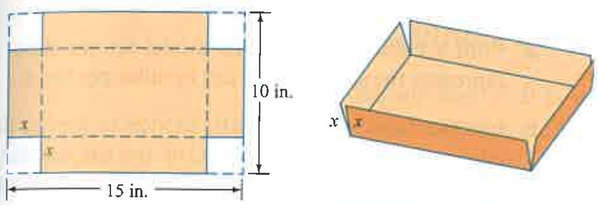
$$P(x) = -0.000001x^3 + 96x - 98{,}000$$

where $0 < x \le 9000$.

a. What is the maximum profit, to the nearest thousand dollars, the company can expect from the sale of its game? $264,000

b. How many games, to the nearest unit, does the company need to produce and sell to obtain the maximum profit? 5657 games

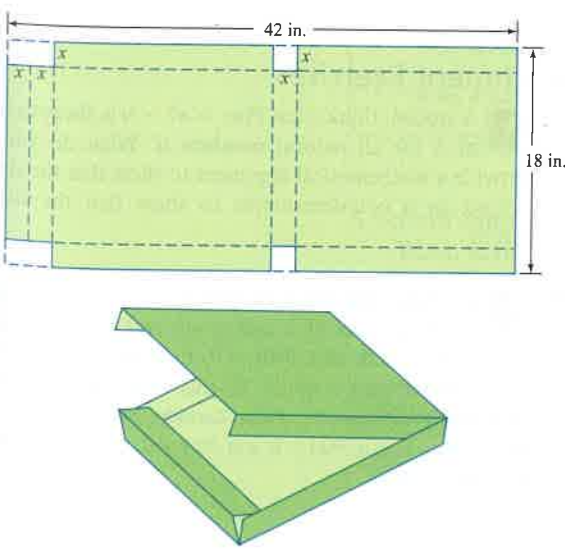
69. **Construction of a Box** A company constructs boxes from rectangular pieces of cardboard that measure 10 inches by 15 inches. An open box is formed by cutting squares that measure x inches by x inches from each corner of the cardboard and folding up the sides, as shown in the following figure.



a. Express the volume V of the box as a function of x.
$V(x) = x(15 - 2x)(10 - 2x) = 4x^3 - 50x^2 + 150x$

b. Determine (to the nearest hundredth of an inch) the x value that maximizes the volume of the box. 1.96 in.

70. **Maximizing Volume** A closed box is to be constructed from a rectangular sheet of cardboard that measures 18 inches by 42 inches. The box is made by cutting rectangles that measure x inches by $2x$ inches from two of the corners and by cutting two squares that measure x inches by x inches from the top and from the bottom of the rectangle, as shown in the following figure. What value of x (to the nearest thousandth of an inch) will produce a box with maximum volume? 3.571 in.



71. **Wind Turbine Power** The power P, in watts, generated by a particular wind turbine with winds blowing at v meters per second is given by the cubic polynomial function

$$P(v) = 4.95v^3$$

a. Find the power generated, to the nearest 10 watts, when the wind speed is 8 meters per second. 2530 W

b. What wind speed, in meters per second, is required to generate 10,000 watts? Round to the nearest tenth. 12.6 m/s

c. If the wind speed is doubled, what effect does this have on the power generated by the turbine?
The power increases by a factor of 8.

d. If the wind speed is tripled, what effect does this have on the power generated by the turbine?
The power increases by a factor of 27.

72. **Weight of a Tiger Shark** The following table shows the estimated weights of tiger sharks of various lengths.

Estimated Weight of Tiger Sharks

Length (in feet, x)	Weight (in pounds)
4	35
5	73
6	132
7	219
8	338
9	496
10	699
11	954

Source: National Oceanic and Atmospheric Administration.

a. Find the cubic regression function that models the data.
$P(x) = 1.005050505x^3 - 4.601731602x^2 + 18.38888889x - 29.38744589$

b. What is the coefficient of determination for this cubic regression?
0.9999992225

c. What does the coefficient of determination indicate concerning how well the cubic regression function models the data? The cubic regression function provides a very good model of the data.

d. Use the cubic regression function to estimate the weight of a tiger shark that has a length of 7.75 feet and the weight of a tiger shark that has a length of 12 feet. Round to the nearest pound. 305 pounds, 1265 pounds

73. **A Cat's Age in Human Years** Many people think cats age about 7 years, in terms of human years, for each calendar year. However, the data in the following table, from *The Cat Owner's Manual,* indicate that the "7-to-1" rule is not accurate for estimating a young cat's age in terms of human years.

Cat's Age in Human Years

Calendar Age, x (in months)	Approximate age (in human years)
2	3
4	6
6	9
8	11
10	13
12	15
18	20
24	24

Source: The Cat Owner's Manual, by Quirk Books.

a. Find a cubic regression function P that models the data, where x represents the age of the cat in months and $P(x)$ represents the cat's approximate age in human years. $P(x) = 0.00072333177x^3 - 0.0481273792x^2 + 1.749672272x - 0.2386484211$

b. What is the coefficient of determination for this cubic regression? 0.9995341654

c. What does the coefficient of determination indicate concerning how well the cubic regression function models the data? The cubic regression function provides a very good model of the data.

d. Use the cubic regression function to estimate the age, in human years, of a 5-month-old cat. Round to the nearest tenth of a year. 7.4 years

74. **Box-Office Receipts** The United States and Canada movie theater box-office receipts are given in the following table, for 2002 to 2011.

Movie Theater Box-Office Receipts

Year	Receipts (billions of dollars)	Year	Receipts (billions of dollars)
2002	9.1	2007	9.6
2003	9.2	2008	9.6
2004	9.3	2009	10.6
2005	8.8	2010	10.6
2006	9.2	2011	10.2

Source: The Motion Picture Association of America.

a. Find a quartic regression function for the data. Use $x = 2$ to represent 2002, $x = 3$ to represent 2003, . . . , and $x = 11$ to represent 2011. $P(x) = -0.0047931235x^4 + 0.1152000777x^3 - 0.9137674825x^2 + 2.856390831x + 6.188636364$

b. Use the quartic regression function to estimate the movie theater box-office receipts for 2010. $10.644639 billion

75. **Fuel Efficiency** The fuel efficiency, in miles per gallon, for a midsize car at various speeds, in miles per hour, is given in the following table.

Fuel Efficiency of a Midsize Car

Speed (mph)	Fuel Efficiency (mpg)	Speed (mph)	Fuel Efficiency (mpg)
0	0.0	40	30.2
5	11.1	45	30.6
10	17.2	50	31.7
15	22.4	55	30.8
20	26.2	60	29.5
25	27.1	65	28.2
30	28.3	70	26.3
35	29.4	75	24.1

a. Find a cubic and a quartic model for the data. Let x represent the speed of the car in miles per hour.

b. Use the cubic and the quartic models to predict the fuel efficiency, in miles per gallon, of the car traveling at a speed of 80 miles per hour. Round to the nearest tenth. Cubic, 24.4 mpg; quartic, 18.5 mpg

c. Which of the fuel efficiency values from **b** is the more realistic value? Answers will vary; however, the downward trend for speeds greater than 50 mph suggests that 18.5 mpg is the more realistic value.

76. Use a graph of $P(x) = 4x^4 - 12x^3 + 13x^2 - 12x + 9$ to determine between which two consecutive integers P has a real zero. Between 1 and 2

77. The point (2, 0) is on the graph of P. What point must be on the graph of $P(x - 3)$? (5, 0)

78. The point (3, 5) is on the graph of P. What point must be on the graph of $P(x + 1) - 2$? (2, 3)

79. Explain how to use the graph of $y = x^3$ to produce the graph of $P(x) = (x - 2)^3 + 1$. Shift the graph of $y = x^3$ horizontally 2 units to the right and vertically upward 1 unit.

Enrichment Exercises

80. A student thinks that $P(n) = n^3 - n$ is always a multiple of 6 for all natural numbers n. What do you think? Provide a mathematical argument to show that the student is correct or a counterexample to show that the student is wrong.

81. Consider the following conjecture. Let P be a polynomial function. If a and b are real numbers such that $a < b$, $P(a) > 0$, and $P(b) > 0$, then $P(x)$ does not have a real zero between a and b. Is this conjecture true or false? Support your answer. False. Consider $P(x) = x^2$, $a = -1$, and $b = 1$, then $a < b$, $P(a) > 0$ and $P(b) > 0$; however, P has a zero between a and b.

SECTION 3.3

- Multiple Zeros of a Polynomial Function
- Rational Zero Theorem
- Upper and Lower Bounds for Real Zeros
- Descartes' Rule of Signs
- Zeros of a Polynomial Function
- Applications of Polynomial Functions

Zeros of Polynomial Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A19.

PS1. Find the zeros of $P(x) = 6x^2 - 25x + 14$. [1.3, 2.4] $\frac{2}{3}, \frac{7}{2}$

PS2. Use synthetic division to divide $2x^3 + 3x^2 + 4x - 7$ by $x + 2$. [3.1] $2x^2 - x + 6 - \frac{19}{x+2}$

PS3. Use synthetic division to divide $3x^4 - 21x^2 - 3x - 5$ by $x - 3$. [3.1] $3x^3 + 9x^2 + 6x + 15 + \frac{40}{x-3}$

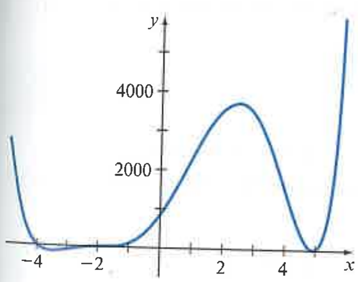
PS4. List all natural numbers that are factors of 12. [P.1] 1, 2, 3, 4, 6, 12

PS5. List all integers that are factors of 27. [P.1] $\pm 1, \pm 3, \pm 9, \pm 27$

PS6. Given $P(x) = 4x^3 - 3x^2 - 2x + 5$, find $P(-x)$. [2.5] $P(-x) = -4x^3 - 3x^2 + 2x + 5$

Multiple Zeros of a Polynomial Function

Recall that if P is a polynomial function then the values of x for which $P(x)$ is equal to 0 are called the *zeros* of P or the **roots** of the equation $P(x) = 0$. A zero of a polynomial function may be a **multiple zero.** For example, $P(x) = x^2 + 6x + 9$ can be expressed in factored form as $(x + 3)(x + 3)$. Setting each factor equal to zero yields $x = -3$ in both cases. Thus $P(x) = x^2 + 6x + 9$ has a zero of -3 that occurs twice. The following definition will be most useful when we are discussing multiple zeros.



$P(x) = (x - 5)^2(x + 2)^3(x + 4)$

Figure 3.18

Definition of Multiple Zeros of a Polynomial Function

If a polynomial function P has $(x - r)$ as a factor exactly k times, then r is a **zero of multiplicity k** of the polynomial function P.

EXAMPLE

The graph of the polynomial function

$$P(x) = (x - 5)^2(x + 2)^3(x + 4)$$

is shown in Figure 3.18. This polynomial function has

- 5 as a zero of multiplicity 2.
- -2 as a zero of multiplicity 3.
- -4 as a zero of multiplicity 1.

A zero of multiplicity 1 is generally referred to as a **simple zero.**

When searching for the zeros of a polynomial function, it is important that we know how many zeros to expect. This question is answered completely in Section 3.4. For the work in this section, the following result is helpful.

Number of Zeros of a Polynomial Function

A polynomial function P of degree n has at most n zeros, where each zero of multiplicity k is counted k times.