3.1 Solving Quadratic Equations

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Learning Target Solve quadratic equations graphically and algebraically.

Success Criteria
- I can solve quadratic equations by graphing.
- I can solve quadratic equations algebraically.
- I can use quadratic equations to solve real-life problems.

EXPLORE IT! Solving Quadratic Equations

Math Practice

Find General Methods
Recall that when $ab = 0$, either a or b must be 0. How does this property help you solve quadratic equations?

Work with a partner.

a. Match each quadratic equation with the graph of its related function. Then use the graph to find the real solutions (if any) of each equation. Explain your reasoning.

i. $x^2 - 2x = 0$

ii. $x^2 - 2x + 1 = 0$

iii. $x^2 - 2x + 2 = 0$

iv. $-x^2 + 2x = 0$

v. $-x^2 + 2x - 1 = 0$

vi. $-x^2 + 2x - 2 = 0$

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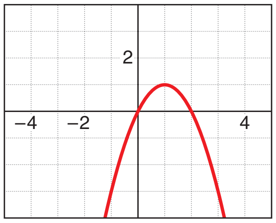

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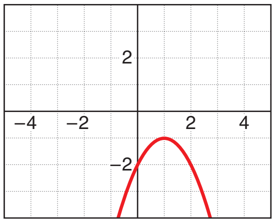

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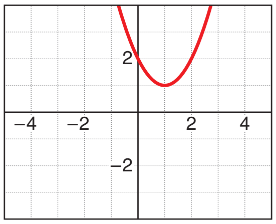

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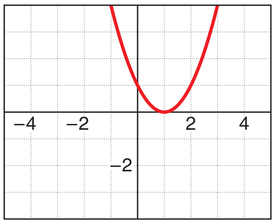

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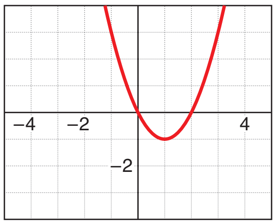

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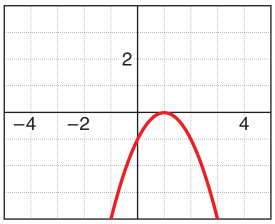

b. How can you use a graph to determine the number of real solutions of a quadratic equation?

c. What algebraic methods can you use to solve the equations in part (a)? Solve each equation using an algebraic method.

Vocabulary

quadratic equation in one variable, *p. 90*
root of an equation, *p. 90*
zero of a function, *p. 92*

STUDY TIP

Quadratic equations can have zero, one, or two real solutions.

Solving Quadratic Equations by Graphing

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A **quadratic equation in one variable** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. A **root of an equation** is a solution of the equation. You can use various methods to solve quadratic equations.

KEY IDEA

Solving Quadratic Equations

By graphing Find the x-intercepts of the graph of the related function $y = ax^2 + bx + c$.

Using square roots Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

By factoring Write the quadratic equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

EXAMPLE 1 Solving Quadratic Equations by Graphing

WATCH

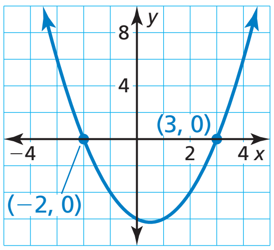
Solve each equation by graphing.

a. $x^2 - x - 6 = 0$

b. $-2x^2 - 2 = 4x$

SOLUTION

a. The equation is in standard form. Graph the related function $y = x^2 - x - 6$.



The x-intercepts are -2 and 3.

▶ The solutions, or roots, are $x = -2$ and $x = 3$.

Check

$x^2 - x - 6 = 0$

$(-2)^2 - (-2) - 6 \stackrel{?}{=} 0$

$4 + 2 - 6 \stackrel{?}{=} 0$

$0 = 0$ ✓

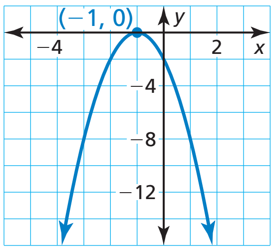
$x^2 - x - 6 = 0$

$3^2 - 3 - 6 \stackrel{?}{=} 0$

$9 - 3 - 6 \stackrel{?}{=} 0$

$0 = 0$ ✓

b. Add $-4x$ to each side to obtain $-2x^2 - 4x - 2 = 0$. Graph the related function $y = -2x^2 - 4x - 2$.



The x-intercept is -1.

▶ The solution, or root, is $x = -1$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation by graphing.

1. $x^2 - 8x + 12 = 0$

2. $4x^2 - 12x + 9 = 0$

3. $-\frac{1}{2}x^2 = 20 - 6x$

4. WRITING Explain how to use graphing to find the roots of the equation $ax^2 + bx + c = 0$.

5. MP STRUCTURE How many roots does $x^2 = 0$ have? Use your answer to determine the number of roots of $(x - 2)^2 = 0$. Explain your reasoning.

Solving Quadratic Equations Algebraically

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When solving quadratic equations using square roots, you can use properties of square roots to write your solutions in different forms. When a radicand in the denominator of a fraction is not a perfect square, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called *rationalizing the denominator*.

EXAMPLE 2 Solving Quadratic Equations Using Square Roots

WATCH

Solve each equation using square roots.

a. $4x^2 - 31 = 49$ **b.** $3x^2 + 9 = 0$ **c.** $\frac{2}{5}(x + 3)^2 = 5$

SOLUTION

a.

$4x^2 - 31 = 49$	Write the equation.
$4x^2 = 80$	Add 31 to each side.
$x^2 = 20$	Divide each side by 4.
$x = \pm\sqrt{20}$	Take square root of each side.
$x = \pm\sqrt{4} \cdot \sqrt{5}$	Product Property of Square Roots
$x = \pm 2\sqrt{5}$	Simplify.

▶ The solutions are $x = 2\sqrt{5}$ and $x = -2\sqrt{5}$.

b.

$3x^2 + 9 = 0$	Write the equation.
$3x^2 = -9$	Subtract 9 from each side.
$x^2 = -3$	Divide each side by 3.

▶ The square of a real number cannot be negative. So, the equation has no real solution.

c.

$\frac{2}{5}(x + 3)^2 = 5$	Write the equation.
$(x + 3)^2 = \frac{25}{2}$	Multiply each side by $\frac{5}{2}$.
$x + 3 = \pm\sqrt{\frac{25}{2}}$	Take square root of each side.
$x = -3 \pm \sqrt{\frac{25}{2}}$	Subtract 3 from each side.
$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}}$	Quotient Property of Square Roots
$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$	Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.
$x = -3 \pm \frac{5\sqrt{2}}{2}$	Simplify.

▶ The solutions are $x = -3 + \frac{5\sqrt{2}}{2}$ and $x = -3 - \frac{5\sqrt{2}}{2}$.

STUDY TIP

Because $\frac{\sqrt{2}}{\sqrt{2}} = 1$, the value of $\frac{\sqrt{25}}{\sqrt{2}}$ does not change when you multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation using square roots.

6. $\frac{2}{3}x^2 + 14 = 20$ **7.** $-2x^2 + 1 = -6$ **8.** $2(x - 4)^2 = -5$

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When the left side of $ax^2 + bx + c = 0$ is factorable, you can solve the equation using the *Zero-Product Property*.

KEY IDEA

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

EXAMPLE 3 Solving a Quadratic Equation by Factoring

WATCH

Solve $x^2 - 4x = 45$ by factoring.

SOLUTION

$x^2 - 4x = 45$	Write the equation.
$x^2 - 4x - 45 = 0$	Write in standard form.
$(x - 9)(x + 5) = 0$	Factor the polynomial.
$x - 9 = 0$ or $x + 5 = 0$	Zero-Product Property
$x = 9$ or $x = -5$	Solve for x.

▶ The solutions are $x = -5$ and $x = 9$.

Math Practice

Understand Mathematical Terms

If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x-intercept of the graph of the function, and k is also a root of the equation $ax^2 + bx + c = 0$.

You know the x-intercepts of the graph of $f(x) = a(x - p)(x - q)$ are p and q. Because the value of the function is zero when $x = p$ and when $x = q$, the numbers p and q are also called *zeros* of the function. A **zero of a function** f is an x-value for which $f(x) = 0$.

EXAMPLE 4 Finding the Zeros of a Quadratic Function

WATCH

Find the zeros of $f(x) = 2x^2 - 11x + 12$.

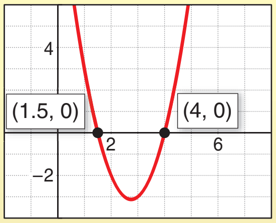
SOLUTION

To find the zeros of the function, find the x-values for which $f(x) = 0$.

$2x^2 - 11x + 12 = 0$	Set $f(x)$ equal to 0.
$(2x - 3)(x - 4) = 0$	Factor the polynomial.
$2x - 3 = 0$ or $x - 4 = 0$	Zero-Product Property
$x = 1.5$ or $x = 4$	Solve for x.

▶ The zeros of the function are $x = 1.5$ and $x = 4$.

Check



SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation by factoring.

9. $x^2 + 12x + 35 = 0$ **10.** $2x^2 - 8x + 6 = 0$ **11.** $3x^2 - 5x = 2$

Find the zero(s) of the function.

12. $f(x) = x^2 - 8x$ **13.** $f(x) = x^2 + 2x - 8$ **14.** $f(x) = 4x^2 + 28x + 49$

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Solving Real-Life Problems

One way to find the maximum value or minimum value of a quadratic function is to first write the function in intercept form $f(x) = a(x - p)(x - q)$. Because the vertex of the function lies on the axis of symmetry, $x = \frac{p + q}{2}$, the maximum value or minimum value occurs at the average of the zeros p and q.

EXAMPLE 5 Modeling Real Life

WATCH

A streaming service company charges \$6 per month and has 15 million subscribers. For each \$1 increase in price, the company loses 1.5 million subscribers. How much should the company charge to maximize monthly revenue? What is the maximum monthly revenue?

SOLUTION

Step 1 Define the variables. Let x represent the price increase and $R(x)$ represent the monthly revenue.

Step 2 Write a verbal model. Then write a quadratic function in intercept form.

Monthly revenue (millions of dollars)	=	Number of subscribers (millions of people)	•	Subscription price (dollars/person)
↓		↓		↓
$R(x)$	=	$(15 - 1.5x)$	•	$(6 + x)$

$$R(x) = (-1.5x + 15)(x + 6)$$

$$R(x) = -1.5(x - 10)(x + 6)$$

Step 3 Identify the zeros and find their average. Then find how much each subscription should cost to maximize monthly revenue.

The zeros of the revenue function are 10 and −6. The average of the zeros is $\frac{10 + (-6)}{2} = 2$.

To maximize revenue, each subscription should cost \$6 + \$2 = \$8.

Step 4 Find the maximum monthly revenue.

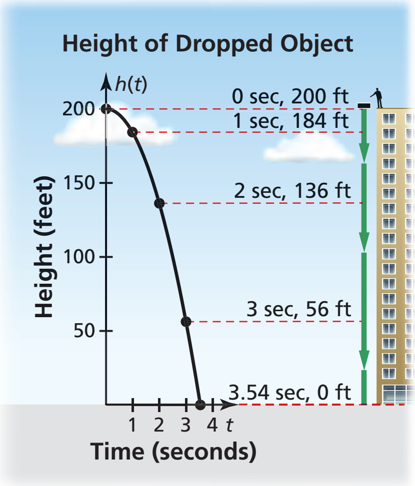
$$R(2) = -1.5(2 - 10)(2 + 6) = 96$$

▶ So, the company should charge \$8 per subscription to maximize monthly revenue. The maximum monthly revenue is \$96 million.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

15. **WHAT IF?** The company charges \$8 per month and has 15 million subscribers. How much should the company charge to maximize monthly revenue? What is the maximum monthly revenue?

16. A pottery store charges \$10 per mug and sells 40 mugs per month. For each \$0.50 decrease in price, the store sells 5 more mugs. What is the maximum monthly profit for mugs when each mug costs \$3 to make?



When an object is dropped, its height (in feet) above the ground after t seconds can be modeled by the function $h(t) = -16t^2 + s_0$, where s_0 is the initial height (in feet) of the object. The graph of $h(t) = -16t^2 + 200$, representing the height of an object dropped from an initial height of 200 feet, is shown at the left.

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The model $h(t) = -16t^2 + s_0$ assumes that the force of air resistance on the object is negligible. Also, this model applies only to objects dropped on Earth. For planets with stronger or weaker gravitational forces, different models are used.

EXAMPLE 6 Modeling Real Life

WATCH

For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet.

a. Write a function h that gives the height (in feet) of the container after t seconds. How long does the container take to hit the ground?

b. Find and interpret $h(1) - h(1.5)$.

SOLUTION

a. The initial height is 50, so the model is $h(t) = -16t^2 + 50$. Find the zeros of the function.

$h(t) = -16t^2 + 50$	Write the function.
$0 = -16t^2 + 50$	Substitute 0 for $h(t)$.
$-50 = -16t^2$	Subtract 50 from each side.
$\frac{-50}{-16} = t^2$	Divide each side by -16.
$\pm\sqrt{\frac{50}{16}} = t$	Take square root of each side.
$\pm 1.8 \approx t$	Use technology.

▶ Reject the negative solution, -1.8, because time must be positive. The container will fall for about 1.8 seconds before it hits the ground.

Math Practice

State the Meaning of Symbols

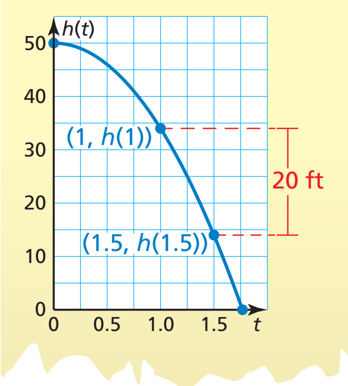
In the model for the height of a dropped object, interpret the term $-16t^2$. What does the negative symbol in the term mean?

b. Find $h(1)$ and $h(1.5)$. These represent the heights after 1 and 1.5 seconds.

$$h(1) = -16(1)^2 + 50$$
$$= -16 + 50 = 34$$
$$h(1.5) = -16(1.5)^2 + 50$$
$$= -16(2.25) + 50$$
$$= -36 + 50 = 14$$
$$h(1) - h(1.5) = 34 - 14 = 20$$

▶ So, the container fell 20 feet between 1 and 1.5 seconds. You can check this by graphing the function. The points appear to be about 20 feet apart. So, the answer is reasonable.

Check



SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

17. WHAT IF? The egg container is dropped from a height of 80 feet. How does this change your answers in parts (a) and (b)?

3.1 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–10, solve the equation by graphing. *Example 1*

1. $x^2 + 3x + 2 = 0$

2. $-x^2 + 2x + 3 = 0$

3. $0 = x^2 - 9$

4. $-8 = -x^2 - 4$

5. $8x = -4 - 4x^2$

6. $3x^2 = 6x - 3$

7. $7 = -x^2 - 4x$

8. $2x = x^2 + 2$

9. $\frac{1}{5}x^2 + 6 = 2x$

10. $3x = \frac{1}{4}x^2 + 5$

In Exercises 11–18, solve the equation using square roots. *Example 2*

11. $s^2 = 144$

12. $a^2 = 81$

13. $(z - 6)^2 = 25$

14. $(p - 4)^2 = 49$

15. $4(x - 1)^2 + 2 = 10$

16. $2(x + 2)^2 - 5 = 8$

17. $\frac{1}{2}r^2 - 10 = \frac{3}{2}r^2$

18. $\frac{1}{5}x^2 + 2 = \frac{3}{5}x^2$

ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in solving the equation.**

19.

✗

$$2(x + 1)^2 + 3 = 21$$
$$2(x + 1)^2 = 18$$
$$(x + 1)^2 = 9$$
$$x + 1 = 3$$
$$x = 2$$

20.

✗

$$-2x^2 - 8 = 0$$
$$-2x^2 = 8$$
$$x^2 = -4$$
$$x = \pm 2$$

In Exercises 21–30, solve the equation by factoring. *Example 3*

21. $0 = x^2 + 6x + 9$

22. $0 = z^2 - 10z + 25$

23. $x^2 - 8x = -12$

24. $x^2 - 11x = -30$

25. $n^2 - 6n = 0$

26. $a^2 - 49 = 0$

27. $3p^2 + 11p = 4$

28. $6t^2 + 14 = -25t$

29. $2w^2 - 16w = 12w - 48$

30. $-y + 28 + y^2 = 2y + 2y^2$

In Exercises 31–38, solve the equation using any method. Explain your choice of method.

31. $u^2 = -9u$

32. $\frac{t^2}{20} + 8 = 15$

33. $-(x + 9)^2 = 64$

34. $-2(x + 2)^2 = 5$

35. $7(x - 4)^2 - 18 = 10$

36. $t^2 + 8t + 16 = 0$

37. $x^2 + 3x + \frac{5}{4} = 0$

38. $x^2 - 1.75 = 0.5$

In Exercises 39–48, find the zero(s) of the function. *Example 4*

39. $g(x) = x^2 + 6x + 8$

40. $f(x) = x^2 - 8x + 16$

41. $h(x) = x^2 + 7x - 30$

42. $g(x) = x^2 + 11x$

43. $g(x) = x^2 + 22x + 121$

44. $h(x) = x^2 + 19x + 84$

45. $f(x) = 2x^2 - 2x - 12$

46. $f(x) = 4x^2 - 12x + 9$

47. $p(x) = 8x^2 - 19x + 6$

48. $q(x) = 10x^2 - 3x - 4$

49. **MODELING REAL LIFE** A restaurant sells 330 sandwiches each day. For each \$0.25 decrease in price, the restaurant sells 15 more sandwiches. How much should the restaurant charge to maximize daily revenue? What is the maximum daily revenue? *Example 5*

50. **MODELING REAL LIFE** An athletic store charges \$120 per pair of basketball shoes and sells 200 pairs per month. For each \$2 increase in price, the store sells two fewer pairs of shoes. How much should the store charge to maximize monthly revenue? What is the maximum monthly revenue?

51. **MODELING REAL LIFE** You drop a seashell into the ocean from a height of 40 feet above the water. *Example 6*

a. Write a function h that gives the height (in feet) of the seashell above the water after t seconds. Interpret each term. How long does the seashell take to hit the water?

b. Find and interpret $h(0.5) - h(1)$.

52. **MODELING REAL LIFE** According to legend, in 1589, the Italian scientist Galileo Galilei dropped rocks of different weights from the top of the Leaning Tower of Pisa to prove his conjecture that the rocks would hit the ground at the same time.

a. The original height of the tower was about 196 feet. Write a function h that gives the height (in feet) of a rock dropped from the top of the original tower after t seconds. Interpret each term. How long does the rock take to hit the ground?

b. Find and interpret $h(1.25) - h(2.5)$.

53. **COLLEGE PREP** Which equations have roots that are equivalent to the x-intercepts of the graph shown?

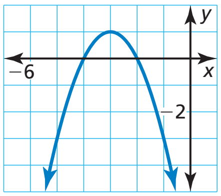
Ⓐ $-x^2 - 6x - 8 = 0$

Ⓑ $0 = (x + 2)(x + 4)$

Ⓒ $0 = -(x + 2)^2 + 4$

Ⓓ $2x^2 - 4x - 6 = 0$

Ⓔ $4(x + 3)^2 - 4 = 0$



54. **COLLEGE PREP** Which graph has x-intercepts that are equivalent to the roots of the equation $\left(x - \frac{3}{2}\right)^2 = \frac{25}{4}$?

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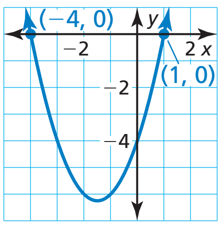

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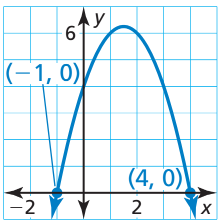

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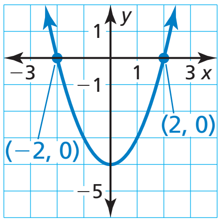

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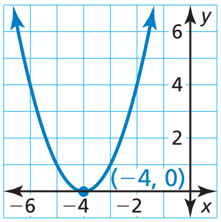

55. **MP** **REASONING** Write a quadratic function in the form $f(x) = x^2 + bx + c$ that has zeros 8 and 11.

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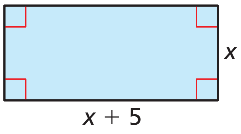
56. **MP** **NUMBER SENSE** Write a quadratic equation in standard form that has roots equidistant from 10.

57. **OPEN-ENDED** Write a quadratic equation that has (a) one real solution, and (b) no real solution.

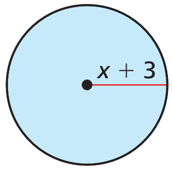
58. **MP** **PRECISION** Describe the relationship among zeros, x-intercepts, and roots.

CONNECTING CONCEPTS **In Exercises 59–62, find the value of x.**

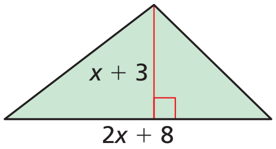
59. Area of rectangle = 36



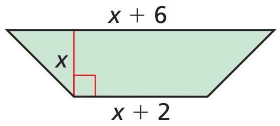
60. Area of circle = 25π



61. Area of triangle = 42

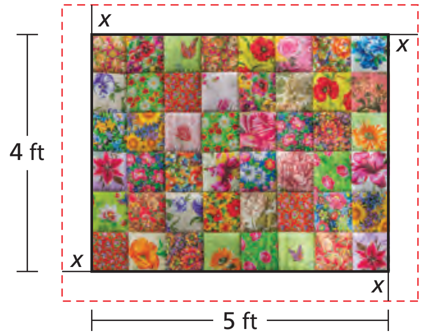


62. Area of trapezoid = 32



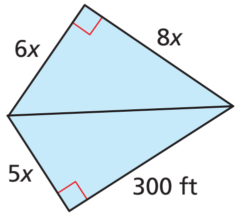
63. **MODELING REAL LIFE** The equation $h = 0.019s^2$ models the height h (in feet) of the largest ocean waves when the wind speed is s knots. Compare the wind speeds required to generate 5-foot waves and 20-foot waves.

64. **MP** **PROBLEM SOLVING** You make a rectangular quilt that is 5 feet by 4 feet. You use the remaining 10 square feet of fabric to add a border of uniform width to the quilt. What is the width of the border?



65. **CRITICAL THINKING** Use an equation to find two consecutive odd integers whose product is 143.

66. CONNECTING CONCEPTS A quadrilateral is divided into two right triangles as shown in the figure. What is the length of each side of the quadrilateral?

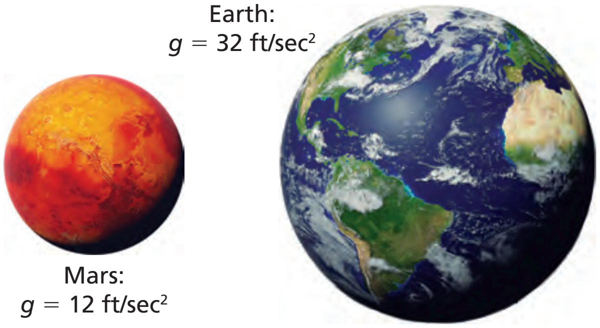


67. ABSTRACT REASONING An equation of the form $ax^2 + bx + c = 0$ has no real solution and a graph of the related function has a vertex that lies in the second quadrant.

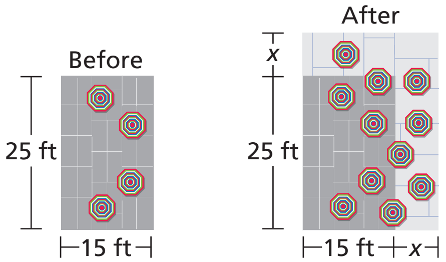
a. Is the value of a positive or negative? Explain your reasoning.

b. The graph is translated so the vertex is in the fourth quadrant. Does the graph have any x-intercepts? Explain.

68. MP REASONING When an object is dropped on *any* planet, its height (in feet) after t seconds can be modeled by the function $h(t) = -\frac{g}{2}t^2 + s_0$, where s_0 is the object's initial height and g is the planet's acceleration due to gravity. Two rocks are dropped from the same initial height on Earth and Mars. Make a conjecture about which rock will hit the ground first. Justify your answer.



69. MP PROBLEM SOLVING A café has an outdoor, rectangular patio. The owner wants to add 329 square feet to the area of the patio by expanding the existing patio as shown. By what distance x should the patio be extended?

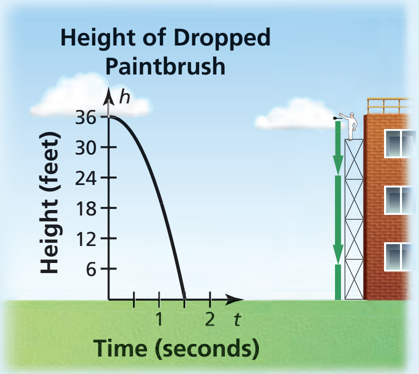


70. MP PROBLEM SOLVING A flea can jump very long distances relative to its size. The path of the jump of a flea can be modeled by the graph of the function $y = -0.189x^2 + 2.462x$, where x is the horizontal distance (in inches) and y is the vertical distance (in inches). Graph the function. Identify the vertex and zeros and interpret their meanings in this situation.

GO DIGITAL

71. MAKING AN ARGUMENT Your friend claims the equation $x^2 + 7x = -49$ can be solved by factoring and has the solutions $x = 0$ and $x = -7$. You solve the equation by graphing the related function and claim there is no solution. Who is correct? Explain.

72. HOW DO YOU SEE IT?
An artist is painting a mural and drops a paintbrush. The graph represents the height h (in feet) of the paintbrush after t seconds.



a. What is the initial height of the paintbrush?

b. How long does the paintbrush take to hit the ground?

73. CRITICAL THINKING The equation $x^2 - 2kx - 75 = 0$ has the solutions $x = k - 10$ and $x = k + 10$, where k is an integer. Find the possible values of k.

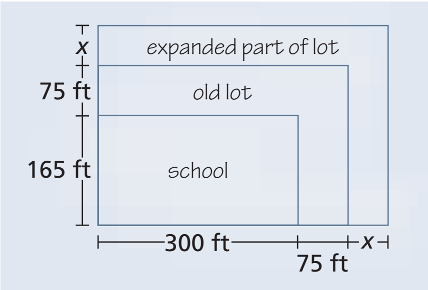
74. ABSTRACT REASONING Factor the expressions $x^2 - 4$ and $x^2 - 9$. Recall that an expression in this form is called a difference of two squares. Use your answers to factor the expression $x^2 - a^2$. Graph the related function $y = x^2 - a^2$. Label the vertex, x-intercepts, and axis of symmetry.

75. DRAWING CONCLUSIONS Consider the expression $x^2 + a^2$, where $a > 0$.

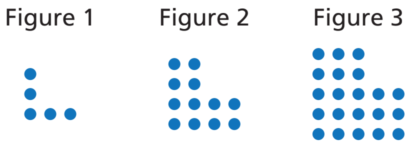
a. You want to rewrite the expression as $(x + m)(x + n)$. Write two equations that m and n must satisfy.

b. Use the equations you wrote in part (a) to solve for m and n. What can you conclude?

76. DIG DEEPER Officials at a school want to double the size of its parking lot by expanding the existing lot as shown. By what distance x should the lot be expanded?



77. MP REPEATED REASONING The first three figures of a pattern are shown. Is there a figure with 480 dots in the pattern? If so, which figure is it? If not, explain why not.



78. **THOUGHT PROVOKING**
Write an equation of the form $ax^4 + bx^2 + c = 0$, where a, b, and c are real numbers, that can be solved by factoring and that has four real solutions. Explain how to solve your equation.

REVIEW & REFRESH

In Exercises 79–82, find the sum or difference.

79. $(x^2 + 2) + (2x^2 - x)$

80. $(-2x + 1) - (-3x^2 + x)$

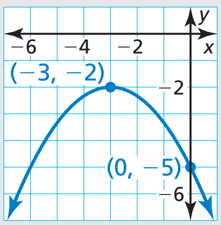
81. $(x^3 + x^2 - 4) + (10 + 3x^2)$

82. $(-3x^3 + x^2 - 12x) - (3x - 6x^2 - 9)$

In Exercises 83 and 84, write an equation of the parabola in vertex form or intercept form.

83. x-intercepts of -8 and 4; passes through $(2, -12)$

84.



In Exercises 85–88, find the product.

85. $11x(-4x^2 + 3x + 8)$

86. $(7 - x)(x - 1)$

87. $(x + 2)(x - 2)$

88. $(3x + 5)^2$

In Exercises 89 and 90, solve the equation using any method. Explain your choice of method.

89. $2x^2 - 15 = 0$

90. $3x^2 - x - 2 = 0$

91. Write the sentence as an absolute value inequality. Then solve the inequality.

A number is more than 9 units from 3.

92. MODELING REAL LIFE The table shows the donations made by 12 people on a fundraising site.

Donations (dollars)			
25	20	25	10
50	15	100	25
50	30	25	75

a. Make a box-and-whisker plot that represents the data. Describe the shape of the distribution.

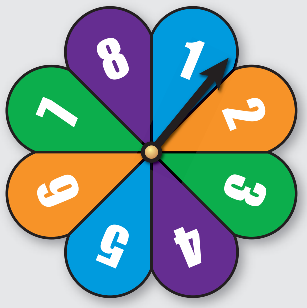
b. Does the data set contain any outliers?

In Exercises 93 and 94, find the minimum or maximum value of the function. Find the domain and range of the function, and when the function is increasing and decreasing.

93. $y = -x^2 - 4x + 6$

94. $y = \frac{1}{2}x^2 - 3x - 2$

95. What is the theoretical probability of spinning a multiple of 4 on the spinner?



96. MP REASONING The equation of a parabola is of the form $x = \frac{1}{4p}y^2$, and the parabola opens left. What can you conclude about the value of p?