71. $(g \circ f)(2c)$

72. $(f \circ g)(3k)$

73. $(g \circ h)(k + 1)$

74. $(h \circ g)(k - 1)$

In Exercises 75 to 78, show that $(f \circ g)(x) = (g \circ f)(x)$.

75. $f(x) = 2x + 3; \quad g(x) = 5x + 12$

76. $f(x) = 4x - 2; \quad g(x) = 7x - 4$

77. $f(x) = \dfrac{6x}{x - 1}; \quad g(x) = \dfrac{5x}{x - 2}$

78. $f(x) = \dfrac{5x}{x + 3}; \quad g(x) = -\dfrac{2x}{x - 4}$

In Exercises 79 to 82, show that

$$(g \circ f)(x) = x \quad \text{and} \quad (f \circ g)(x) = x$$

79. $f(x) = 2x + 3, \quad g(x) = \dfrac{x - 3}{2}$

80. $f(x) = 4x - 5, \quad g(x) = \dfrac{x + 5}{4}$

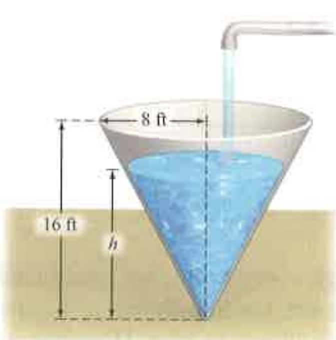
81. $f(x) = \dfrac{4}{x + 1}, \quad g(x) = \dfrac{4 - x}{x}$

82. $f(x) = \dfrac{2}{1 - x}, \quad g(x) = \dfrac{x - 2}{x}$

83. **Conversion Functions** The function $F(x) = \dfrac{x}{12}$ converts x inches to feet. The function $Y(x) = \dfrac{x}{3}$ converts x feet to yards. Explain the meaning of $(Y \circ F)(x)$.

84. **Conversion Functions** The function $F(x) = 3x$ converts x yards to feet. The function $I(x) = 12x$ converts x feet to inches. Explain the meaning of $(I \circ F)(x)$.

85. **Water Tank** A water tank has the shape of a right circular cone with height 16 feet and radius 8 feet. Water is running into the tank so that the radius r (in feet) of the surface of the water is given by $r = 1.5t$, where t is the time (in minutes) that the water has been running. See the accompanying diagram.



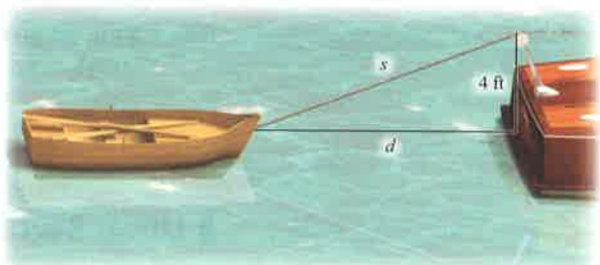
a. The area A of the surface of the water is $A = \pi r^2$. Find $A(t)$ and use it to determine the area of the surface of the water when $t = 2$ minutes.

b. The volume V of the water is given by $V = \dfrac{1}{3}\pi r^2 h$. Find $V(t)$ and use it to determine the volume of the water when $t = 3$ minutes. (*Hint:* The height of the water in the cone is always twice the radius of the surface of the water.)

86. **Scaling a Rectangle** Rework Example 7 of this section with the scaling as follows: The upper right corner of the original rectangle is pulled to the *left* at 0.5 inch per second and downward at 0.2 inch per second.

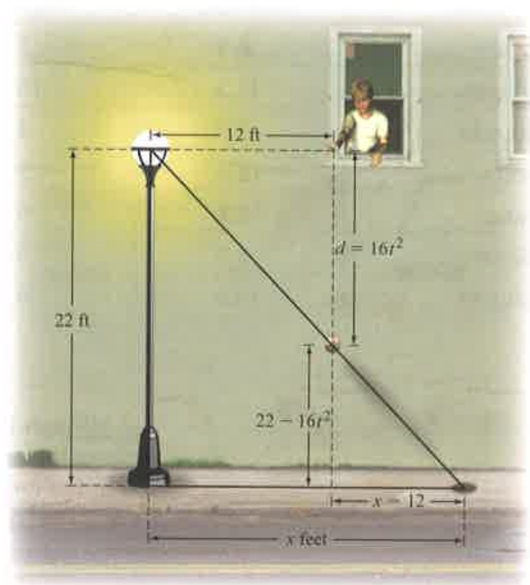
Enrichment Exercises

87. **Towing a Boat** A boat is towed by a rope that runs through a pulley that is 4 feet above the point where the rope is tied to the boat. The length (in feet) of the rope from the boat to the pulley is given by $s = 48 - t$, where t is the time in seconds that the boat has been in tow. The horizontal distance from the pulley to the boat is d.



a. Find $d(t)$. **b.** Evaluate $s(35)$ and $d(35)$.

88. **Shadow Position** The light from a lamppost casts a shadow from a ball that was dropped from a height of 22 feet above the ground, as shown in the diagram. The distance d, in feet, the ball has dropped t seconds after it is released is given by $d = 16t^2$. Find the distance x, in feet, of the shadow from the base of the lamppost as a function of time t.



SECTION 2.7

Modeling Data Using Regression

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A14.

PS1. Find the slope and the y-intercept of the graph of $y = -\frac{x}{3} + 4$. [2.3]

PS2. Find the slope and the y-intercept of the graph of $3x - 4y = 12$. [2.3]

PS3. Find the equation of the line that has a slope of -0.45 and a y-intercept of $(0, 2.3)$. [2.3]

PS4. Find the equation of the line that passes through the point $P(3, -4)$ and has a slope of $-\frac{2}{3}$. [2.3]

PS5. If $f(x) = 3x^2 + 4x - 1$, find $f(2)$. [2.2]

PS6. You are given $P_1(2, -1)$ and $P_2(4, 14)$. If $f(x) = x^2 - 3$, find $|f(x_1) - y_1| + |f(x_2) - y_2|$. [2.2]

Linear Regression Models

The data in the table given below show the population of selected states and the number of professional sports teams (Major League Baseball, National Football League, National Basketball Association, Women's National Basketball Association, National Hockey League) in those states. A scatter plot of the data is shown in Figure 2.103 on page 237.

Number of Professional Sports Teams for Selected States

State	Population (millions)	Number of Teams	State	Population (millions)	Number of Teams
Arizona	6.4	5	Minnesota	5.3	5
California	37.3	16	New Jersey	8.8	3
Colorado	5.0	4	New York	19.4	9
Florida	18.8	9	North Carolina	9.5	3
Illinois	12.8	6	Ohio	11.5	7
Indiana	6.5	3	Pennsylvania	12.7	7
Michigan	10.0	4	Texas	25.1	10

Although there is no one line that passes through every point, we could find an approximate linear model for these data. For instance, the line shown in Figure 2.104 in red approximates the data better than the line shown in blue. However, as Figure 2.105 shows, there are many other lines we could draw that seem to approximate the data.

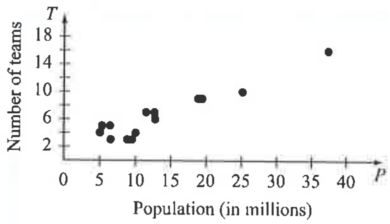


Figure 2.103

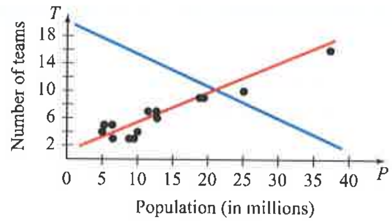


Figure 2.104

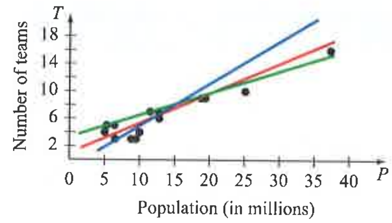


Figure 2.105

To find the line that "best" approximates the data, **regression analysis** is used. This type of analysis produces the linear function whose graph is called the **line of best fit** or the **least-squares regression line.**[1]

Definition of the Least-Squares Regression Line

The **least-squares regression line** is the line that minimizes the sum of the squares of the vertical deviations of all data points from the line.

To help understand this definition, consider the data set

$$S = \{(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)\}$$

as shown in Figure 2.106. As we will show later, the least-squares regression line for this data set is $y = 1.1x + 0.5$, also shown in Figure 2.106. If we evaluate this linear function at the x-coordinates of the data set S, we obtain the set of ordered pairs $T = \{(1, 1.6), (2, 2.7), (3, 3.8), (4, 4.9), (5, 6)\}$. The vertical deviations are the differences between the y-coordinates in S and the y-coordinates in T. From the definition, we must calculate the sum of the squares of these deviations.

$$(2 - 1.6)^2 + (3 - 2.7)^2 + (3 - 3.8)^2 + (4 - 4.9)^2 + (7 - 6)^2 = 2.7$$

Because $y = 1.1x + 0.5$ is the least-squares regression line, for no other line is the sum of the squares of the deviations less than 2.7. For instance, if we consider the equation $y = 1.25x + 0.75$, which is the equation of the line through the two points $P_1(1, 2)$ and $P_2(5, 7)$ of the data set, the sum of the squared deviations is larger than 2.7. See Figure 2.107.

$$(2 - 2)^2 + (3 - 3.25)^2 + (3 - 4.5)^2 + (4 - 5.75)^2 + (7 - 7)^2 = 5.375$$

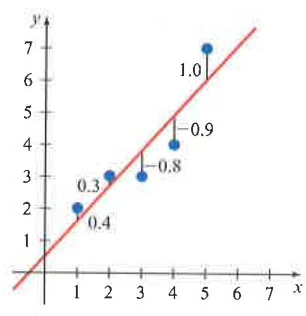


Figure 2.106

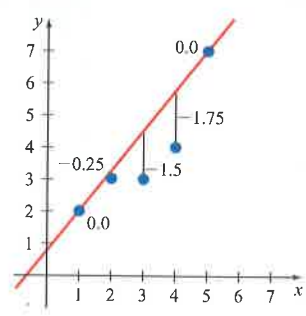


Figure 2.107

[1]The least-squares regression line is also called the *least-squares line* or the *regression line.*

Integrating Technology

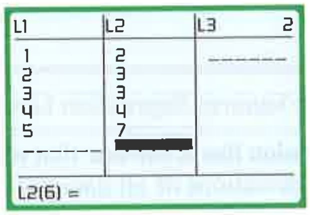
The equations used to calculate a regression line are somewhat cumbersome. Fortunately, these equations are preprogrammed into most graphing calculators. We will now illustrate the technique for a TI-83/TI-83 Plus/TI-84 Plus calculator using data set S given on page 237.

First ensure that DiagnosticOn is enabled. This is accomplished using the following keystrokes.

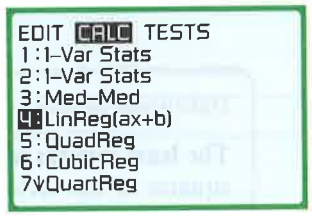
[2ND] CATALOG (Scroll to DiagnosticOn) [ENTER] [ENTER]

Press [STAT]. Select EDIT. Press [ENTER].

If necessary, delete any data in L1 and L2. Enter the given data.

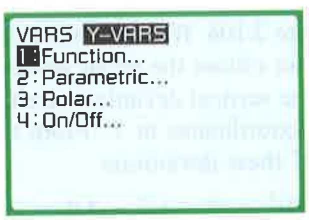


Press [STAT]. Select CALC. Select 4, LinReg(ax+b). Press [ENTER].

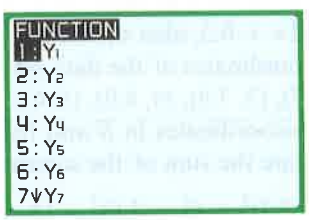


If your data are in L1 and L2, scroll to Store RegEQ.

Press [VARS]. Select Y-VARS. Press [ENTER].



Select 1. Press [ENTER].

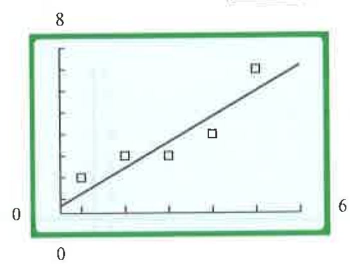


Scroll to Calculate and press [ENTER]. From the screen below, the equation of the regression line is $y = 1.1x + 0.5$.

If you need a graph of the regression line, continue as follows. Press [2nd] [STAT PLOT].

Press [ENTER]. Move the cursor to On.

Press [2nd] QUIT. This will return you to the home screen. Use WINDOW to adjust the axes for the data. Press [GRAPH].



Remember to return to [STAT PLOT] and select Off to resume normal equation graphing.

If you used the keystrokes we have shown here, the regression line will be stored in Y1. This is helpful if you wish to graph the regression line. However, if it is not necessary to graph the regression line, then at step 4, scroll to Calculate and press [ENTER]. The result will be the screen showing the results of the regression calculations.

EXAMPLE 1 Find a Regression Equation

Find the regression equation for the data on the population of a state and the number of professional sports teams in that state given on page 236. How many sports teams are predicted for Indiana, whose population is approximately 6.5 million? Round to the nearest whole number.

Solution

Using your calculator, enter the data from the table. Then have the calculator produce the values for the regression equation. Your results should be similar to those shown in Figure 2.108. The equation of the regression line is

$$y = 0.3794402856x + 1.374845857$$

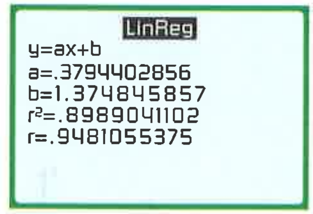


Figure 2.108

To find the number of professional sports teams the regression equation predicts for Indiana, evaluate the regression equation for $x = 6.5$.

$$\begin{aligned} y &= 0.3794402856x + 1.374845857 \\ &= 0.3794402856(6.5) + 1.374845857 \\ &\approx 3.841207713 \end{aligned}$$

The equation predicts that Indiana should have four sports teams.

▶ Try Exercise 18, page 243

Integrating Technology

If you followed the steps we gave on page 238 and stored the regression equation from Example 1 in Y1, then you can evaluate the regression equation using the following keystrokes:

[VARS] ▶ [ENTER] [ENTER] [(] 6.5 [)] [ENTER]

Correlation Coefficient

The scatter plot of a state's population and the corresponding number of professional sports teams is shown in Figure 2.109, along with the graph of the regression line. Note that the slope of the regression line is positive. This indicates that as a state's population increases, the number of teams increases. Note also that for these data the value of r on the regression calculation screen was positive, $r \approx 0.9481$.

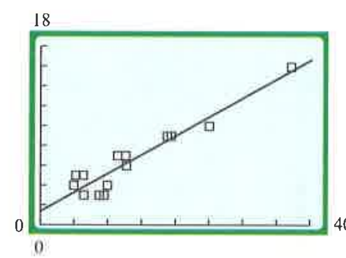


Figure 2.109

Now consider the data in the next table, which shows the trade-in value of a 2010 Corvette for various odometer readings. The graph in Figure 2.110 shows the scatter plot and the regression line.

Trade-in Value of 2010 Corvette Coupe, June 2012

Odometer Reading (thousands of mi)	Trade-in Value ($)
25	72,950
30	72,390
40	71,520
45	71,160
55	71,080

Source: Edmonds.com, June 2012.

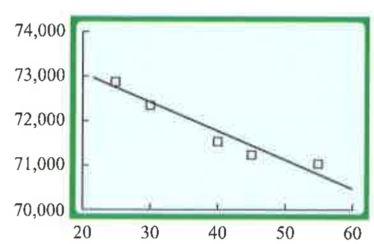


Figure 2.110

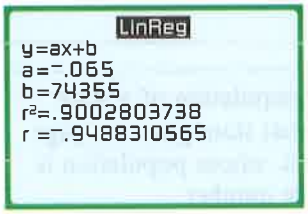
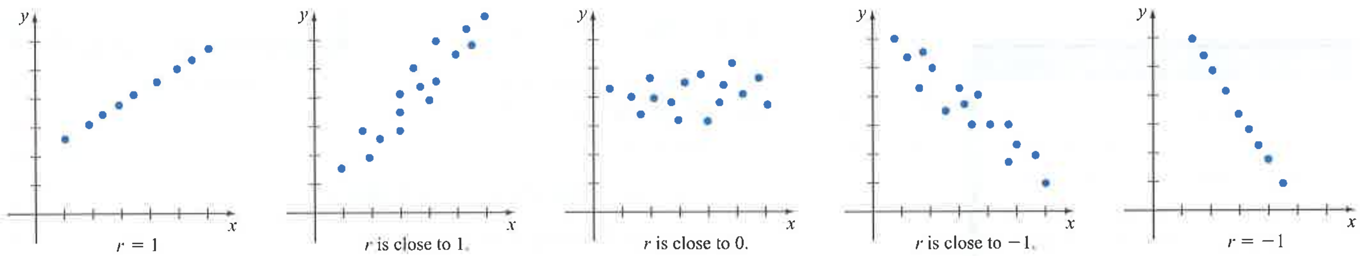


Figure 2.111

In this case, the slope of the regression line is negative. This means that as the odometer reading increases, the trade-in value of the car decreases. Note also that the value of r is negative, with $r \approx -0.9488$. See Figure 2.111.

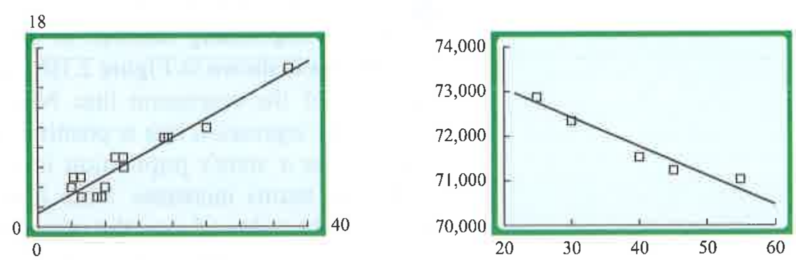
Linear Correlation Coefficient

The **linear correlation coefficient** r is a measure of how close the points of a data set can be modeled by a straight line. If $r = -1$, then the points of the data set can be modeled *exactly* by a straight line with negative slope. If $r = 1$, then the data set can be modeled *exactly* by a straight line with positive slope. For all data sets, $-1 \leq r \leq 1$.



If $r \neq 1$ or $r \neq -1$, then the data set *cannot* be modeled exactly by a straight line. The further the value of r is from 1 or -1 (or, in other words, the closer the value of r to zero), the more the ordered pairs of the data set deviate from a straight line.

The graphs below show the points of the data sets and the graphs of the regression lines for the state population and sports teams data and the odometer reading and trade-in data. Note the values of r and the closeness of the data points to the regression lines.



$r \approx 0.9481$ $r \approx -0.9488$

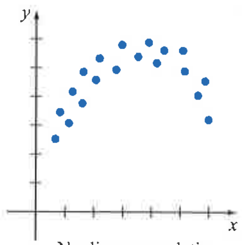
A researcher calculates a regression line to determine the relationship between two variables. The researcher wants to know whether a change in one variable produces a predictable change in the second variable. The value of r^2 tells the researcher the extent of that relationship.

Coefficient of Determination

The **coefficient of determination** r^2 measures the proportion of the variation in the dependent variable that is explained by the regression equation.

For the population and sports team data, $r^2 \approx 0.8989$. This means that approximately 90% of the total variation in the dependent variable (number of teams) can be attributed to the state population. This also means that population alone does not predict with certainty the number of sports teams. Other factors, such as climate, are involved in the number of sports teams.

Question • What is the coefficient of determination for the odometer reading and trade-in value data (see page 239), and what is its significance?



Nonlinear correlation between x and y

Quadratic Regression Models

To this point our focus has been on *linear* regression equations. However, there may be a nonlinear relationship between two quantities. The scatter plot to the left suggests that a quadratic function might be a better model of the data than a linear model. As we proceed through this text, various function models will be discussed.

EXAMPLE 2 Find a Quadratic Regression Model

The height h, in feet, of a basketball d feet from a player after the basketball has been released toward the basket is given in the table below. Find a regression model for these data.

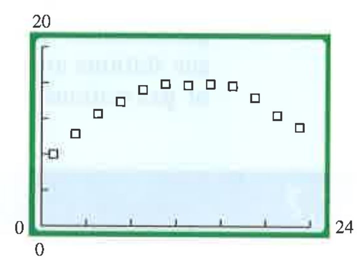
Distance (in feet)	1	3	5	7	9	11	13	15	17	19	21	23
Height (in feet)	8	10.3	12.6	13.9	15	15.9	15.8	15.9	15.7	14.4	12.4	11.1

Note

An alternate method for finding a regression equation for a data set using WolframAlpha is given in the **Exploring Concepts with Technology** on page 247.

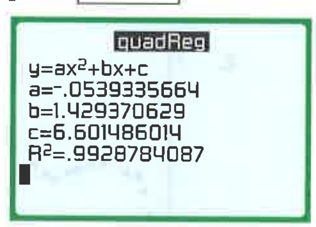
Solution

1. **Construct a scatter plot** for these data. Enter the data into your calculator as explained on page 238.



From the scatter plot, it appears that there is a nonlinear relationship between the variables.

2. Find the regression equation using QuadReg in the STAT CALC menu. For a TI-83/TI-83 Plus/T1-84 Plus calculator, press STAT ► 5 ENTER. Scroll to Calculate and press ENTER.



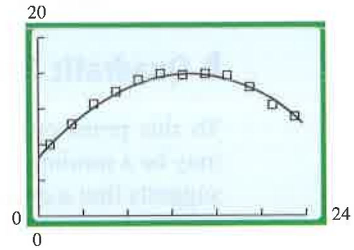
Note

For nonlinear regression calculations, the value of r is not shown on a TI-83/T1-83 Plus/T1-84 Plus graphing calculator. In such cases, the coefficient of determination is used to determine how well the data fit the model.

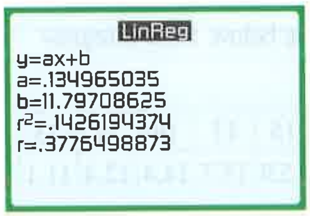
3. **Examine the coefficient of determination.** The coefficient of determination is approximately 0.993. Because this number is fairly close to 1, the regression

Answer • $r^2 \approx 0.90028$. This means that approximately 90% of the total variation in trade-in value can be attributed to the odometer reading.

equation $y = -0.0539335664x^2 + 1.429370629x + 6.601486014$ provides a good model of the data. The following calculator screen shows the scatter diagram and the parabola that is the graph of the regression equation.



Try Exercise 34, page 246



In Example 2, we could have calculated the *linear* regression model for the data. The results are shown at the left. Note that the coefficient of determination for this calculation is approximately 0.419. Because this number is less than the coefficient of determination for the quadratic model, we choose a quadratic model of the data rather than a linear model.

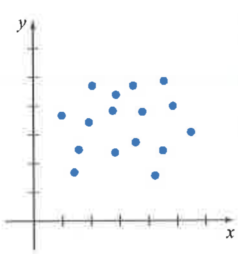
A final note: The regression line equation does not *prove* that the changes in the dependent variable are *caused* by the independent variable. For instance, suppose various cities throughout the United States were randomly selected and the numbers of gas stations (independent variable) and restaurants (dependent variable) were recorded in a table. If we calculated the regression equation for these data, we would find that r would be close to 1. However, this does not mean that gas stations *cause* restaurants to be built. The primary cause is that there are fewer gas stations and restaurants in cities with small populations and greater numbers of gas stations and restaurants in cities with large populations.

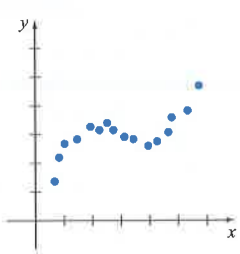
EXERCISE SET 2.7

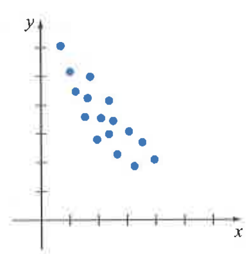
Use a graphing calculator for this Exercise Set.

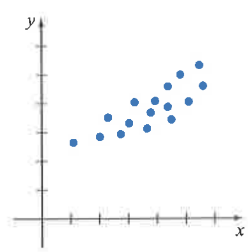
Concept Check

In Exercises 1 to 4, determine whether the scatter plot suggests a linear relationship between x and y, a nonlinear relationship between x and y, or no relationship between x and y.

1. 

2. 

3. 

4. 

In Exercises 5 and 6, determine for which scatter plot, A or B, the coefficient of determination is closer to 1.

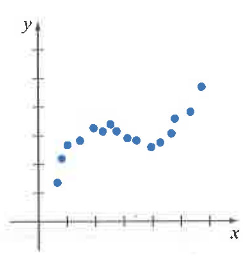
5. 

Figure A

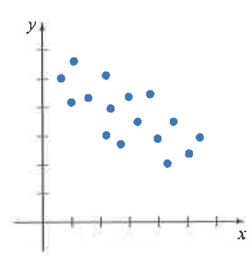


Figure B

Indicates Try It Exercises

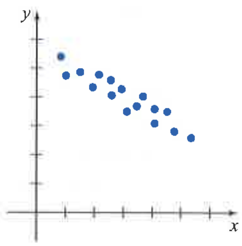
6. 

Figure A

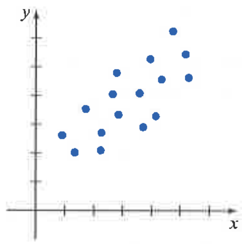


Figure B

In Exercises 7 to 12, find the linear regression equation for the given set.

7. {(2, 6), (3, 6), (4, 8), (6, 11), (8, 18)}

8. {(2, −3), (3, −4), (4, −9), (5, −10), (7, −12)}

9. {(−3, 11.8), (−1, 9.5), (0, 8.6), (2, 8.7), (5, 5.4)}

10. {(−7, −11.7), (−5, −9.8), (−3, −8.1), (1, −5.9), (2, −5.7)}

11. {(1.3, −4.1), (2.6, −0.9), (5.4, 1.2), (6.2, 7.6), (7.5, 10.5)}

12. {(−1.5, 8.1), (−0.5, 6.2), (3.0, −2.3), (5.4, −7.1), (6.1, −9.6)}

In Exercises 13 to 16, find a quadratic model for the given data.

13. {(1, −1), (2, 1), (4, 8), (5, 14), (6, 25)}

14. {(−2, −5), (−1, 0), (0, 1), (1, 4), (2, 4)}

15. {(1.5, −2.2), (2.2, −4.8), (3.4, −11.2), (5.1, −20.6), (6.3, −28.7)}

16. {(−2, −1), (−1, −3.1), (0, −2.9), (1, 0.8), (2, 6.8), (3, 15.9)}

17. **Archeology** The data below show the length of the humerus and the total wingspan, in centimeters, of several pterosaurs, which are extinct flying reptiles of the order Pterosauria. (*Source:* Southwest Educational Development Laboratory.)

Pterosaur Data

Humerus (cm)	Wingspan (cm)
24	600
32	750
22	430
17	370
13	270
4.4	68
3.2	53
1.5	24

Humerus (cm)	Wingspan (cm)
20	500
27	570
15	300
15	310
9	240
4.4	55
2.9	50

a. Compute the linear regression equation for these data.

b. On the basis of this model, what is the projected wingspan of the pterosaur *Quetzalcoatlus northropi,* which is thought to have been the largest of the prehistoric birds, if its humerus is 54 centimeters? Round to the nearest centimeter.

18. **Sports** The data in the table below show the distance, in feet, a ball travels for various bat speeds, in miles per hour.

Bat Speed (mph)	Distance (ft)
40	200
45	213
50	242
60	275
70	297
75	326
80	335

a. Find the linear regression equation for these data.

b. Using the regression model, what is the expected distance a ball will travel when the bat speed is 58 miles per hour? Round to the nearest foot.

19. **Sports** The table below shows the number of strokes per minute that a rower makes and the speed of the boat in meters per second.

Strokes per Min	Speed (m/s)
30	4.1
31	4.2
33	4.4
34	4.5
36	4.7
39	5.1

a. Find the linear regression equation for these data.

b. Using the regression model, what is the expected speed of the boat when the rowing rate is 32 strokes per minute? Round to the nearest tenth of a meter per second.

20. **Biology** A medical researcher wants to determine the effect of pH (a measure of alkalinity or acidity, with pure water having a pH of 7) on the growth of a bacteria culture. The following table gives the measurements of different cultures, in thousands of bacteria, after 8 hours.

pH	Number of Bacteria (thousands)
4	116
5	120
6	131
7	136
8	141
9	151
10	148
11	163

a. Find the linear regression equation for these data.

b. Using the regression model, what is the expected number of bacteria when the pH is 7.5? Round to the nearest thousand bacteria.

21. **Health** The body mass index (BMI) of a person is a measure of the person's ideal body weight. The table below shows the BMI for different weights for a person 5 feet 6 inches tall.

BMI Data for Person 5′ 6″ Tall

Weight (lb)	BMI	Weight (lb)	BMI
110	17	160	25
120	19	170	27
125	20	180	29
135	21	190	30
140	22	200	32
145	23	205	33
150	24	215	34

a. Compute the linear regression equation for these data.

b. On the basis of the model, what is the estimated BMI for a person 5 feet 6 inches tall whose weight is 158 pounds?

22. **Health** The BMI (see Exercise 21) of a person depends on height, as well as weight. The table below shows the changes in BMI for a 150-pound person as height, in inches, changes.

BMI Data for 150-Pound Person

Height (in.)	BMI	Height (in.)	BMI
60	29	71	21
62	27	72	20
64	25	73	19
66	24	74	19
67	23	75	18
68	23	76	18
70	21		

a. Compute the linear regression equation for these data.

b. On the basis of the model, what is the estimated BMI for a 150-pound person who is 5 feet 8 inches tall?

23. **Industrial Engineering** Permanent-magnet direct-current motors are used in a variety of industrial applications. For these motors to be effective, there must be a strong linear relationship between the current (in amps, A) supplied to the motor and the resulting torque (in newton-centimeters, N-cm) produced by the motor. A randomly selected motor is chosen from a production line and tested, with the following results.

Direct-Current Motor Data at 12 Volts

Current (A)	Torque (N-cm)	Current (A)	Torque (N-cm)
7.3	9.4	8.5	8.6
11.9	2.8	7.9	4.3
5.6	5.6	14.5	9.5
14.2	4.9	12.7	8.3
7.9	7.0	10.6	4.7

Based on the data in this table, is the chosen motor effective? Explain.

24. **Health Sciences** The average remaining lifetime for men in the United States is given in the following table. (*Source:* U.S. National Center for Health Statistics.)

Average Remaining Lifetime for Men

Age	Years	Age	Years
0	75.5	65	17.2
15	61.3	75	10.6
35	42.6		

Based on the data in this table, is there a strong linear correlation between a man's age and the average remaining lifetime for that man? Explain.

25. **Health Sciences** The average remaining lifetime for women in the United States is given in the following table. (*Source:* U.S. National Center for Health Statistics.)

Average Remaining Lifetime for Women

Age	Years	Age	Years
0	80.5	65	19.9
15	66.1	75	12.5
35	46.7		

a. Based on the data in this table, is there a strong linear correlation between a woman's age and the average remaining lifetime for that woman?

b. Compute the linear regression equation for these data.

c. On the basis of the model, what is the estimated remaining lifetime of a woman of age 25? Round to the nearest number of years.

26. **Biology** The table below gives the body lengths, in centimeters, and the highest observed flying speeds, in meters per second, of various animals.

Species	Length (cm)	Flying Speed (m/s)
Horsefly	1.3	6.6
Hummingbird	8.1	11.2
Dragonfly	8.5	10.0
Willow warbler	11.0	12.0
Common pintail	56.0	22.8

Use a linear regression model to estimate the flying speed of a whimbrel whose length is 41 centimeters. Round to the nearest meter per second. (*Source:* Based on data from Leiva, *Algebra 2: Explorations and Applications,* p. 76, McDougal Littell, Boston; copyright 1997.)

27. **Health** The table below shows the number of calories burned in 1 hour when running at various speeds.

Running Speed (mph)	Calories Burned
10	1126
10.9	1267
5	563
5.2	633
6	704
6.7	774
7	809
8	950
8.6	985
9	1056
7.5	880

a. Are the data positively or negatively correlated?

b. Use a linear regression model to estimate the number of calories a person who runs at 9.5 mph for 1 hour will burn. Round to the nearest number of calories.

28. **Traffic Safety** A traffic safety institute measured the braking distance, in feet, of a car traveling at certain speeds in miles per hour. The data from one of those tests are given in the following table.

Speed (mph)	Breaking Distance (ft)
20	23.9
30	33.7
40	40.0
50	41.7
60	46.8
70	48.9
80	49.0

a. Find the quadratic regression equation for these data.

b. Using the regression model, what is the expected braking distance when a car is traveling at 65 mph? Round to the nearest tenth of a foot.

29. **Biology** The survival of certain larvae after hatching depends on the temperature (in degrees Celsius) of the surrounding environment. The following table shows the number of larvae that survive at various temperatures. Find a quadratic model for these data.

Larvae Surviving for Various Temperatures

Temp. (°C)	Number Surviving	Temp. (°C)	Number Surviving
20	40	26	68
21	47	27	67
22	52	28	64
23	61	29	62
24	64	30	61
25	64		

30. **Meteorology** The table below, based on data from the National Weather Service, gives the mean temperature, in degrees Celsius, for each of the months shown for Chicago, Illinois, in 2011.

Month, January corresponds to 1, x	Temperature, y	Month, January corresponds to 1, x	Temperature, y
1	–5.2	7	25.5
3	3.1	9	17.3
5	13.9	11	7.2

Find the quadratic regression model for these data. Use the model to predict the mean temperature for August 2011. Round to the nearest tenth.

31. Ethanol Fuel Consumption The table below shows the ethanol consumption, in thousands of gallons, for selected years from 2005 to 2009.

Years Since 2005, x	Ethanol Consumption, thousands of gallons, y
0	52,881
2	75,126
3	86,756
4	98,907

Find the quadratic regression model for these data. Use the model to predict what ethanol consumption was in 2006. Round to the nearest thousand gallons. (*Source: Statistical Abstract of the United States, 2012*, Table 1097, page 688.)

32. Organic Farmland The table below shows the number of acres, in thousands, of certified organic farmland.

Years Since 2000, x	Acres, in thousands, y	Years Since 2000, x	Acres, in thousands, y
0	6592	5	8493
1	6949	6	9469
2	7323	7	11,352
3	8035	8	12,941
4	8021		

Find the quadratic regression model for these data. (*Source: Statistical Abstract of the United States, 2012*, Table 832, page 539.)

33. Automotive Engineering The fuel efficiency, in miles per gallon, for a certain midsize car at various speeds, in miles per hour, is given in the table below.

Fuel Efficiency of a Midsize Car

mph	mpg	mph	mpg
25	29	55	31
30	32	60	28
35	33	65	24
40	35	70	19
45	34	75	17
50	33		

a. Find a quadratic model for these data.

b. Use the model to predict the fuel efficiency of this car when it is traveling at a speed of 50 mph. Round to the nearest tenth mile per gallon.

34. Biology The data in the following table show the oxygen consumption, in milliliters per minute, of a bird flying level at various speeds, in kilometers per hour.

Oxygen Consumption

Speed (km/h)	Consumption (ml/min)
20	32
25	27
28	22
35	21
42	26
50	34

a. Find a quadratic model for these data.

b. Use the model to determine the speed at which the bird has minimum oxygen consumption.

Enrichment Exercises

35. Physics Galileo (1564–1642) studied acceleration due to gravity by allowing balls of various weights to roll down an incline. This allowed him to time the descent of a ball more accurately than by just dropping the ball. The data in the following table show some results of such an experiment using balls of different masses. Time t is measured in seconds; distance s is measured in centimeters.

Distance Traveled for Balls of Various Weights

5-lb Ball		10-lb Ball		15-lb Ball	
t	s	t	s	t	s
2	2	3	5	3	5
4	10	6	22	5	15
6	22	9	49	7	30
8	39	12	87	9	49
10	61	15	137	11	75
12	86	18	197	13	103
14	120			15	137
16	156				

a. Find a quadratic model for each of the balls.

b. On the basis of a similar experiment, Galileo concluded that, if air resistance is excluded, all falling objects fall with the same acceleration. Explain how one could make such a conclusion from the regression equations.

36. Astronomy In 1929, Edwin Hubble published a paper that revolutionized astronomy ("A Relationship Between Distance and Radial Velocity Among Extra-Galactic Nebulae," *Proceedings of the National Academy of Science*, page 168). His paper dealt with the distance an extragalactic nebula was from the Milky Way galaxy and the nebula's velocity with respect to the Milky Way. The data are given in the following table. Distance is measured in megaparsecs (1 megaparsec equals 1.918×10^{19} miles), and velocity (called the *recession velocity*) is measured in kilometers per second. A negative velocity means the nebula is moving toward the Milky Way; a positive velocity means the nebula is moving away from the Milky Way.

Recession Velocities

Distance	Velocity
0.032	170
0.034	290
0.214	−130
0.263	−70
0.9	650
0.9	150
0.9	500
1.0	920
0.275	−185
0.275	−220
0.45	200
0.5	290
0.5	270
0.63	200
0.8	300
0.9	−30
1.1	450
1.1	500
1.4	500
1.7	960
2.0	500
2.0	850
2.0	800
2.0	1090

a. Find the linear regression model for these data.

b. On the basis of this model, what is the recession velocity of a nebula that is 1.5 megaparsecs from the Milky Way?

Scan the following QR code to access WolframAlpha on a mobile device.

www.wolframalpha.com

Note

WolframAlpha is not just an online computational engine. It is also a knowledge engine. This means that in addition to performing mathematical procedures, it can provide answers to many questions that pertain to factual information.

WolframAlpha is different from most online search engines in that it finds answers to questions by searching through its very large database of factual information, rather than searching the Web for sites that might have information related to your question.

Apple's voice-powered iPhone assistant Siri (Speech Interpretation and Recognition Interface) often relies on WolframAlpha to answer questions that require factual information.

Sources: Wikipedia and CNET.

Exploring Concepts with Technology

Use WolframAlpha to Determine Linear and Quadratic Regressions

The online computational knowledge engine WolframAlpha, available at www.wolframalpha.com, can be used to determine linear and quadratic regression functions for a data set. WolframAlpha was conceived by British scientist Stephen Wolfram and developed by Wolfram Research. WolframAlpha runs on computers, tablets, and smartphones, although entering the necessary input data on a smartphone is a tedious process.

To find the linear regression function for the data set

$$S = \{(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)\},$$

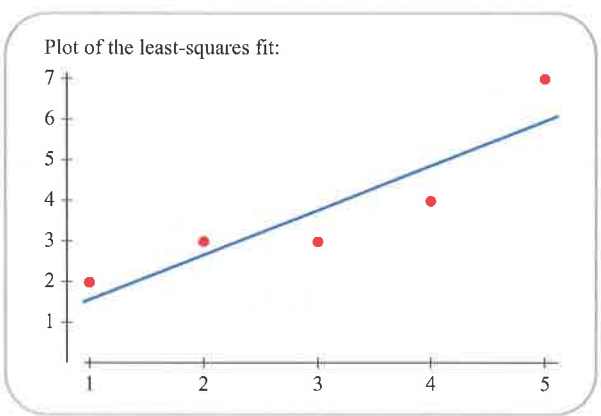
enter the following text into WolframAlpha's input field.

```
linear fit {(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)}
```

Click on the equal sign icon, at the far right of the input field, to display

$$P(x) = 1.1x + 0.5$$

as the linear regression function. A graph of the linear regression function and a scatter plot of the data are also provided, as shown below.



(continued)

To determine the quadratic regression function for a data set, type "quadratic fit" followed by the data set.

You can learn about other procedures that can be performed by WolframAlpha by clicking on the "Examples" link that appears under the yellow input field.

CHAPTER 2 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

2.1 Two-Dimensional Coordinate System and Graphs

Distance Formula The distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	See Example 1, page 154, and then try Exercise 2, page 252.
Midpoint Formula The coordinates of the midpoint of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.	See Example 1, page 154, and then try Exercise 4, page 252.
Graph of an Equation The graph of an equation in the two variables x and y is the graph of all ordered pairs that satisfy the equation.	See Examples 2 and 3, pages 155 and 156, and then try Exercise 7, page 252.
x-Intercepts and y-Intercepts If $(x_1, 0)$ satisfies an equation in two variables, then the point $P(x_1, 0)$ is an x-intercept of the graph of the equation. If $(0, y_1)$ satisfies an equation in two variables, then the point $P(0, y_1)$ is a y-intercept of the graph of the equation.	See Example 5, page 158, and then try Exercise 9, page 252.
Equation of a Circle The standard form of the equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.	See Examples 6 and 7, pages 160 and 161, and then try Exercises 14 and 16, page 252.

2.2 Introduction to Functions

Definition of a Function A function is a set of ordered pairs in which no two ordered pairs have the same first coordinate and different second coordinates.	See Example 1, page 166, and then try Exercises 18 and 20, page 252.
Evaluate a Function To evaluate a function, replace the independent variable with a number in the domain of the function and then simplify the resulting numerical expression.	See Example 2, page 166, and then try Exercise 22, page 252.
Piecewise-Defined Function A piecewise-defined function is represented by more than one expression.	See Example 3, page 167, and then try Exercise 23, page 252.
Domain and Range of a Function The domain of a function is the set of all first coordinates of the ordered pairs of the function. The range of a function is the set of all second coordinates of the ordered pairs of the function.	See Example 4, page 168, and then try Exercise 26, page 252. See Example 6, page 170, and then try Exercise 29, page 252.
Graph a Function The graph of a function is the graph of all ordered pairs of the function.	See Example 5, page 170, and then try Exercise 31, page 252.