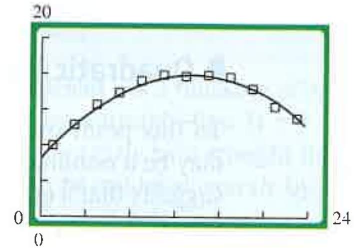
equation $y = -0.0539335664x^2 + 1.429370629x + 6.601486014$ provides a good model of the data. The following calculator screen shows the scatter diagram and the parabola that is the graph of the regression equation.



▶ Try Exercise 34, page 246

```
LinReg
y=ax+b
a=.134965035
b=11.79708625
r²=.1426194374
r=.3776498873
```

In Example 2, we could have calculated the *linear* regression model for the c The results are shown at the left. Note that the coefficient of determination for calculation is approximately 0.419. Because this number is less than the coeffic of determination for the quadratic model, we choose a quadratic model of the rather than a linear model.

A final note: The regression line equation does not *prove* that the chang the dependent variable are *caused* by the independent variable. For instance, pose various cities throughout the United States were randomly selected and numbers of gas stations (independent variable) and restaurants (dependent able) were recorded in a table. If we calculated the regression equation for t data, we would find that r would be close to 1. However, this does not mean gas stations *cause* restaurants to be built. The primary cause is that there are fe gas stations and restaurants in cities with small populations and greater num of gas stations and restaurants in cities with large populations.

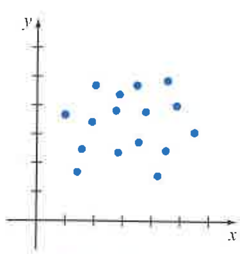
EXERCISE SET 2.7

Use a graphing calculator for this Exercise Set.

Concept Check

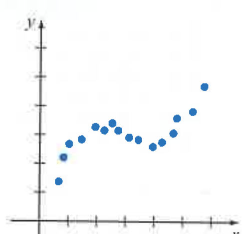
In Exercises 1 to 4, determine whether the scatter plot suggests a linear relationship between *x* and *y*, a nonlinear relationship between *x* and *y*, or no relationship between *x* and *y*.

1.



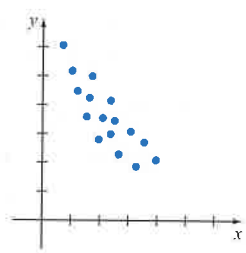
No relationship

2.



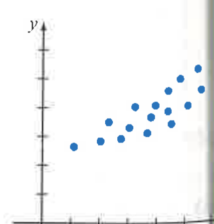
Nonlinear

3.



Linear

4.



Linear

In Exercises 5 and 6, determine for which scatter plot, A or B, the coefficient of determination is closer to 1.

5.

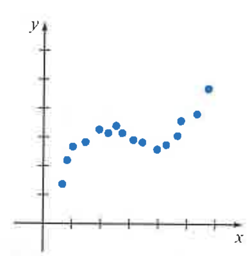


Figure A

Figure A

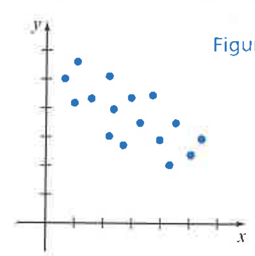


Figure B

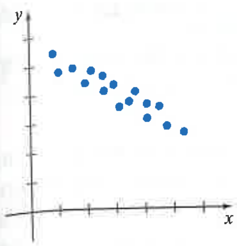
6. 

Figure A

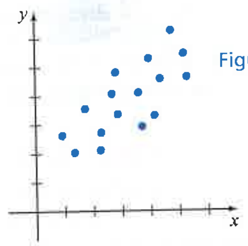


Figure B

Figure A

In Exercises 7 to 12, find the linear regression equation for the given set.

7. {(2, 6), (3, 6), (4, 8), (6, 11), (8, 18)}
$y = 2.00862069x + 0.5603448276$

8. {(2, −3), (3, −4), (4, −9), (5, −10), (7, −12)}
$y = -1.918918919x + 0.4594594595$

9. {(−3, 11.8), (−1, 9.5), (0, 8.6), (2, 8.7), (5, 5.4)}
$y = -0.7231182796x + 9.233870968$

10. {(−7, −11.7), (−5, −9.8), (−3, −8.1), (1, −5.9), (2, −5.7)}
$y = 0.6591216216x - 6.658108108$

11. {(1.3, −4.1), (2.6, −0.9), (5.4, 1.2), (6.2, 7.6), (7.5, 10.5)}
$y = 2.222641509x - 7.364150943$

12. {(−1.5, 8.1), (−0.5, 6.2), (3.0, −2.3), (5.4, −7.1), (6.1, −9.6)}
$y = -2.301587302x + 4.813968254$

In Exercises 13 to 16, find a quadratic model for the given data.

13. {(1, −1), (2, 1), (4, 8), (5, 14), (6, 25)}
$y = 1.095779221x^2 - 2.69642857x + 1.136363636$

14. {(−2, −5), (−1, 0), (0, 1), (1, 4), (2, 4)}
$y = -0.5714285714x^2 + 2.2x + 1.942857143$

15. {(1.5, −2.2), (2.2, −4.8), (3.4, −11.2), (5.1, −20.6), (6.3, −28.7)}
$y = -0.2987274717x^2 - 3.20998141x + 3.416463667$

16. {(−2, −1), (−1, −3.1), (0, −2.9), (1, 0.8), (2, 6.8), (3, 15.9)}
$y = 1.414285714x^2 + 1.954285714x - 2.705714286$

17. **Archeology** The data below show the length of the humerus and the total wingspan, in centimeters, of several pterosaurs, which are extinct flying reptiles of the order Pterosauria. (*Source:* Southwest Educational Development Laboratory.)

Pterosaur Data

Humerus (cm)	Wingspan (cm)	Humerus (cm)	Wingspan (cm)
24	600	20	500
32	750	27	570
22	430	15	300
17	370	15	310
13	270	9	240
4.4	68	4.4	55
3.2	53	2.9	50
1.5	24		

a. Compute the linear regression equation for these data.
$y = 23.55706665x - 24.4271215$

b. On the basis of this model, what is the projected wingspan of the pterosaur *Quetzalcoatlus northropi*, which is thought to have been the largest of the prehistoric birds, if its humerus is 54 centimeters? Round to the nearest centimeter. 1248 cm

18. **Sports** The data in the table below show the distance, in feet, a ball travels for various bat speeds, in miles per hour.

Bat Speed (mph)	Distance (ft)
40	200
45	213
50	242
60	275
70	297
75	326
80	335

a. Find the linear regression equation for these data.
$y = 3.410344828x + 65.09359606$

b. Using the regression model, what is the expected distance a ball will travel when the bat speed is 58 miles per hour? Round to the nearest foot. 263 ft

19. **Sports** The table below shows the number of strokes per minute that a rower makes and the speed of the boat in meters per second.

Strokes per Min	Speed (m/s)
30	4.1
31	4.2
33	4.4
34	4.5
36	4.7
39	5.1

a. Find the linear regression equation for these data.
$y = 0.1094224924x + 0.7978723404$

b. Using the regression model, what is the expected speed of the boat when the rowing rate is 32 strokes per minute? Round to the nearest tenth of a meter per second. 4.3 m/s

20. **Biology** A medical researcher wants to determine the effect of pH (a measure of alkalinity or acidity, with pure water having a pH of 7) on the growth of a bacteria culture. The following table gives the measurements of different cultures, in thousands of bacteria, after 8 hours.

pH	Number of Bacteria (thousands)
4	116
5	120
6	131
7	136
8	141
9	151
10	148
11	163

a. Find the linear regression equation for these data.
$y = 6.357142857x + 90.57142857$

b. Using the regression model, what is the expected number of bacteria when the pH is 7.5? Round to the nearest thousand bacteria. 138,000 bacteria

21. **Health** The body mass index (BMI) of a person is a measure of the person's ideal body weight. The table below shows the BMI for different weights for a person 5 feet 6 inches tall.

BMI Data for Person 5′ 6″ Tall

Weight (lb)	BMI	Weight (lb)	BMI
110	17	160	25
120	19	170	27
125	20	180	29
135	21	190	30
140	22	200	32
145	23	205	33
150	24	215	34

a. Compute the linear regression equation for these data.
$y = 0.1628623408x - 0.6875682232$

b. On the basis of the model, what is the estimated BMI for a person 5 feet 6 inches tall whose weight is 158 pounds? 25

22. **Health** The BMI (see Exercise 21) of a person depends on height, as well as weight. The table below shows the changes in BMI for a 150-pound person as height, in inches, changes.

BMI Data for 150-Pound Person

Height (in.)	BMI	Height (in.)	BMI
60	29	71	21
62	27	72	20
64	25	73	19
66	24	74	19
67	23	75	18
68	23	76	18
70	21		

a. Compute the linear regression equation for these data.
$y = -0.6800298805x + 69.05129482$

b. On the basis of the model, what is the estimated BMI for a 150-pound person who is 5 feet 8 inches tall? 23

23. **Industrial Engineering** Permanent-magnet direct-current motors are used in a variety of industrial applications. For these motors to be effective, there must be a strong linear relationship between the current (in amps, A) supplied to the motor and the resulting torque (in newton-centimeters, N-cm) produced by the motor. A randomly selected motor is chosen from a production line and tested, with the following results.

Direct-Current Motor Data at 12 Volts

Current (A)	Torque (N-cm)	Current (A)	Torque (N-cm)
7.3	9.4	8.5	8.6
11.9	2.8	7.9	4.3
5.6	5.6	14.5	9.5
14.2	4.9	12.7	8.3
7.9	7.0	10.6	4.7

Based on the data in this table, is the chosen motor effective? Explain.

No, because the linear correlation coefficient is close to 0.

24. **Health Sciences** The average remaining lifetime for men in the United States is given in the following table. (*Source:* U.S. National Center for Health Statistics.)

Average Remaining Lifetime for Men

Age	Years	Age	Years
0	75.5	65	17.2
15	61.3	75	10.6
35	42.6		

Based on the data in this table, is there a strong linear correlation between a man's age and the average remaining lifetime for that man? Explain.

Yes, because the linear correlation coefficient is close to −1.

25. **Health Sciences** The average remaining lifetime for women in the United States is given in the following table. (*Source:* U.S. National Center for Health Statistics.)

Average Remaining Lifetime for Women

Age	Years	Age	Years
0	80.5	65	19.9
15	66.1	75	12.5
35	46.7		

a. Based on the data in this table, is there a strong linear correlation between a woman's age and the average remaining lifetime for that woman?

Yes, there is a strong linear correlation.

b. Compute the linear regression equation for these data. $y = -0.911\overline{6}x + 79.78\overline{3}$

c. On the basis of the model, what is the estimated remaining lifetime of a woman of age 25? Round to the nearest number of years. 57 years

26. **Biology** The table below gives the body lengths, in centimeters, and the highest observed flying speeds, in meters per second, of various animals.

Species	Length (cm)	Flying Speed (m/s)
Horsefly	1.3	6.6
Hummingbird	8.1	11.2
Dragonfly	8.5	10.0
Willow warbler	11.0	12.0
Common pintail	56.0	22.8

Use a linear regression model to estimate the flying speed of a whimbrel whose length is 41 centimeters. Round to the nearest meter per second. (*Source:* Based on data from Leiva, *Algebra 2: Explorations and Applications,* p. 76, McDougal Littell, Boston; copyright 1997.) 19 m/s

27. **Health** The table below shows the number of calories burned in 1 hour when running at various speeds.

Running Speed (mph)	Calories Burned
10	1126
10.9	1267
5	563
5.2	633
6	704
6.7	774
7	809
8	950
8.6	985
9	1056
7.5	880

a. Are the data positively or negatively correlated? Positively

b. Use a linear regression model to estimate the number of calories a person who runs at 9.5 mph for 1 hour will burn. Round to the nearest number of calories. 1098 calories

28. **Traffic Safety** A traffic safety institute measured the braking distance, in feet, of a car traveling at certain speeds in miles per hour. The data from one of those tests are given in the following table.

Speed (mph)	Breaking Distance (ft)
20	23.9
30	33.7
40	40.0
50	41.7
60	46.8
70	48.9
80	49.0

a. Find the quadratic regression equation for these data. $y = -0.0074642857x^2 + 1.148214286x + 4.807142857$

b. Using the regression model, what is the expected braking distance when a car is traveling at 65 mph? Round to the nearest tenth of a foot. 47.9 ft

29. **Biology** The survival of certain larvae after hatching depends on the temperature (in degrees Celsius) of the surrounding environment. The following table shows the number of larvae that survive at various temperatures. Find a quadratic model for these data.

Larvae Surviving for Various Temperatures

Temp. (°C)	Number Surviving	Temp. (°C)	Number Surviving
20	40	26	68
21	47	27	67
22	52	28	64
23	61	29	62
24	64	30	61
25	64		

$y = -0.6328671329x^2 + 33.61608392x - 379.4405594$

30. **Meteorology** The table below, based on data from the National Weather Service, gives the mean temperature, in degrees Celsius, for each of the months shown for Chicago, Illinois, in 2011.

Month, January corresponds to 1, x	Temperature, y	Month, January corresponds to 1, x	Temperature, y
1	−5.2	7	25.5
3	3.1	9	17.3
5	13.9	11	7.2

Find the quadratic regression model for these data. Use the model to predict the mean temperature for August 2011. Round to the nearest tenth. $y = -0.75x^2 + 10.66x - 17.91$; 19.4 °C

31. **Ethanol Fuel Consumption** The table below shows the ethanol consumption, in thousands of gallons, for selected years from 2005 to 2009.

Years Since 2005, x	Ethanol Consumption, thousands of gallons, y
0	52,881
2	75,126
3	86,756
4	98,907

Find the quadratic regression model for these data. Use the model to predict what ethanol consumption was in 2006. Round to the nearest thousand gallons. (*Source: Statistical Abstract of the United States, 2012*, Table 1097, page 688.) $y = 198.2272727x^2 - 10{,}708.60909x + 52{,}885.98182$; 63,793 thousand gallons

32. **Organic Farmland** The table below shows the number of acres, in thousands, of certified organic farmland.

Years Since 2000, x	Acres, in thousands, y	Years Since 2000, x	Acres, in thousands, y
0	6592	5	8493
1	6949	6	9469
2	7323	7	11,352
3	8035	8	12,941
4	8021		

Find the quadratic regression model for these data. (*Source: Statistical Abstract of the United States, 2012*, Table 832, page 539.) $y = 107.4664502x^2 - 137.1482684x + 6909.909091$

33. **Automotive Engineering** The fuel efficiency, in miles per gallon, for a certain midsize car at various speeds, in miles per hour, is given in the table below.

Fuel Efficiency of a Midsize Car

mph	mpg	mph	mpg
25	29	55	31
30	32	60	28
35	33	65	24
40	35	70	19
45	34	75	17
50	33		

a. Find a quadratic model for these data. $y = -0.0165034965x^2 + 1.366713287x + 5.685314685$

b. Use the model to predict the fuel efficiency of this car when it is traveling at a speed of 50 mph. Round to the nearest tenth mile per gallon. 32.8 mpg

34. **Biology** The data in the following table show the oxygen consumption, in milliliters per minute, of a bird flying level at various speeds, in kilometers per hour.

Oxygen Consumption

Speed (km/h)	Consumption (ml/min)
20	32
25	27
28	22
35	21
42	26
50	34

a. Find a quadratic model for these data. $y = 0.05208x^2 - 3.56026x + 82.32999$

b. Use the model to determine the speed at which the bird has minimum oxygen consumption. ≈34 km/h

Enrichment Exercises

35. **Physics** Galileo (1564–1642) studied acceleration due to gravity by allowing balls of various weights to roll down an incline. This allowed him to time the descent of a ball more accurately than by just dropping the ball. The data in the following table show some results of such an experiment using balls of different masses. Time t is measured in seconds; distance s is measured in centimeters.

Distance Traveled for Balls of Various Weights

5-lb Ball		10-lb Ball		15-lb Ball	
t	s	t	s	t	s
2	2	3	5	3	5
4	10	6	22	5	15
6	22	9	49	7	30
8	39	12	87	9	49
10	61	15	137	11	75
12	86	18	197	13	103
14	120			15	137
16	156				

a. Find a quadratic model for each of the balls. Answer on the next page.

b. On the basis of a similar experiment, Galileo concluded that, if air resistance is excluded, all falling objects fall with the same acceleration. Explain how one could make such a conclusion from the regression equations. All the regression equations are approximately the same. Therefore, the equations of motion of the three masses are the same.

36. **Astronomy** In 1929, Edwin Hubble published a paper that revolutionized astronomy ("A Relationship Between Distance and Radial Velocity Among Extra-Galactic Nebulae," *Proceedings of the National Academy of Science*, page 168). His paper dealt with the distance an extragalactic nebula was from the Milky Way galaxy and the nebula's velocity with respect to the Milky Way. The data are given in the following table. Distance is measured in megaparsecs (1 megaparsec equals 1.918×10^{19} miles), and velocity (called the *recession velocity*) is measured in kilometers per second. A negative velocity means the nebula is moving toward the Milky Way; a positive velocity means the nebula is moving away from the Milky Way.

Recession Velocities

Distance	Velocity	Distance	Velocity
0.032	170	0.9	650
0.034	290	0.9	150
0.214	−130	0.9	500
0.263	−70	1.0	920

Distance	Velocity	Distance	Velocity
0.275	−185	1.1	450
0.275	−220	1.1	500
0.45	200	1.4	500
0.5	290	1.7	960
0.5	270	2.0	500
0.63	200	2.0	850
0.8	300	2.0	800
0.9	−30	2.0	1090

a. Find the linear regression model for these data.
$y = 454.1584409x - 40.78364910$

b. On the basis of this model, what is the recession velocity of a nebula that is 1.5 megaparsecs from the Milky Way?
640 km/s

35. a. 5 lb: $s = 0.6130952381t^2 - 0.0714285714t + 0.1071428571$
10 lb: $s = 0.6091269841t^2 - 0.0011904762t - 0.3$
15 lb: $s = 0.5922619048t^2 + 0.3571428571t - 1.520833333$

Scan the following QR code to access WolframAlpha on a mobile device.

www.wolframalpha.com

Exploring Concepts with Technology

Use WolframAlpha to Determine Linear and Quadratic Regressions

The online computational knowledge engine WolframAlpha, available at www.wolframalpha.com, can be used to determine linear and quadratic regression functions for a data set. WolframAlpha was conceived by British scientist Stephen Wolfram and developed by Wolfram Research. WolframAlpha runs on computers, tablets, and smartphones, although entering the necessary input data on a smartphone is a tedious process.

To find the linear regression function for the data set

$$S = \{(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)\},$$

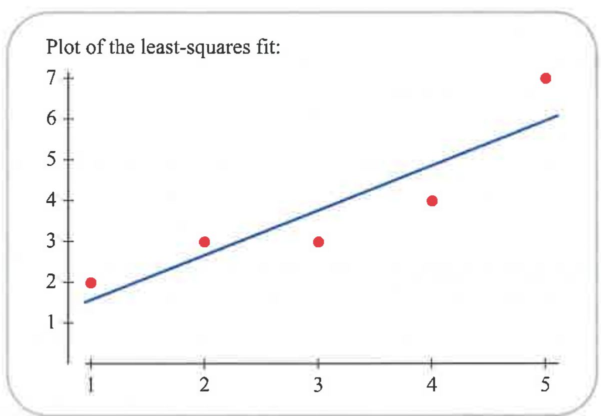
enter the following text into WolframAlpha's input field.

linear fit {(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)}

Click on the equal sign icon, at the far right of the input field, to display

$$P(x) = 1.1x + 0.5$$

as the linear regression function. A graph of the linear regression function and a scatter plot of the data are also provided, as shown below.



(continued)

Note

WolframAlpha is not just an online computational engine. It is also a knowledge engine. This means that in addition to performing mathematical procedures, it can provide answers to many questions that pertain to factual information.

WolframAlpha is different from most online search engines in that it finds answers to questions by searching through its very large database of factual information, rather than searching the Web for sites that might have information related to your question.

Apple's voice-powered iPhone assistant Siri (Speech Interpretation and Recognition Interface) often relies on WolframAlpha to answer questions that require factual information.

Sources: Wikipedia and CNET.