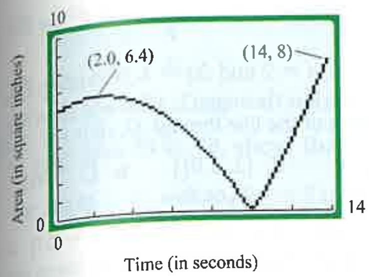


$A = (3 + 0.5t)|2 - 0.2t|$

Figure 2.102

b. $A = lw = (3 + 0.5t)|2 - 0.2t|$

c. Use a graphing utility to determine that A is increasing on $[0, 2]$ and on $[10, 14]$ and that A is decreasing on $[2, 10]$. See Figure 2.102.

d. The highest point on the graph of A occurs when $t = 14$ seconds. See Figure 2.102.

Try Exercise 86, page 235

You may be inclined to think that if the area of a rectangle is decreasing then its perimeter is also decreasing, but this is not always the case. For example, the area of the scaled rectangle in Example 7 was shown to decrease on [2, 10] even though its perimeter was always increasing.

Answers to Exercises 9–20 are on pages AA8–AA9.

EXERCISE SET 2.6

Concept Check

In Exercises 1 to 4, let $f(-2) = 3$ and $g(-2) = -6$. Find each of the following.

1. $(f + g)(-2)$ -3
2. $(f - g)(-2)$ 9
3. $(f \cdot g)(-2)$ -18
4. $\dfrac{f}{g}(-2)$ $-\dfrac{1}{2}$
5. Suppose $f(-5) = 7$ and $g(7) = -2$. What is the value of $g[f(-5)]$? -2
6. If $f[g(0)] = 3$ and $g(0) = 4$, what is the value of $f(4)$? 3

In Exercises 7 and 8, find $f(2 + h)$.

7. $f(x) = 3x - 4$ $2 + 3h$
8. $f(x) = x^2 + 1$ $5 + 4h + h^2$

In Exercises 9 to 20, use the given functions f and g to find $f + g$, $f - g$, fg, and $\dfrac{f}{g}$. State the domain of each.

9. $f(x) = x^2 - 2x - 15, \quad g(x) = x + 3$
10. $f(x) = x^2 - 25, \quad g(x) = x - 5$
11. $f(x) = 2x + 8, \quad g(x) = x + 4$
12. $f(x) = 5x - 15, \quad g(x) = x - 3$
13. $f(x) = x^3 - 2x^2 + 7x, \quad g(x) = x$
14. $f(x) = x^2 - 5x - 8, \quad g(x) = -x$
15. $f(x) = 4x - 7, \quad g(x) = 2x^2 + 3x - 5$
16. $f(x) = 6x + 10, \quad g(x) = 3x^2 + x - 10$
17. $f(x) = \sqrt{x - 3}, \quad g(x) = x$
18. $f(x) = \sqrt{x - 4}, \quad g(x) = -x$
19. $f(x) = \sqrt{4 - x^2}, \quad g(x) = 2 + x$
20. $f(x) = \sqrt{x^2 - 9}, \quad g(x) = x - 3$

In Exercises 21 to 36, evaluate the indicated function, where $f(x) = x^2 - 3x + 2$ and $g(x) = 2x - 4$.

21. $(f + g)(5)$ 18
22. $(f + g)(-7)$ 54
23. $(f + g)\left(\dfrac{1}{2}\right)$ $-\dfrac{9}{4}$
24. $(f + g)\left(\dfrac{2}{3}\right)$ $-\dfrac{20}{9}$
25. $(f - g)(-3)$ 30
26. $(f - g)(24)$ 462
27. $(f - g)(-1)$ 12
28. $(f - g)(0)$ 6
29. $(fg)(7)$ 300
30. $(fg)(-3)$ -200
31. $(fg)\left(\dfrac{2}{5}\right)$ $-\dfrac{384}{125}$
32. $(fg)(-100)$ $-2{,}101{,}608$
33. $\left(\dfrac{f}{g}\right)(-4)$ $-\dfrac{5}{2}$
34. $\left(\dfrac{f}{g}\right)(11)$ 5
35. $\left(\dfrac{f}{g}\right)\left(\dfrac{1}{2}\right)$ $-\dfrac{1}{4}$
36. $\left(\dfrac{f}{g}\right)\left(\dfrac{1}{4}\right)$ $-\dfrac{3}{8}$

Indicates Try It Exercises

In Exercises 37 to 44, find the difference quotient of the given function.

37. $f(x) = 2x + 4$ 2

38. $f(x) = 4x - 5$ 4

39. $f(x) = x^2 - 6$ $2x + h$

40. $f(x) = x^2 + 11$ $2x + h$

41. $f(x) = 2x^2 + 4x - 3$ $4x + 2h + 4$

42. $f(x) = 2x^2 - 5x + 7$ $4x + 2h - 5$

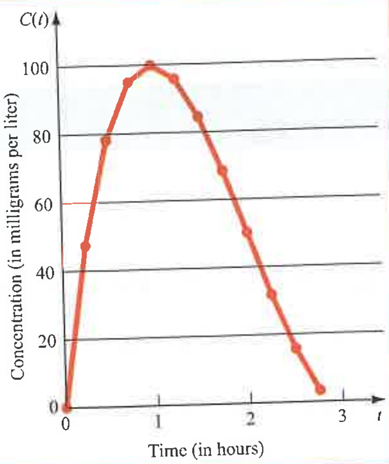
43. $f(x) = -4x^2 + 6$ $-8x - 4h$

44. $f(x) = -5x^2 - 4x$ $-10x - 5h - 4$

45. **Concentration of a Medication** The concentration $C(t)$ (in milligrams per liter) of a medication in a patient's blood is given by the data in the following table.

Concentration of Medication in Patient's Blood

t (h)	$C(t)$ (mg/L)
0	0
0.25	47.3
0.50	78.1
0.75	94.9
1.00	99.8
1.25	95.7
1.50	84.4
1.75	68.4
2.00	50.1
2.25	31.6
2.50	15.6
2.75	4.3



The **average rate of change** of the concentration over the time interval from $t = a$ to $t = a + \Delta t$ is

$$\frac{C(a + \Delta t) - C(a)}{\Delta t}$$

Use the data in the table to evaluate the average rate of change for each of the following time intervals.

a. [0, 1] (*Hint:* In this case, $a = 0$ and $\Delta t = 1$.) Compare this result to the slope of the line through (0, $C(0)$) and (1, $C(1)$). 99.8 (mg/L)/h; This is identical to the slope of the line through (0, $C(0)$) and (1, $C(1)$).

b. [0, 0.5] 156.2 (mg/L)/h

c. [1, 2] −49.7 (mg/L)/h

d. [1, 1.5] −30.8 (mg/L)/h

e. [1, 1.25] −16.4 (mg/L)/h

f. The data in the table can be modeled by the function $Con(t) = 25t^3 - 150t^2 + 225t$. Use $Con(t)$ to verify that the average rate of change over $[1, 1 + \Delta t]$ is given by $-75(\Delta t) + 25(\Delta t)^2$. What does the average rate of change over $[1, 1 + \Delta t]$ seem to approach as Δt approaches 0? 0 (mg/L)/h

46. **Ball Rolling on a Ramp** The distance traveled by a ball rolling down a ramp is given by $s(t) = 6t^2$, where t is the time in seconds after the ball is released and $s(t)$ is measured in feet. The ball travels 6 feet in 1 second and 24 feet in 2 seconds. Use the difference quotient for average velocity given on page 228 to evaluate the average velocity for each of the following time intervals.

a. [2, 3] (*Hint:* In this case, $a = 2$ and $\Delta t = 1$.) Compare this result to the slope of the line through (2, $s(2)$) and (3, $s(3)$). 30 ft/s; This is identical to the slope of the line through (2, $s(2)$) and (3, $s(3)$).

b. [2, 2.5] 27 ft/s

c. [2, 2.1] 24.6 ft/s

d. [2, 2.01] 24.06 ft/s

e. [2, 2.001] 24.006 ft/s

f. Verify that the average velocity over $[2, 2 + \Delta t]$ is $24 + 6(\Delta t)$. What does the average velocity seem to approach as Δt approaches 0? 24 ft/s

In Exercises 47 to 58, find $(g \circ f)(x)$ and $(f \circ g)(x)$ for the given functions f and g.

47. $f(x) = 3x + 5,\quad g(x) = 2x - 7$
$(g \circ f)(x) = 6x + 3,\ (f \circ g)(x) = 6x - 16$

48. $f(x) = 2x - 7,\quad g(x) = 3x + 2$
$(g \circ f)(x) = 6x - 19,\ (f \circ g)(x) = 6x - 3$

49. $f(x) = x^2 + 4x - 1,\quad g(x) = x + 2$
$(g \circ f)(x) = x^2 + 4x + 1,\ (f \circ g)(x) = x^2 + 8x + 11$

50. $f(x) = x^2 - 11x,\quad g(x) = 2x + 3$
$(g \circ f)(x) = 2x^2 - 22x + 3,\ (f \circ g)(x) = 4x^2 - 10x - 24$

51. $f(x) = x^3 + 2x,\quad g(x) = -5x$
$(g \circ f)(x) = -5x^3 - 10x,\ (f \circ g)(x) = -125x^3 - 10x$

52. $f(x) = -x^3 - 7,\quad g(x) = x + 1$
$(g \circ f)(x) = -x^3 - 6,\ (f \circ g)(x) = -x^3 - 3x^2 - 3x - 8$

53. $f(x) = \dfrac{2}{x + 1},\quad g(x) = 3x - 5$ $(g \circ f)(x) = \dfrac{1 - 5x}{x + 1}$, $(f \circ g)(x) = \dfrac{2}{3x - 4}$

54. $f(x) = \sqrt{x + 4},\quad g(x) = \dfrac{1}{x}$
$(g \circ f)(x) = \dfrac{\sqrt{x + 4}}{x + 4},\ (f \circ g)(x) = \dfrac{\sqrt{x + 4x^2}}{x}$

55. $f(x) = \dfrac{1}{x^2},\quad g(x) = \sqrt{x - 1}$ $(g \circ f)(x) = \dfrac{\sqrt{1 - x^2}}{|x|}$, $(f \circ g)(x) = \dfrac{1}{x - 1}$

56. $f(x) = \dfrac{6}{x - 2},\quad g(x) = \dfrac{3}{5x}$
$(g \circ f)(x) = \dfrac{x - 2}{10},\ (f \circ g)(x) = \dfrac{30x}{3 - 10x}$

57. $f(x) = \dfrac{3}{|5 - x|},\quad g(x) = -\dfrac{2}{x}$ $(g \circ f)(x) = -\dfrac{2|5 - x|}{3}$, $(f \circ g)(x) = \dfrac{3|x|}{|5x + 2|}$

58. $f(x) = |2x + 1|,\quad g(x) = 3x^2 - 1$
$(g \circ f)(x) = 12x^2 + 12x + 2,\ (f \circ g)(x) = |6x^2 - 1|$

In Exercises 59 to 74, evaluate each composite function, where $f(x) = 2x + 3$, $g(x) = x^2 - 5x$, and $h(x) = 4 - 3x^2$.

59. $(g \circ f)(4)$ 66

60. $(f \circ g)(4)$ −5

61. $(f \circ g)(-3)$ 51

62. $(g \circ f)(-1)$ −4

63. $(g \circ h)(0)$ −4

64. $(h \circ g)(0)$ 4

65. $(f \circ f)(8)$ 41

66. $(f \circ f)(-8)$ −23

67. $(h \circ g)\left(\dfrac{2}{5}\right)$ $-\dfrac{3848}{625}$

68. $(g \circ h)\left(-\dfrac{1}{3}\right)$ $-\dfrac{44}{9}$

69. $(g \circ f)(\sqrt{3})$ $6 + 2\sqrt{3}$

70. $(f \circ g)(\sqrt{2})$ $7 - 10\sqrt{2}$

71. $(g \circ f)(2c)$ $16c^2 + 4c - 6$

72. $(f \circ g)(3k)$ $18k^2 - 30k + 3$

73. $(g \circ h)(k + 1)$ $9k^4 + 36k^3 + 45k^2 + 18k - 4$

74. $(h \circ g)(k - 1)$ $-3k^4 + 42k^3 - 183k^2 + 252k - 104$

In Exercises 75 to 78, show that $(f \circ g)(x) = (g \circ f)(x)$.

75. $f(x) = 2x + 3;\quad g(x) = 5x + 12$

76. $f(x) = 4x - 2;\quad g(x) = 7x - 4$

77. $f(x) = \dfrac{6x}{x - 1};\quad g(x) = \dfrac{5x}{x - 2}$

78. $f(x) = \dfrac{5x}{x + 3};\quad g(x) = -\dfrac{2x}{x - 4}$

In Exercises 79 to 82, show that

$$(g \circ f)(x) = x \quad \text{and} \quad (f \circ g)(x) = x$$

79. $f(x) = 2x + 3,\quad g(x) = \dfrac{x - 3}{2}$

80. $f(x) = 4x - 5,\quad g(x) = \dfrac{x + 5}{4}$

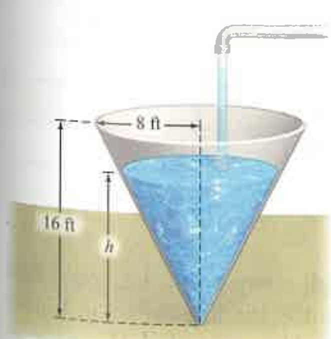
81. $f(x) = \dfrac{4}{x + 1},\quad g(x) = \dfrac{4 - x}{x}$

82. $f(x) = \dfrac{2}{1 - x},\quad g(x) = \dfrac{x - 2}{x}$

83. **Conversion Functions** The function $F(x) = \dfrac{x}{12}$ converts x inches to feet. The function $Y(x) = \dfrac{x}{3}$ converts x feet to yards. Explain the meaning of $(Y \circ F)(x)$.
$(Y \circ F)(x)$ converts x inches to yards.

84. **Conversion Functions** The function $F(x) = 3x$ converts x yards to feet. The function $I(x) = 12x$ converts x feet to inches. Explain the meaning of $(I \circ F)(x)$.
$(I \circ F)(x)$ converts x yards to inches.

85. **Water Tank** A water tank has the shape of a right circular cone with height 16 feet and radius 8 feet. Water is running into the tank so that the radius r (in feet) of the surface of the water is given by $r = 1.5t$, where t is the time (in minutes) that the water has been running. See the accompanying diagram.



86. a. $l = |3 - 0.5t|, w = |2 - 0.2t|$

b. $A = lw = |3 - 0.5t||2 - 0.2t|$

$= |(3 - 0.5t)(2 - 0.2t)|$

c. A is increasing on [6, 8] and on [10, 14]; A is decreasing on [0, 6] and on [8, 10].

d. The highest point on the graph of A for $0 \le t \le 14$ occurs when $t = 0$ s.

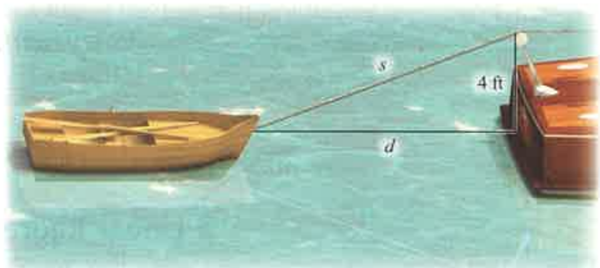
a. The area A of the surface of the water is $A = \pi r^2$. Find $A(t)$ and use it to determine the area of the surface of the water when $t = 2$ minutes. $A(t) = \pi(1.5t)^2$, $A(2) = 9\pi \text{ ft}^2 \approx 28.27 \text{ ft}^2$

b. The volume V of the water is given by $V = \dfrac{1}{3}\pi r^2 h$. Find $V(t)$ and use it to determine the volume of the water when $t = 3$ minutes. (*Hint:* The height of the water in the cone is always twice the radius of the surface of the water.)
$V(t) = 2.25\pi t^3$, $V(3) = 60.75\pi \text{ ft}^3 \approx 190.85 \text{ ft}^3$

86. **Scaling a Rectangle** Rework Example 7 of this section with the scaling as follows: The upper right corner of the original rectangle is pulled to the *left* at 0.5 inch per second and downward at 0.2 inch per second. Answer in lower left column.

Enrichment Exercises

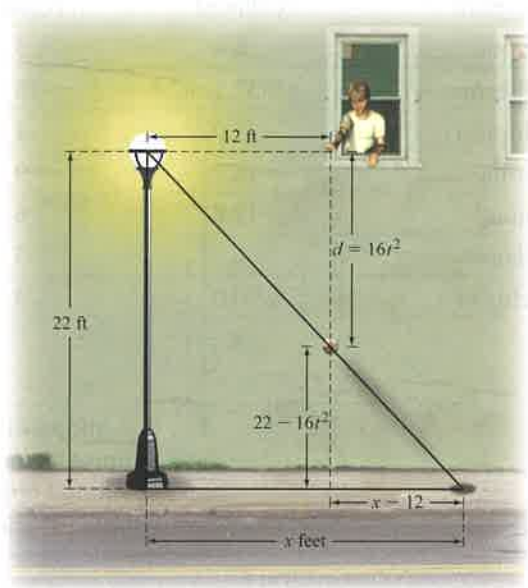
87. **Towing a Boat** A boat is towed by a rope that runs through a pulley that is 4 feet above the point where the rope is tied to the boat. The length (in feet) of the rope from the boat to the pulley is given by $s = 48 - t$, where t is the time in seconds that the boat has been in tow. The horizontal distance from the pulley to the boat is d.



a. Find $d(t)$. $d(t) = \sqrt{(48 - t)^2 - 4^2}$

b. Evaluate $s(35)$ and $d(35)$. $s(35) = 13$ ft, $d(35) \approx 12.37$ ft

88. **Shadow Position** The light from a lamppost casts a shadow from a ball that was dropped from a height of 22 feet above the ground, as shown in the diagram. The distance d, in feet, the ball has dropped t seconds after it is released is given by $d = 16t^2$. Find the distance x, in feet, of the shadow from the base of the lamppost as a function of time t. $x = \dfrac{33}{2t^2}$



SECTION 2.7

Modeling Data Using Regression

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A14.

PS1. Find the slope and the y-intercept of the graph of $y = -\frac{x}{3} + 4$. [2.3] Slope, $-\frac{1}{3}$; y-intercept, (0, 4)

PS2. Find the slope and the y-intercept of the graph of $3x - 4y = 12$. [2.3] Slope, $\frac{3}{4}$; y-intercept, (0, −3)

PS3. Find the equation of the line that has a slope of −0.45 and a y-intercept of (0, 2.3). [2.3] $y = -0.45x + 2.3$

PS4. Find the equation of the line that passes through the point $P(3, -4)$ and has a slope of $-\frac{2}{3}$. [2.3] $y = -\frac{2}{3}x - 2$

PS5. If $f(x) = 3x^2 + 4x - 1$, find $f(2)$. [2.2] 19

PS6. You are given $P_1(2, -1)$ and $P_2(4, 14)$. If $f(x) = x^2 - 3$, find $|f(x_1) - y_1| + |f(x_2) - y_2|$. [2.2] 3

Linear Regression Models

The data in the table given below show the population of selected states and the number of professional sports teams (Major League Baseball, National Football League, National Basketball Association, Women's National Basketball Association, National Hockey League) in those states. A scatter plot of the data is shown in Figure 2.103 on page 237.

Number of Professional Sports Teams for Selected States

State	Population (millions)	Number of Teams	State	Population (millions)	Number of Teams
Arizona	6.4	5	Minnesota	5.3	5
California	37.3	16	New Jersey	8.8	3
Colorado	5.0	4	New York	19.4	9
Florida	18.8	9	North Carolina	9.5	3
Illinois	12.8	6	Ohio	11.5	7
Indiana	6.5	3	Pennsylvania	12.7	7
Michigan	10.0	4	Texas	25.1	10

Although there is no one line that passes through every point, we could find an approximate linear model for these data. For instance, the line shown in Figure 2.104 in red approximates the data better than the line shown in blue. However, as Figure 2.105 shows, there are many other lines we could draw that seem to approximate the data.