62. $P(x) = -\frac{x^2}{14{,}000} + 1.68x - 4000$

Business In Exercises 63 and 64, determine the profit function for the given revenue function and cost function. Also determine the break-even point or points.

63. $R(x) = x(102.50 - 0.1x)$; $C(x) = 52.50x + 1840$

64. $R(x) = x(210 - 0.25x)$; $C(x) = 78x + 6399$

65. **Tour Cost** A charter bus company has determined that the cost, in dollars, of providing x people with a tour is

$$C(x) = 180 + 2.50x$$

A full tour consists of 60 people. The ticket price per person is \$15 plus \$0.25 for each unsold ticket. Determine

a. The revenue function

b. The profit function

c. The company's maximum profit

d. The number of ticket sales that yields the maximum profit

66. **Delivery Cost** An air freight company has determined that the cost, in dollars, of delivering x parcels per flight is

$$C(x) = 2025 + 7x$$

The price per parcel, in dollars, the company charges to send x parcels is

$$p(x) = 22 - 0.01x$$

Determine

a. The revenue function

b. The profit function

c. The company's maximum profit

d. The price per parcel that yields the maximum profit

e. The minimum number of parcels the air freight company must ship to break even

67. **Gasoline Sales** A gasoline station can sell 10,000 gallons of gas per day at a price of \$3.95 per gallon. The station manager estimates that for each \$0.05 price reduction per gallon, 500 more gallons of gasoline can be sold each day. The cost per gallon of gas to the station is \$2.75. Find the price at which the station should sell a gallon of gasoline to maximize its daily profit on gasoline sales.

68. **Ticket Prices** Expand Your Wings Airlines finds that if it prices a Los Angeles to New York one-way ticket at \$390, it can sell 350 seats on its airplane. The airline estimates that for each \$10 price reduction per ticket, 25 more tickets can be sold each day. The cost to the airline for each ticket is \$150. Find the price at which the airline should sell a ticket to maximize its daily profit on ticket sales for the Los Angeles to New York flight.

69. **Projectile** If the initial velocity of a projectile is 128 feet per second, then its height, in feet, is a function of time t, in seconds, given by the equation $h(t) = -16t^2 + 128t$.

a. Find the time t when the projectile achieves its maximum height.

b. Find the maximum height of the projectile.

c. Find the time t when the projectile hits the ground.

70. **Projectile** The height in feet of a projectile with an initial velocity of 64 feet per second and an initial height of 80 feet is a function of time t in seconds given by

$$h(t) = -16t^2 + 64t + 80$$

a. Find the maximum height of the projectile.

b. Find the time t when the projectile achieves its maximum height.

c. Find the time t when the projectile has a height of 0 feet.

71. **Fire Management** The height of a stream of water from the nozzle of a fire hose can be modeled by

$$y(x) = -0.014x^2 + 1.19x + 5$$

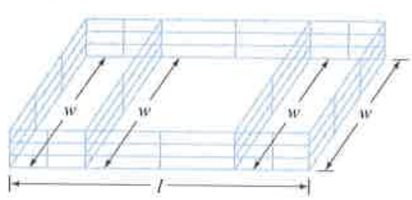
where $y(x)$ is the height, in feet, of the stream x feet from the firefighter. What is the maximum height that the stream of water from this nozzle can reach? Round to the nearest foot.

72. **Astronaut Training** A weightless environment can be created in an airplane by flying in a series of parabolic paths. This is one method that NASA uses to train astronauts for the experience of weightlessness. Suppose the height h, in feet, of NASA's airplane is modeled by $h(t) = -6.6t^2 + 430t + 28{,}000$, where t is the number of seconds after the plane enters its parabolic path. Find the maximum height of the plane to the nearest 1000 feet.

Enrichment Exercises

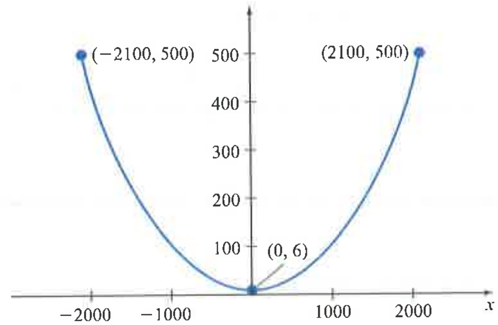
73. **Sports** For a serve to be legal in tennis, the ball must be at least 3 feet high when it is 39 feet from the server and it must land in a spot that is less than 60 feet from the server. Does the path of a ball given by $h(x) = -0.002x^2 - 0.03x + 8$, where $h(x)$ is the height of the ball (in feet) x feet from the server, satisfy the conditions of a legal serve?

74. **Rectangular Enclosure** A farmer uses 1200 feet of fence to enclose a rectangular region and to subdivide the region into three smaller rectangular regions by placing fences parallel to one of the sides. Find the dimensions that produce the greatest enclosed area.



75. Norman Window A Norman window has the shape of a rectangle surmounted by a semicircle. The exterior perimeter of the window shown in the figure is 48 feet. Find the height h and the radius r that will allow the maximum amount of light to enter the window. (*Hint:* Write the area of the window as a quadratic function of the radius r.)

76. Golden Gate Bridge The suspension cables of the main span of the Golden Gate Bridge are in the shape of a parabola. If a coordinate system is drawn as shown, find the quadratic function that models a suspension cable for the main span of the bridge.



SECTION 2.5

Symmetry
Even and Odd Functions
Translations of Graphs
Reflections of Graphs
Compressing and Stretching of Graphs

Properties of Graphs

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A11.

PS1. For the graph of the parabola whose equation is $f(x) = x^2 + 4x - 6$, what is the equation of the axis of symmetry? [2.4]

PS2. For $f(x) = \dfrac{3x^4}{x^2 + 1}$, show that $f(-3) = f(3)$. [2.2]

PS3. For $f(x) = 2x^3 - 5x$, show that $f(-2) = -f(2)$. [2.2]

PS4. Let $f(x) = x^2$ and $g(x) = x + 3$. Find $f(a) - g(a)$ for $a = -2, -1, 0, 1, 2$. [2.2]

PS5. What is the midpoint of the line segment between $P(-a, b)$ and $Q(a, b)$? [2.1]

PS6. What is the midpoint of the line segment between $P(-a, -b)$ and $Q(a, b)$? [2.1]

Symmetry

A graph is **symmetric with respect to the y-axis** if whenever the point given by (x, y) is on the graph, then $(-x, y)$ is also on the graph. The graph in Figure 2.62 is symmetric with respect to the y-axis. A graph is **symmetric with respect to the x-axis** if whenever the point given by (x, y) is on the graph, then $(x, -y)$ is also on the graph. The graph in Figure 2.63 is symmetric with respect to the x-axis.

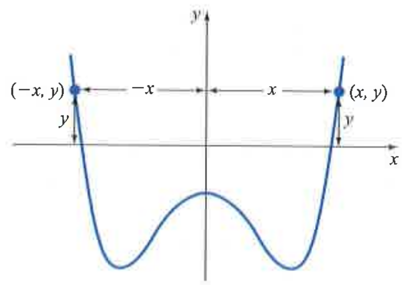


Figure 2.62
Symmetry with respect to the y-axis

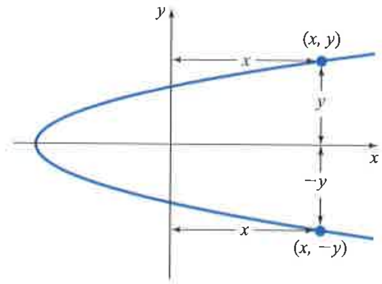


Figure 2.63
Symmetry with respect to the x-axis

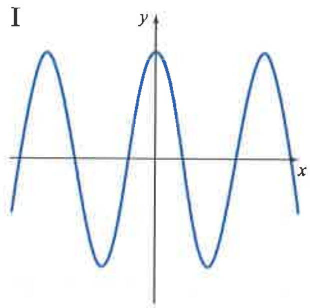
Tests for Symmetry with Respect to a Coordinate Axis

The graph of an equation is symmetric with respect to

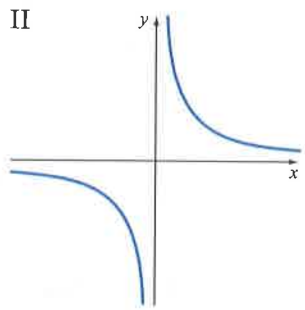
- the y-axis if the replacement of x with $-x$ leaves the equation unaltered.
- the x-axis if the replacement of y with $-y$ leaves the equation unaltered.

Question • Which of the graphs below, I, II, or III, is **a.** symmetric with respect to the x-axis? **b.** symmetric with respect to the y-axis?

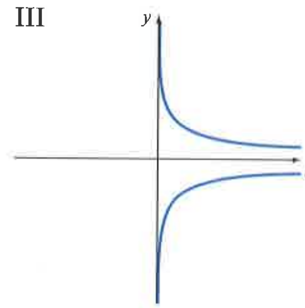
I



II



III



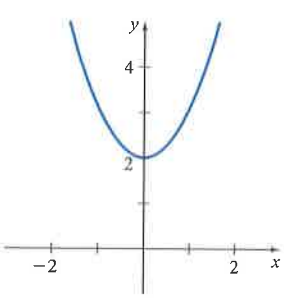
EXAMPLE 1 Determine Symmetries of a Graph

Determine whether the graph of the given equation has symmetry with respect to either the x- or the y-axis.

a. $y = x^2 + 2$ **b.** $x = |y| - 2$

Solution

a. The equation $y = x^2 + 2$ *is unaltered* by the replacement of x with $-x$. That is, the simplification of $y = (-x)^2 + 2$ yields the original equation $y = x^2 + 2$. Thus the graph of $y = x^2 + 2$ is symmetric with respect to the y-axis. However, the equation $y = x^2 + 2$ *is altered* by the replacement of y with $-y$. That is, the simplification of $-y = x^2 + 2$, which is $y = -x^2 - 2$, *does not* yield the original equation $y = x^2 + 2$. The graph of $y = x^2 + 2$ is not symmetric with respect to the x-axis. See Figure 2.64.



$y = x^2 + 2$

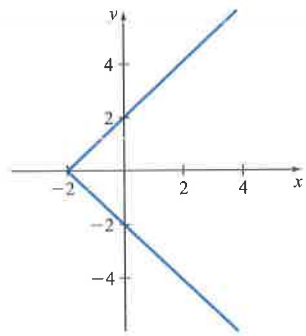
Figure 2.64

b. The equation $x = |y| - 2$ *is altered* by the replacement of x with $-x$. That is, the simplification of $-x = |y| - 2$, which is $x = -|y| + 2$, *does not*

(continued)

Answer • **a.** III is symmetric with respect to the x-axis.
b. I is symmetric with respect to the y-axis.

yield the original equation $x = |y| - 2$. This implies that the graph of $x = |y| - 2$ is not symmetric with respect to the y-axis. However, the equation $x = |y| - 2$ *is unaltered* by the replacement of y with $-y$. That is, the simplification of $x = |-y| - 2$ yields the original equation $x = |y| - 2$. The graph of $x = |y| - 2$ is symmetric with respect to the x-axis. See Figure 2.65.



$x = |y| - 2$

Figure 2.65

Try Exercise 18, page 222

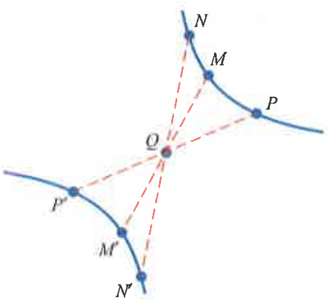


Figure 2.66

Definition of Symmetry with Respect to a Point

A graph is **symmetric with respect to a point** Q if for each point P on the graph, there is a point P' on the graph such that Q is the midpoint of the line segment PP'.

The graph in Figure 2.66 is symmetric with respect to the point Q. For any point P on the graph, there exists a point P' on the graph such that Q is the midpoint of $P'P$.

When we discuss symmetry with respect to a point, we frequently use the origin. A graph is symmetric with respect to the origin if whenever the point given by (x, y) is on the graph, then $(-x, -y)$ is also on the graph. The graph in Figure 2.67 is symmetric with respect to the origin.

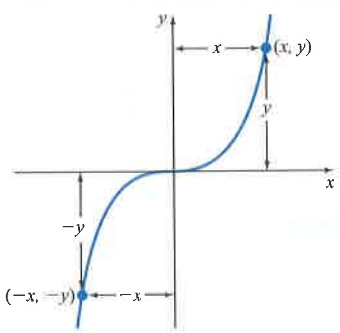


Figure 2.67

Symmetry with respect to the origin

Test for Symmetry with Respect to the Origin

The graph of an equation is symmetric with respect to the origin if the replacement of x with $-x$ and of y with $-y$ leaves the equation unaltered.

EXAMPLE 2 Determine Symmetry with Respect to the Origin

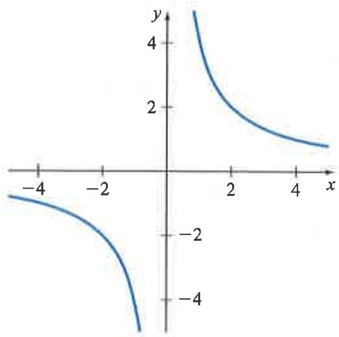
Determine whether the graph of each equation has symmetry with respect to the origin.

a. $xy = 4$ **b.** $y = x^3 + 1$

Solution

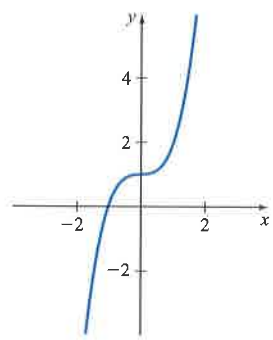
a. The equation $xy = 4$ is unaltered by the replacement of x with $-x$ and y with $-y$. That is, the simplification of $(-x)(-y) = 4$ yields the original equation $xy = 4$. Thus the graph of $xy = 4$ is symmetric with respect to the origin. See Figure 2.68.

b. The equation $y = x^3 + 1$ *is altered* by the replacement of x with $-x$ and y with $-y$. That is, the simplification of $-y = (-x)^3 + 1$, which is $y = x^3 - 1$, *does not* yield the original equation $y = x^3 + 1$. Thus the graph of $y = x^3 + 1$ is not symmetric with respect to the origin. See Figure 2.69.



$xy = 4$

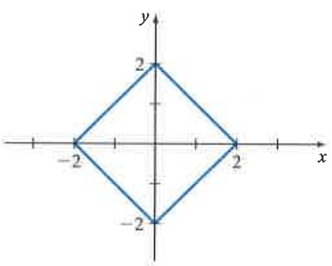
Figure 2.68



$y = x^3 + 1$

Figure 2.69

Try Exercise 28, page 222



$|x| + |y| = 2$

Figure 2.70

Some graphs have more than one symmetry. For example, the graph of $|x| + |y| = 2$ has symmetry with respect to the x-axis, the y-axis, and the origin. Figure 2.70 is the graph of $|x| + |y| = 2$.

Even and Odd Functions

Some functions are classified as either *even* or *odd*.

Definition of Even and Odd Functions

The function f is an **even function** if

$$f(-x) = f(x) \quad \text{for all } x \text{ in the domain of } f$$

The function f is an **odd function** if

$$f(-x) = -f(x) \quad \text{for all } x \text{ in the domain of } f$$

EXAMPLE 3 Identify Even or Odd Functions

Determine whether each function is even, odd, or neither.

a. $f(x) = x^3$ **b.** $F(x) = |x|$ **c.** $h(x) = x^4 + 2x$

Solution

Replace x with $-x$ and simplify.

a. $f(-x) = (-x)^3 = -x^3 = -(x^3) = -f(x)$

Because $f(-x) = -f(x)$, this function is an odd function.

b. $F(-x) = |-x| = |x| = F(x)$

Because $F(-x) = F(x)$, this function is an even function.

c. $h(-x) = (-x)^4 + 2(-x) = x^4 - 2x$

This function is neither an even nor an odd function because

$$h(-x) = x^4 - 2x$$

which is not equal to either $h(x)$ or $-h(x)$.

Try Exercise 36, page 222

The following properties are results of the tests for symmetry:

- The graph of an even function is symmetric with respect to the y-axis.
- The graph of an odd function is symmetric with respect to the origin.

The graph of f in Figure 2.71 is symmetric with respect to the y-axis. It is the graph of an even function. The graph of g in Figure 2.72 is symmetric with respect to the origin. It is the graph of an odd function. The graph of h in Figure 2.73 is not symmetric with respect to the y-axis and is not symmetric with respect to the origin. It is neither an even nor an odd function.

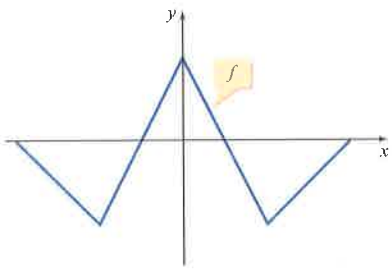


Figure 2.71

The graph of an even function is symmetric with respect to the y-axis.

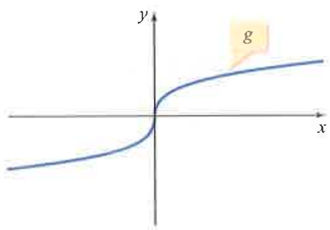


Figure 2.72

The graph of an odd function is symmetric with respect to the origin.

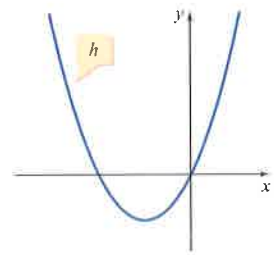


Figure 2.73

The graph of a function that is neither even nor odd is not symmetric with respect to the y-axis or the origin.

Translations of Graphs

The shape of a graph may be exactly the same as the shape of another graph; only their positions in the xy-plane may differ. For example, the graph of $y = f(x) + 2$ is the graph of $y = f(x)$ with each point moved up vertically 2 units. The graph of $y = f(x) - 3$ is the graph of $y = f(x)$ with each point moved down vertically 3 units. See Figure 2.74.

The graphs of $y = f(x) + 2$ and $y = f(x) - 3$ in Figure 2.74 are called *vertical translations* of the graph of $y = f(x)$.

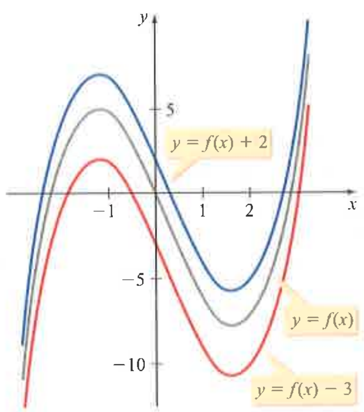


Figure 2.74

Vertical Translation of a Graph

If f is a function and c is a positive constant, then the graph of

- $y = f(x) + c$ is a vertical shift c units upward of the graph of $y = f(x)$.
- $y = f(x) - c$ is a vertical shift c units downward of the graph of $y = f(x)$.

In Figure 2.75, the graph of $y = h(x + 3)$ is the graph of $y = h(x)$ with each point shifted to the left horizontally 3 units. Similarly, the graph of $y = h(x - 3)$ is the graph of $y = h(x)$ with each point shifted to the right horizontally 3 units.

The graphs of $y = h(x + 3)$ and $y = h(x - 3)$ in Figure 2.75 are called *horizontal translations* of the graph of $y = h(x)$.

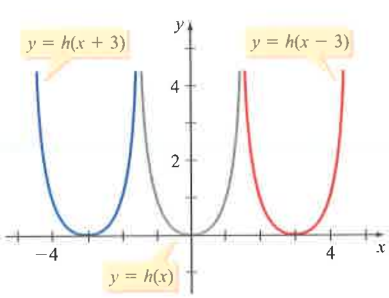


Figure 2.75

Horizontal Translation of a Graph

If f is a function and c is a positive constant, then the graph of

- $y = f(x + c)$ is a horizontal shift c units to the left of the graph of $y = f(x)$.
- $y = f(x - c)$ is a horizontal shift c units to the right of the graph of $y = f(x)$.

EXAMPLE 4 Graph Using a Vertical Translation

Use the graph of f in Figure 2.76 to sketch a graph of each function.

a. $g(x) = f(x) + 3$

b. $h(x) = f(x) - 2$

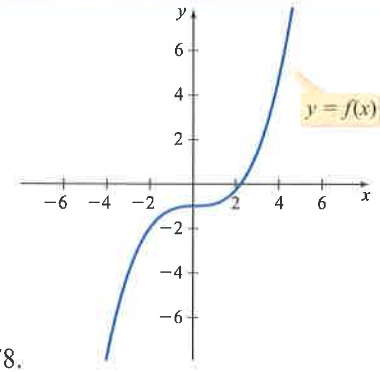


Figure 2.76

Solution

a. The graph of g is a vertical shift 3 units upward of the graph of f. See Figure 2.77.

b. The graph of h is a vertical shift 2 units downward of the graph of f. See Figure 2.78.

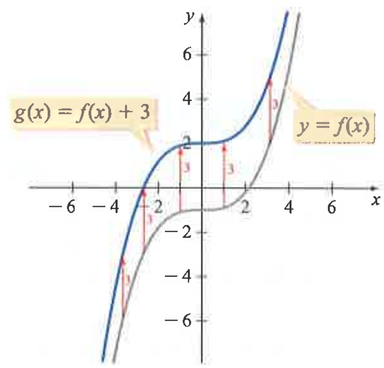


Figure 2.77

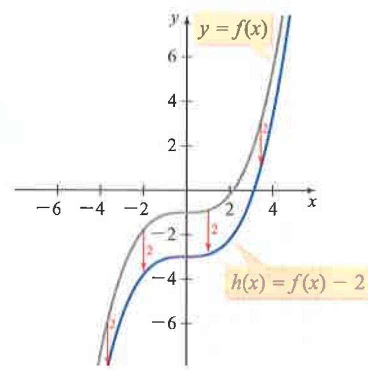


Figure 2.78

▶ Try Exercise 50a, page 223

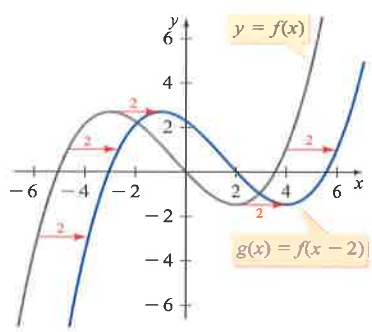


Figure 2.80

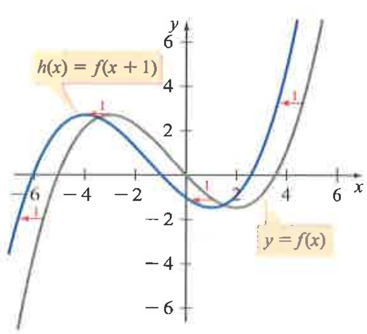


Figure 2.81

EXAMPLE 5 Graph Using a Horizontal Translation

Use the graph in Figure 2.79 to sketch a graph of each function.

a. $g(x) = f(x - 2)$

b. $h(x) = f(x + 1)$

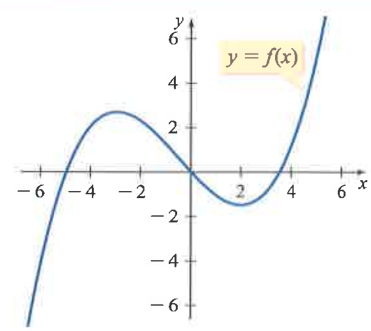


Figure 2.79

Solution

a. The graph of g is a horizontal shift 2 units to the right of the graph of f. See Figure 2.80.

b. The graph of h is a horizontal shift 1 unit to the left of the graph of f. See Figure 2.81.

▶ Try Exercise 50b, page 223

It is possible to have both a vertical and a horizontal translation of a graph.

EXAMPLE 6 Graph a Function Using a Horizontal and a Vertical Translation

Given the graph of f in Figure 2.82, sketch a graph of $g(x) = f(x + 1) - 3$.

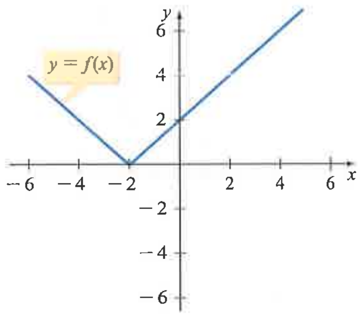


Figure 2.82

Solution

The graph of g is a horizontal shift 1 unit to the left and a vertical shift 3 units down of the graph of f as shown in Figure 2.83.

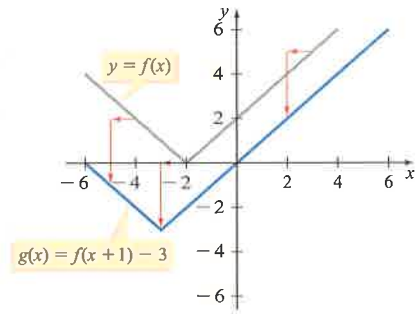


Figure 2.83

▶ Try Exercise 54, page 223

Integrating Technology

Scan the QR code to access this demostration.

http://www.wadsworthmedia.com/math/precalculus_simulations/Translation.html

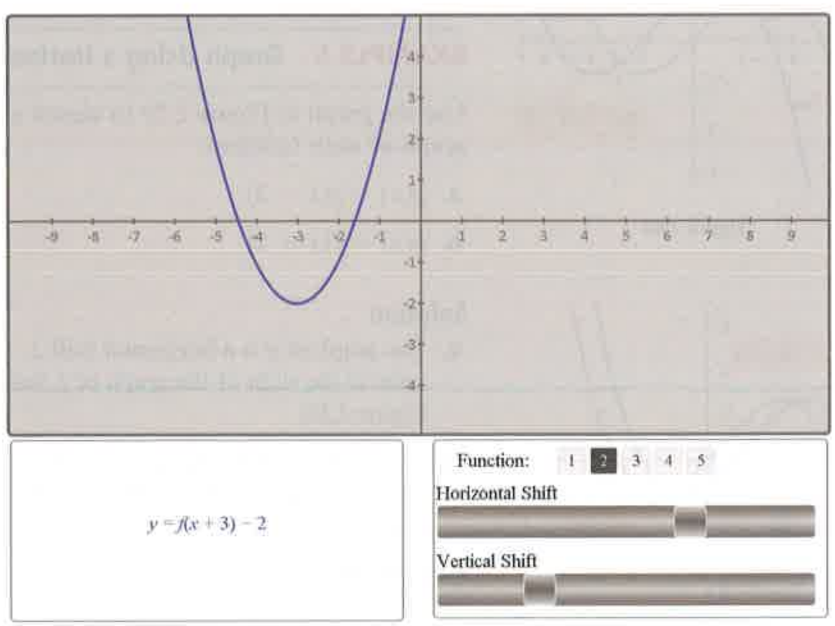
Vertical and Horizontal Translation of Graphs

An interactive demonstration that allows you to explore horizontal and vertical translations of the graph of a function is available online. You can access this demonstration by scanning the QR code at the left or by entering

http://www.wadsworthmedia.com/math/precalculus_simulations/Translation.html

in a search engine.

There are five different functions from which to choose. The graph below shows $y = f(x + 3) - 2$ for function 2.



Suggestions for things to try:

- Select function 3. Move the horizontal shift slide. Note how the position of the graph changes. Select other functions and verify that moving the horizontal shift slider affects the graph in the same way.
- Select another function and move the vertical shift slide. Note how the position of the graph changes. Select other functions and verify that moving the vertical shift slider affects the graph in the same way.

Reflections of Graphs

The graph of $y = -f(x)$ cannot be obtained from the graph of $y = f(x)$ by a combination of vertical and/or horizontal shifts. Figure 2.84 illustrates that the graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the x-axis.

The graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ across the y-axis, as shown in Figure 2.85.

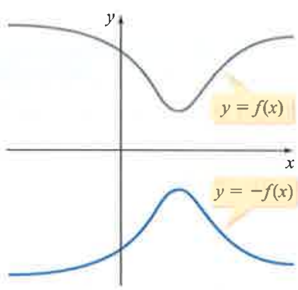


Figure 2.84

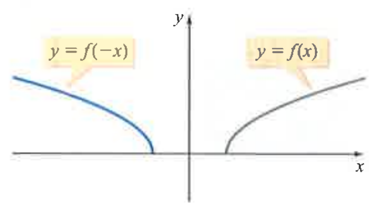


Figure 2.85

Reflections

The graph of

- $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis.
- $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis.

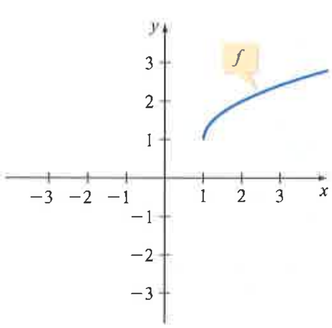


Figure 2.86

EXAMPLE 7 Graph by Using Reflections

Use reflections of the graph of $y = f(x)$, shown in Figure 2.86, to graph the following.

a. $g(x) = -f(x)$ **b.** $h(x) = f(-x)$

Solution

a. Because $g(x) = -f(x)$, the graph of g is the graph of f reflected across the x-axis. See Figure 2.87.

b. Because $h(x) = f(-x)$, the graph of h is the graph of f reflected across the y-axis. See Figure 2.88.

(continued)

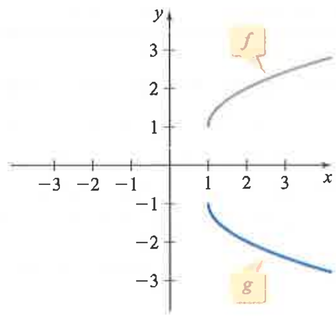


Figure 2.87

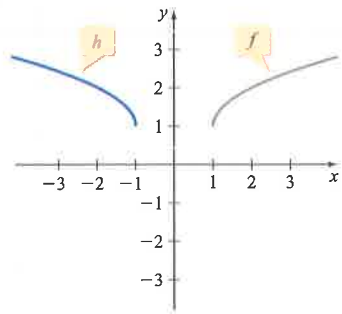


Figure 2.88

▶ Try Exercise 62, page 224

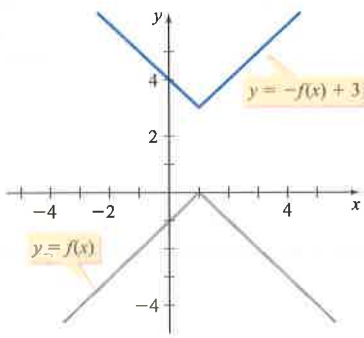


Figure 2.89

Some graphs of functions can be constructed by using a combination of translations and reflections. For instance, in Figure 2.89 the graph of $y = -f(x) + 3$ was obtained by reflecting the graph of $y = f(x)$ across the x-axis and then shifting that graph up 3 units.

Scan the QR code to access this demostration.

http://www.wadsworthmedia.com/math/precalculus_simulations/TranslationReflection.html

Integrating Technology

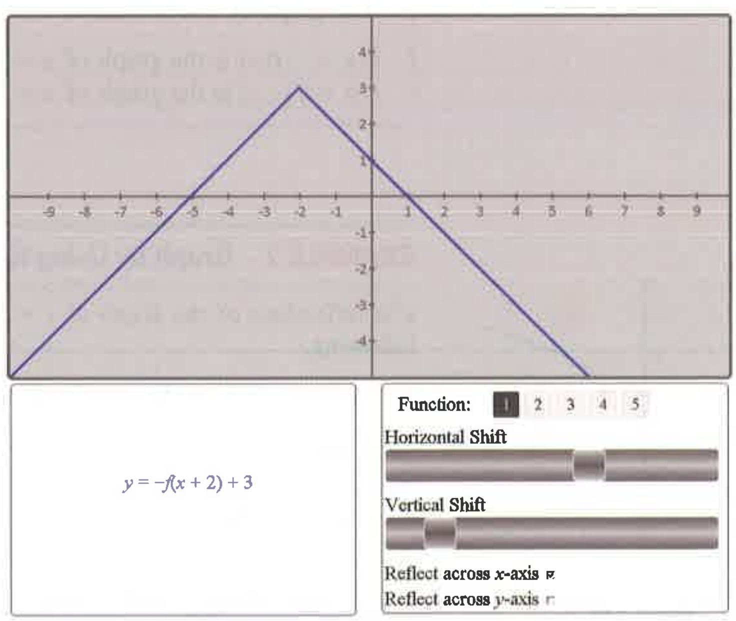
Translation and Reflecting Graphs

An interactive demonstration that allows you explore vertical and horizontal translations of the graph of a function along with reflecting the graph across the x- or y-axis is available online. You can access this demonstration by scanning the QR code given at the left or by entering

http://www.wadsworthmedia.com/math/precalculus_simulations/TranslationReflection.html

in a search engine.

There are five different functions from which to choose. The graph below shows the reflection of $y = f(x + 2) - 3$ across the x-axis. The graph shown is the graph of $y = -f(x + 2) + 3$



Suggestions for things to try:

- Make sure **Reflect across *x*-axis** and **Reflect across *y*-axis** are not checked. Select various functions and move the horizontal or vertical shift sliders. Then click on **Reflect across *x*-axis**.
- Make sure **Reflect across *x*-axis** and **Reflect across *y*-axis** are not checked. Select various functions and move the horizontal or vertical shift sliders. Then click on **Reflect across *y*-axis**.

Compressing and Stretching of Graphs

The graph of the equation $y = c \cdot f(x)$ for $c \neq 1$ vertically compresses or stretches the graph of $y = f(x)$. To determine the points on the graph of $y = c \cdot f(x)$, multiply the y-coordinate of each point on the graph of $y = f(x)$ by c. For example, Figure 2.90 shows that the graph of $y = \frac{1}{2}|x|$ can be obtained by plotting points that have a y-coordinate that is one-half of the y-coordinate of those points that make up the graph of $y = |x|$.

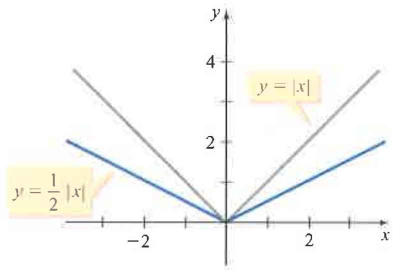


Figure 2.90

If $0 < c < 1$, then the graph of $y = c \cdot f(x)$ is obtained by *compressing* the graph of $y = f(x)$. Figure 2.90 illustrates the vertical compressing of the graph of $y = |x|$ toward the x-axis to form the graph of $y = \frac{1}{2}|x|$.

If $c > 1$, then the graph of $y = c \cdot f(x)$ is obtained by *stretching* the graph of $y = f(x)$. For example, if $f(x) = |x|$, then we obtain the graph of

$$y = 2f(x) = 2|x|$$

by stretching the graph of f away from the x-axis. See Figure 2.91.

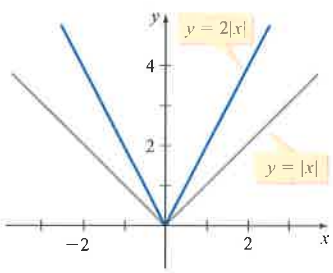


Figure 2.91

Vertical Stretching and Compressing of Graphs

Assume that f is a function and c is a positive constant. Then,

- if $c > 1$, the graph of $y = c \cdot f(x)$ is the graph of $y = f(x)$ *stretched* vertically away from the x-axis by a factor of c.
- if $0 < c < 1$, the graph of $y = c \cdot f(x)$ is the graph of $y = f(x)$ *compressed* vertically toward the x-axis by a factor of c.

EXAMPLE 8 Graph by Using Vertical Stretching or Compressing

Use the graph of f in Figure 2.92 to sketch a graph of the following.

a. $g(x) = 2f(x)$ **b.** $h(x) = \frac{1}{2}f(x)$

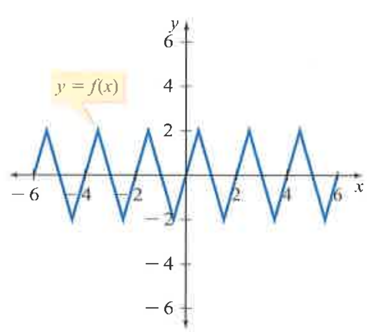


Figure 2.92

Solution

a. The y-coordinates of the graph of g are twice the y-coordinates of the graph of f. Therefore, the graph of g is stretched away from the x-axis, as shown in Figure 2.93.

b. The y-coordinates of the graph of h are one-half the y-coordinates of the graph of f. Therefore, the graph of h is compressed toward the x-axis, as shown in Figure 2.94.

(continued)

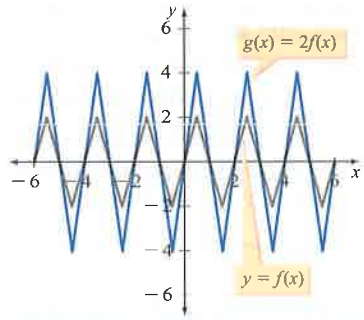


Figure 2.93

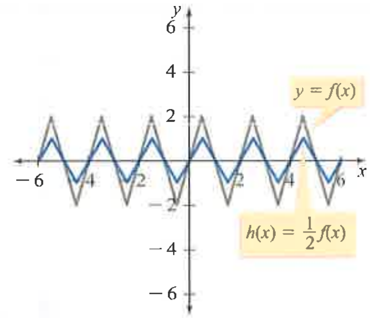


Figure 2.94

▶ Try Exercise 64, page 224

Some functions can be graphed by using horizontal compressing or stretching of a given graph. The procedure uses the following concept.

Horizontal Compressing and Stretching of Graphs

Assume that f is a function and c is a positive constant. Then,

- if $c > 1$, the graph of $y = f(c \cdot x)$ is the graph of $y = f(x)$ *compressed* horizontally toward the y-axis by a factor of $\frac{1}{c}$.
- if $0 < c < 1$, the graph of $y = f(c \cdot x)$ is the graph of $y = f(x)$ *stretched* horizontally away from the y-axis by a factor of $\frac{1}{c}$.

If the point (x, y) is on the graph of $y = f(x)$, then the graph of $y = f(cx)$ will contain the point $\left(\frac{1}{c}x, y\right)$.

EXAMPLE 9 Graph by Using Horizontal Compressing or Stretching

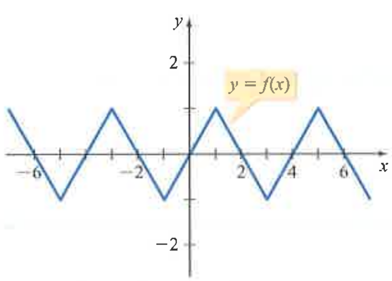


Figure 2.95

Use the graph of $y = f(x)$, shown in Figure 2.95, to graph the following.

a. $y = f(2x)$ **b.** $y = f\left(\frac{1}{3}x\right)$

Solution

a. Because $2 > 1$, the graph of $y = f(2x)$ is a horizontal compression of the graph of $y = f(x)$ by a factor of $\frac{1}{2}$. For example, the point $(2, 0)$ on the graph of $y = f(x)$ becomes the point $(1, 0)$ on the graph of $y = f(2x)$. See Figure 2.96.

b. Because $0 < \frac{1}{3} < 1$, the graph of $y = f\left(\frac{1}{3}x\right)$ is a horizontal stretching of the graph of $y = f(x)$ by a factor of 3. For example, the point $(1, 1)$ on the graph of $y = f(x)$ becomes the point $(3, 1)$ on the graph of $y = f\left(\frac{1}{3}x\right)$. See Figure 2.97.

(continued)

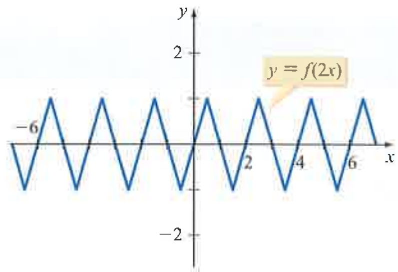


Figure 2.96

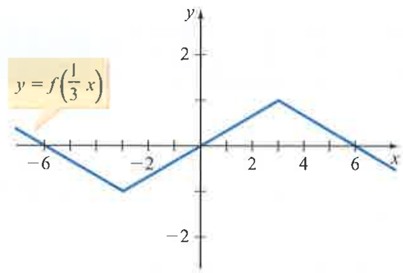


Figure 2.97

Try Exercise 68, page 224

Scan the QR code to access this demostration.

http://www.wadsworthmedia.com/math/precalculus_simulations/compressStretch.html

Integrating Technology

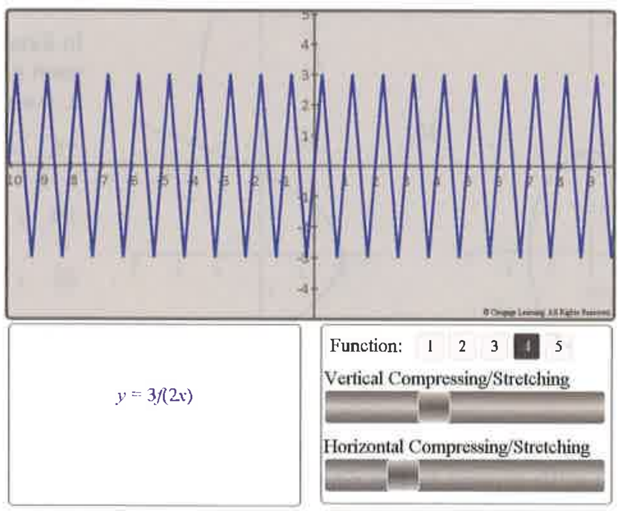
Vertical and Horizontal Compressing and Stretching of Graphs

An interactive demonstration that allows you to explore vertical and horizontal compressing and stretching the graph of a function is available online. You can access this demonstration by scanning the QR code at the left or by entering

http://www.wadsworthmedia.com/math/precalculus_simulations/compressStretch.html

in a search engine.

There are five different functions from which to choose. The graph below shows $y = 3f(2x)$ for function 4, a triangle wave.



Suggestions for things to try:

- Select function 3. Move the **Vertical Compressing/Stretching** slide. Note how the position of the graph changes. Select other functions and verify that moving the **Vertical Compressing/Stretching** slide affects the graph in the same way.
- Select another function and move the **Horizontal Compressing/Stretching** slide. Note how the position of the graph changes. Select other functions and verify that moving the **Horizontal Compressing/Stretching** slide affects the graph in the same way.

EXERCISE SET 2.5

Concept Check

In Exercises 1 to 6, plot the image of the given point with respect to

a. **the y-axis. Label this point A.**
b. **the x-axis. Label this point B.**
c. **the origin. Label this point C.**

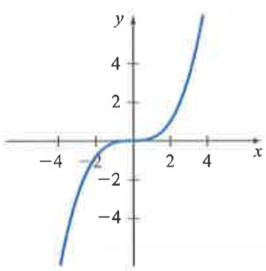
1. $P(5, -3)$ **2.** $Q(-4, 1)$

3. $R(-2, 3)$ **4.** $S(-5, 3)$

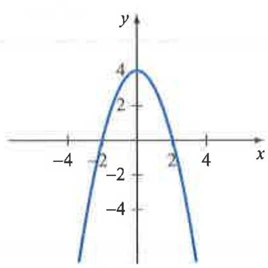
5. $T(-4, -5)$ **6.** $U(5, 1)$

In Exercises 7 to 10, determine whether the graph is symmetric with respect to the x-axis, the y-axis, the origin, or none of these. A graph may have more than one type of symmetry.

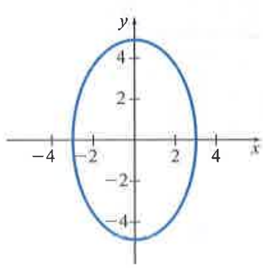
7.



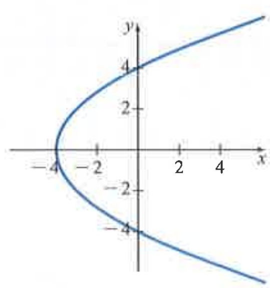
8.



9.

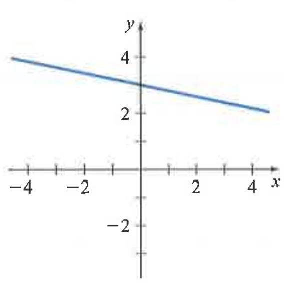


10.

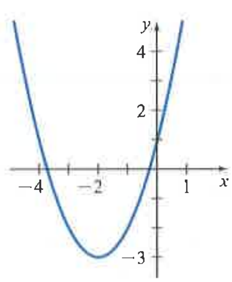


In Exercises 11 and 12, sketch a graph that is symmetric to the given graph with respect to the x-axis.

11.

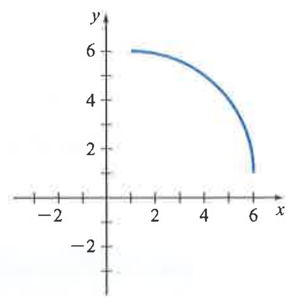


12.

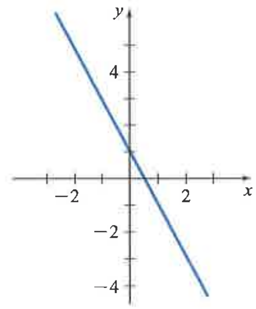


In Exercises 13 and 14, sketch a graph that is symmetric to the given graph with respect to the y-axis.

13.

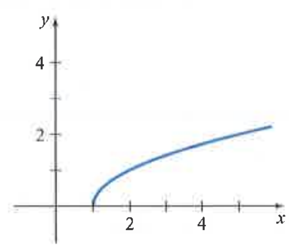


14.

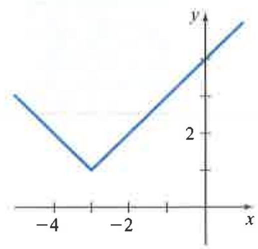


In Exercises 15 and 16, sketch a graph that is symmetric to the given graph with respect to the origin.

15.



16.



In Exercises 17 to 25, determine whether the graph of each equation is symmetric with respect to the a. x-axis, b. y-axis.

17. $y = 2x^2 - 5$ **18.** $x = 3y^2 - 7$ **19.** $y = x^3 + 2$

20. $y = x^5 - 3x$ **21.** $x^2 + y^2 = 9$ **22.** $x^2 - y^2 = 10$

23. $x^2 = y^4$ **24.** $xy = 8$ **25.** $|x| - |y| = 6$

In Exercises 26 to 34, determine whether the graph of each equation is symmetric with respect to the origin.

26. $y = x + 1$ **27.** $y = 3x - 2$ **28.** $y = x^3 - x$

29. $y = -x^3$ **30.** $y = \dfrac{9}{x}$ **31.** $x^2 + y^2 = 10$

32. $x^2 - y^2 = 4$ **33.** $y = \dfrac{x}{|x|}$ **34.** $|y| = |x|$

In Exercises 35 to 48, determine whether the given function is an even function, an odd function, or neither.

35. $g(x) = x^2 - 7$ **36.** $h(x) = x^2 + 1$

37. $F(x) = x^5 + x^3$ **38.** $G(x) = 2x^5 - 10$

39. $H(x) = 3|x|$ **40.** $T(x) = |x| + 2$

Indicates Try It Exercises

41. $f(x) = 1$

42. $k(x) = x^2 + 4x + 8$

43. $r(x) = \sqrt{x^2 + 4}$

44. $u(x) = \sqrt{3 - x^2}$

45. $s(x) = 16x^2$

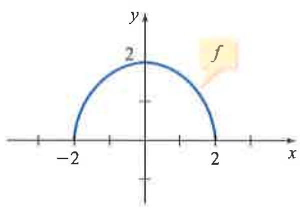
46. $v(x) = 16x^2 + x$

47. $w(x) = 4 + \sqrt[3]{x}$

48. $z(x) = \dfrac{x^3}{x^2 + 1}$

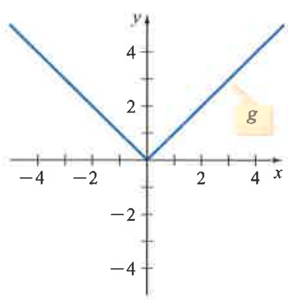
49. Use the graph of f to sketch the graph of

a. $y = f(x) + 3$ **b.** $y = f(x - 3)$



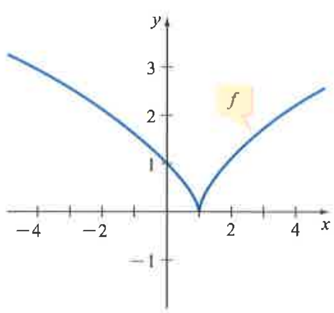
50. Use the graph of g to sketch the graph of

a. $y = g(x) - 2$ **b.** $y = g(x - 3)$



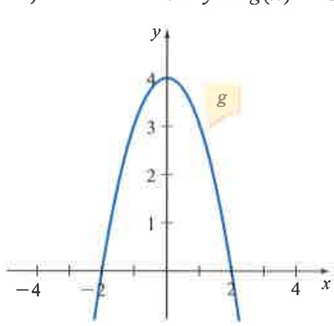
51. Use the graph of f to sketch the graph of

a. $y = f(x + 2)$ **b.** $y = f(x) + 2$



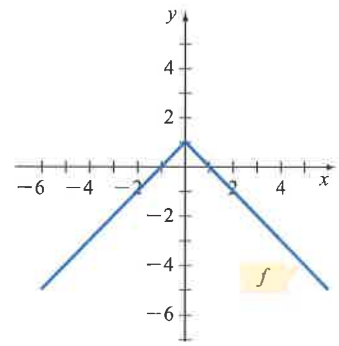
52. Use the graph of g to sketch the graph of

a. $y = g(x - 1)$ **b.** $y = g(x) - 1$



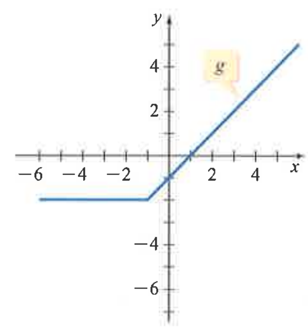
53. Use the graph of f to sketch a graph of

a. $y = f(x - 2) + 1$ **b.** $y = f(x + 3) - 2$



54. Use the graph of g to sketch a graph of

a. $y = g(x + 3) + 2$ **b.** $y = g(x - 2) - 1$



55. Let f be a function such that $f(-2) = 5$, $f(0) = -2$, and $f(1) = 0$. Give the coordinates of three points on the graph of

a. $y = f(x + 3)$ **b.** $y = f(x) + 1$

56. Let g be a function such that $g(-3) = -1$, $g(1) = -3$, and $g(4) = 2$. Give the coordinates of three points on the graph of

a. $y = g(x - 2)$ **b.** $y = g(x) - 2$

57. Let f be a function such that $f(-1) = 3$ and $f(2) = -4$. Give the coordinates of two points on the graph of

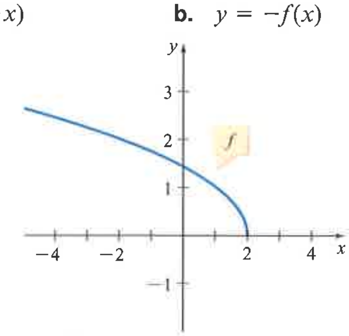
a. $y = f(-x)$ **b.** $y = -f(x)$

58. Let g be a function such that $g(4) = -5$ and $g(-3) = 2$. Give the coordinates of two points on the graph of

a. $y = -g(x)$ **b.** $y = g(-x)$

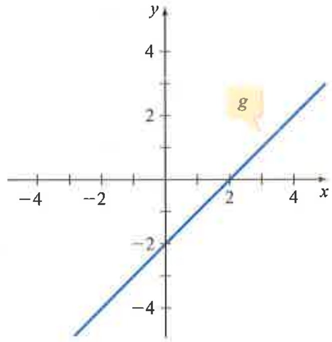
59. Use the graph of f to sketch the graph of

a. $y = f(-x)$ **b.** $y = -f(x)$



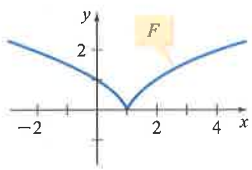
60. Use the graph of g to sketch the graph of

a. $y = -g(x)$ **b.** $y = g(-x)$



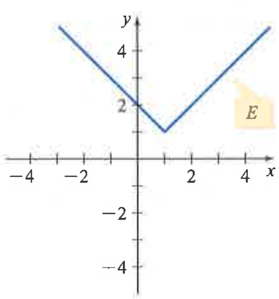
61. Use the graph of F to sketch the graph of

a. $y = -F(x)$ **b.** $y = F(-x)$



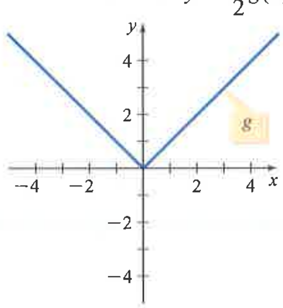
62. Use the graph of E to sketch the graph of

a. $y = -E(x)$ **b.** $y = E(-x)$



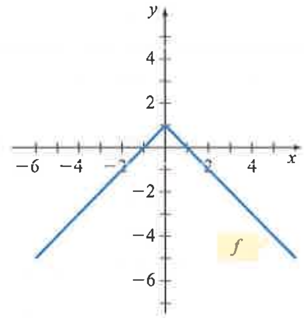
63. Use the graph of $y = g(x)$ to sketch the graph of

a. $y = 2g(x)$ **b.** $y = \frac{1}{2}g(x)$



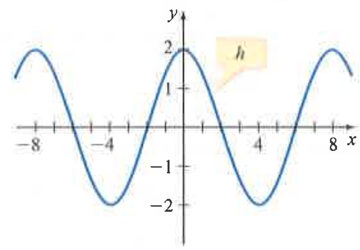
64. Use the graph of $y = f(x)$ to sketch the graph of

a. $y = 2f(x)$ **b.** $y = \frac{1}{2}f(x)$



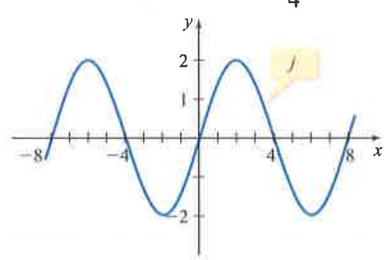
65. Use the graph of $y = h(x)$ to sketch the graph of

a. $y = 3h(x)$ **b.** $y = \frac{1}{2}h(x)$



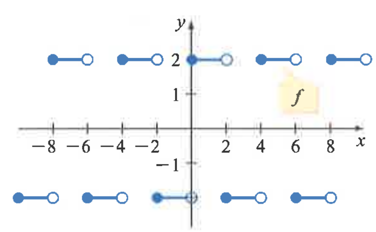
66. Use the graph of $y = j(x)$ to sketch the graph of

a. $y = 4j(x)$ **b.** $y = \frac{1}{4}j(x)$



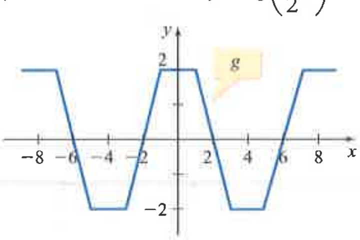
67. Use the graph of $y = f(x)$ to sketch the graph of

a. $y = f(2x)$ **b.** $y = f\left(\frac{1}{3}x\right)$



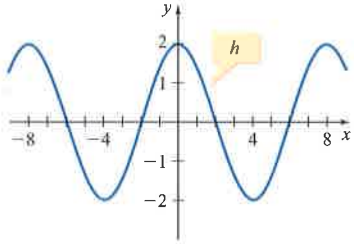
68. Use the graph of $y = g(x)$ to sketch the graph of

a. $y = g(2x)$ **b.** $y = g\left(\frac{1}{2}x\right)$



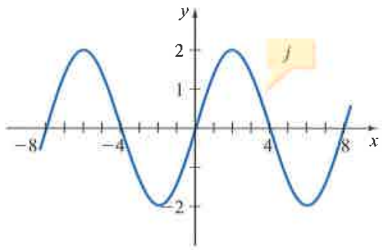
69. Use the graph of $y = h(x)$ to sketch the graph of

a. $y = h(2x)$ **b.** $y = h\left(\frac{1}{2}x\right)$

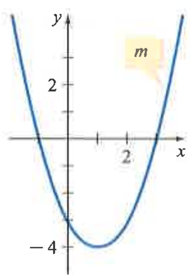


70. Use the graph of $y = j(x)$ to sketch the graph of

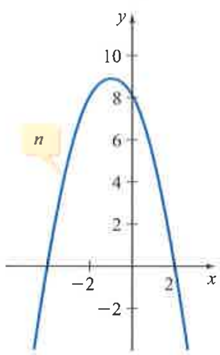
a. $y = j(2x)$ **b.** $y = j\left(\frac{1}{3}x\right)$



71. Use the graph of m to sketch the graph of $y = -\frac{1}{2}m(x) + 3$.

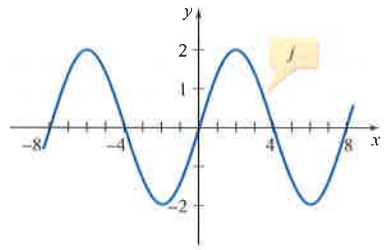


72. Use the graph of n to sketch the graph of $y = \frac{1}{2}n(x) + 1$.



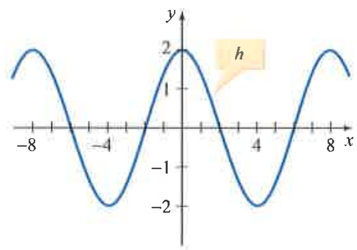
73. Use the graph of $y = j(x)$ to sketch the graph of

a. $y = -\frac{1}{2}j(x) + 1$ **b.** $y = 2j(x) - 1$



74. Use the graph of $y = h(x)$ to sketch the graph of

a. $y = \frac{1}{2}h(x) - 1$ **b.** $y = -2h(x) + 1$



In Exercises 75 to 78, use a graphing utility to graph a function containing a variable *c*. One way to do this is to use a list, a very powerful tool in many branches of mathematics. A list is enclosed in braces. For instance, to graph $y = x^2 + c$ for $c = -2, 0,$ and 3, enter Y1 = x^2 + {−2, 0, 3}. Then press Graph.

75. On the same coordinate axes, graph

$$G(x) = \sqrt[3]{x} + c$$

for $c = 0, -1,$ and 3.

76. On the same coordinate axes, graph

$$H(x) = \sqrt[3]{x + c}$$

for $c = 0, -1,$ and 3.

77. On the same coordinate axes, graph

$$L(x) = cx^2$$

for $c = 1, \frac{1}{2},$ and 2.

78. On the same coordinate axes, graph

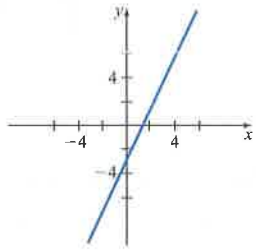
$$M(x) = c\sqrt{x^2 - 4}$$

for $c = 1, \frac{1}{3},$ and 3.

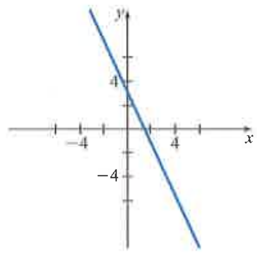
Enrichment Exercises

In Exercises 79 to 82, draw the graph that results from applying the operations to the given graph.

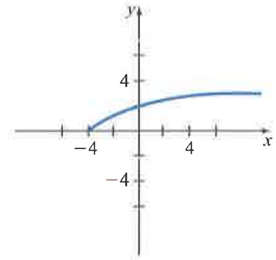
79. Reflect the graph about the y-axis and then about the origin.



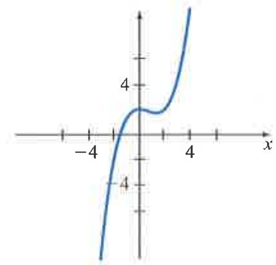
80. Reflect the graph about the origin and then about the y-axis.



81. Reflect the graph about the y-axis and then about the x-axis.



82. Reflect the graph about the x-axis and then about the origin.



SECTION 2.6

Operations on Functions
Difference Quotient
Composition of Functions

Algebra of Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A13.

PS1. Subtract: $(2x^2 + 3x - 4) - (x^2 + 3x - 5)$ [P.3]

PS2. Multiply: $(3x^2 - x + 2)(2x - 3)$ [P.3]

In Exercises PS3 and PS4, find each of the following for $f(x) = 2x^2 - 5x + 2$.

PS3. $f(3a)$ [2.2]

PS4. $f(2 + h)$ [2.2]

In Exercises PS5 and PS6, find the domain of each function.

PS5. $F(x) = \dfrac{x}{x-1}$ [2.2]

PS6. $r(x) = \sqrt{2x - 8}$ [2.2]

Operations on Functions

Functions can be defined in terms of other functions. For example, the function defined by $h(x) = x^2 + 8x$ is the sum of

$$f(x) = x^2 \qquad \text{and} \qquad g(x) = 8x$$

Thus, if we are given any two functions f and g, we can define the four new functions $f + g$, $f - g$, fg, and $\dfrac{f}{g}$ as follows.

Definitions of Operations on Functions

If f and g are functions with domains D_f and D_g, then we define the sum, difference, product, and quotient of f and g as

Sum	$(f + g)(x) = f(x) + g(x)$	Domain: $D_f \cap D_g$
Difference	$(f - g)(x) = f(x) - g(x)$	Domain: $D_f \cap D_g$