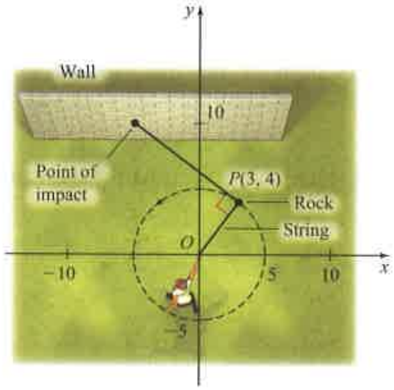
91. Point of Impact A rock attached to a string is whirled horizontally about the origin in a counterclockwise circular path with radius 5 feet. When the string breaks, the rock travels on a linear path perpendicular to the radius OP and hits a wall located at $y = 10$ feet.



If the string breaks when the rock is at $P(3, 4)$, find the x-coordinate of the point at which the rock hits the wall.

92. Point of Impact A rock attached to a string is whirled horizontally about the origin in a counterclockwise circular path with radius 4 feet. When the string breaks, the rock travels on a linear path perpendicular to the radius OP and hits a wall located at $y = 14$ feet. If the string breaks when the rock is at $P(\sqrt{15}, 1)$, find the x-coordinate of the point at which the rock hits the wall.

Enrichment Exercises

93. Find the value of a in the domain of $f(x) = 2x + 3$ for which $f(a) = -1$.

94. Find the value of a in the domain of $f(x) = 4 - 3x$ for which $f(a) = 7$.

95. Find the value of a in the domain of $f(x) = 1 - 4x$ for which $f(a) = 3$.

96. Find the value of a in the domain of $f(x) = \dfrac{2x}{3} + 2$ for which $f(a) = 4$.

97. Slope of a Secant Line The graph of $f(x) = x^2 + 1$ is shown in the next column along with points P and Q. The secant line PQ is also shown.

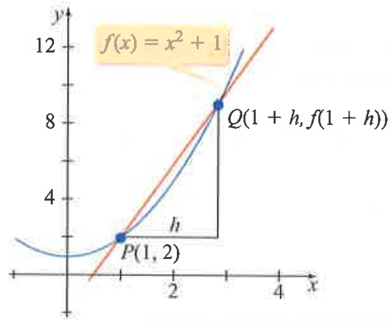
a. If $h = 1$, determine the coordinates of Q and the slope of the line PQ.

b. If $h = 0.1$, determine the coordinates of Q and the slope of the line PQ.

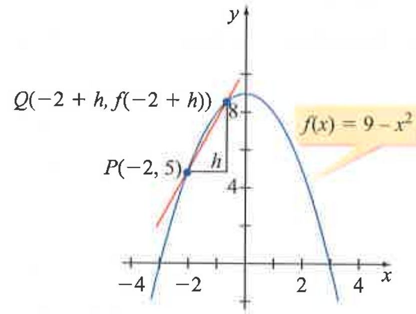
c. If $h = 0.01$, determine the coordinates of Q and the slope of the line PQ.

d. As h approaches 0, what value does the slope of the line PQ seem to be approaching?

e. Show that the slope of the line PQ is $2 + h$.



98. Slope of a Secant Line The graph of $f(x) = 9 - x^2$ is shown below along with points P and Q. The secant line PQ is also shown.



a. If $h = 1$, determine the coordinates of Q and the slope of the line PQ.

b. If $h = 0.1$, determine the coordinates of Q and the slope of the line PQ.

c. If $h = 0.01$, determine the coordinates of Q and the slope of the line PQ.

d. As h approaches 0, what value does the slope of the line PQ seem to be approaching?

e. Show that the slope of the line PQ is $4 - h$.

99. Determine the point $P(x, y)$ on the graph of the equation $y = x^2$ such that the slope of the line through the point $(3, 9)$ and P is $\frac{15}{2}$.

100. Determine the point $P(x, y)$ on the graph of the equation $y = \sqrt{x + 1}$ such that the slope of the line through the point $(3, 2)$ and P is $\frac{3}{8}$.

MID-CHAPTER 2 QUIZ

1. Find the coordinates of the midpoint and the length of the line segment between $P_1(-3, 4)$ and $P_2(1, -2)$.

2. Find the coordinates of the center and the radius of the circle whose equation is $x^2 + y^2 - 6x + 4y - 2 = 0$.

3. Evaluate $f(x) = x^2 - 6x + 1$ when $x = -3$.

4. Write the domain of $f(x) = \sqrt{2 - x}$ in interval notation.

5. Find the zeros of $f(x) = x^2 - x - 12$.

6. Find the slope of the line between the points $P_1(8, -2)$ and $P_2(-2, 3)$.

7. Find the equation of the line parallel to the graph of $2x + 3y = 5$ and passing through $P_1(3, -1)$.

8. Graph $f(x) = -\frac{2}{3}x + 1$ by using the slope and y-intercept.

SECTION 2.4

Quadratic Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A10.

PS1. Factor: $3x^2 + 10x - 8$ [P.4]

PS2. Complete the square of $x^2 - 8x$. Write the resulting trinomial as the square of a binomial. [1.3]

PS3. Find $f(-3)$ for $f(x) = 2x^2 - 5x - 7$. [2.2]

PS4. Solve for x: $2x^2 - x = 1$ [1.3]

PS5. Solve for x: $x^2 + 3x - 2 = 0$ [1.3]

PS6. Suppose that $h = -16t^2 + 64t + 5$. Find two values of t for which $h = 53$. [1.3]

Some applications can be modeled by a *quadratic function.*

Definition of a Quadratic Function

A **quadratic function** of x is a function that can be represented by an equation of the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$.

EXAMPLE

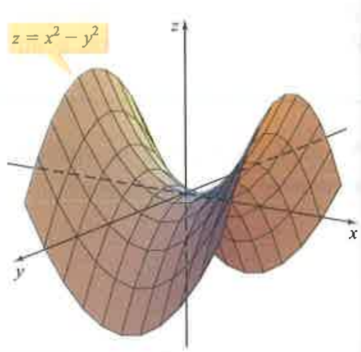
$f(x) = 2x^2 - 3x + 1$ • $a = 2, b = -3, c = 1$

$g(x) = -x^2 - 5$ • $a = -1, b = 0, c = -5$

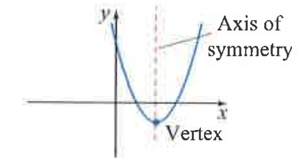
$h(x) = x^2 + 5x$ • $a = 1, b = 5, c = 0$

Note

The equation $z = x^2 - y^2$ defines z as a quadratic function of x and y. The graph of $z = x^2 - y^2$ is the *saddle* shown in the figure below. You will study quadratic functions involving two or more independent variables in calculus.

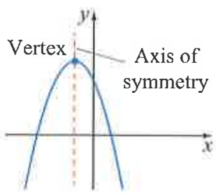


The graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, is a *parabola*. The graph opens up when $a > 0$, as in Figure 2.51a, and opens down when $a < 0$, as in Figure 2.51b. The **vertex of a parabola** is the lowest point on a parabola that opens up or the highest point on a parabola that opens down. The graph of a parabola has an **axis of symmetry**, a vertical line through the vertex such that if the parabola were folded along that line, the two parts of the graph would match up.



$f(x) = ax^2 - bx + c, a > 0$

Figure 2.51a



$f(x) = ax^2 + bx + c, a < 0$

Figure 2.51b

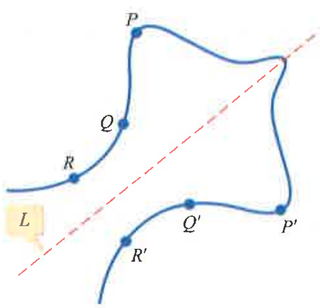


Figure 2.52

Note

The axis of symmetry is a line. When you are asked to determine the axis of symmetry, the answer is an equation, not just a number.

We now give a more formal definition of what it means for a graph to be symmetric about a line.

Definition of Symmetry with Respect to a Line

A graph is **symmetric with respect to a line** L if for each point P on the graph there is a point P' on the graph such that the line L is the perpendicular bisector of the line segment PP'.

The graph in Figure 2.52 is symmetric with respect to the line L. Note that the graph has the property that if the paper is folded along the dotted line, the point P will coincide with the point P', the point Q will coincide with the point Q', and the point R will coincide with the point R'. One part of the graph is a **mirror image** of the rest of the graph across the line L.

Standard Form of a Quadratic Function

The graph of a parabola can be drawn by finding the vertex and the axis of symmetry. Then find a few points on the graph of the parabola on one side of the axis of symmetry and use symmetry with respect to that axis to draw the graph. To do this, we write $f(x) = ax^2 + bx + c, a \neq 0$, in what is called the *standard form of a quadratic function.*

Standard Form of a Quadratic Function

Every quadratic function f given by $f(x) = ax^2 + bx + c$ can be written in the **standard form of a quadratic function,**

$$f(x) = a(x - h)^2 + k, \qquad a \neq 0$$

The graph of f is a parabola with vertex (h, k). The parabola opens up if $a > 0$, and it opens down if $a < 0$. The vertical line $x = h$ is the axis of symmetry of the parabola.

EXAMPLE

$f(x) = (x - 3)^2 - 4$ • $a = 1 > 0$; parabola opens up
Vertex $(3, -4)$; axis of symmetry $x = 3$

$f(x) = -2(x + 1)^2 + 1$ • $a = -2 < 0$; parabola opens down
Vertex $(-1, 1)$; axis of symmetry $x = -1$

EXAMPLE 1 Find the Standard Form of a Quadratic Function

Use the technique of completing the square to find the standard form of $g(x) = 2x^2 - 12x + 19$. Sketch the graph.

Solution

$g(x) = 2x^2 - 12x + 19$

$= 2(x^2 - 6x) + 19$ • Factor 2 from the variable terms.

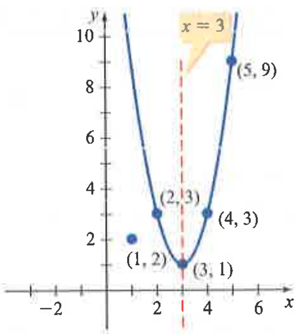
$= 2(x^2 - 6x + 9 - 9) + 19$ • Complete the square.

$= 2(x^2 - 6x + 9) - 2(9) + 19$ • Regroup.

$= 2(x - 3)^2 - 18 + 19$ • Factor and simplify.

$= 2(x - 3)^2 + 1$ • Use standard form.

(continued)



$g(x) = 2x^2 - 12x + 19$

Figure 2.53

The vertex is (3,1). The axis of symmetry is $x = 3$. Because $a > 0$, the parabola opens up. Evaluate the function for two values of x to the right of the axis of symmetry, say $x = 4$ and $x = 5$. Plot these points and use symmetry to draw the graph. See Figure 2.53.

▶ **Try Exercise 10, page 206**

We can write $f(x) = ax^2 + bx + c$ in standard form by completing the square of $ax^2 + bx + c$. This will allow us to derive a general expression for the vertex and axis of symmetry of $f(x) = ax^2 + bx + c$.

$$f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c \qquad \text{• Factor } a \text{ from } ax^2 + bx.$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a} \qquad \text{• Complete the square by adding and subtracting } \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 = \frac{b^2}{4a^2}.$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \qquad \text{• Factor and simplify.}$$

Thus $f(x) = ax^2 + bx + c$ written in standard form is $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$. Comparing this last expression with $f(x) = a(x - h)^2 + k$, we see that the coordinates of the vertex are $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$.

Note that by evaluating $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$ at $x = -\frac{b}{2a}$, we have

$$f\left(-\frac{b}{2a}\right) = a\left(-\frac{b}{2a} + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

$$= a(0) + \frac{4ac - b^2}{4a} = \frac{4ac - b^2}{4a}$$

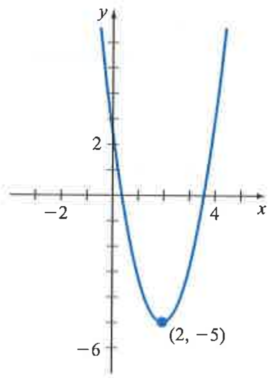
That is, the y-coordinate of the vertex is $f\left(-\frac{b}{2a}\right)$. This result is summarized by the following formula.

Vertex Formula

The coordinates of the vertex of $f(x) = ax^2 + bx + c$ are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The vertex formula can be used to write the standard form of the equation of a parabola. We have

$$h = -\frac{b}{2a} \quad \text{and} \quad k = f\left(-\frac{b}{2a}\right)$$



$f(x) = 2x^2 - 8x + 3$

Figure 2.54

EXAMPLE 2 Find the Vertex and Standard Form of a Quadratic Function

Use the vertex formula to find the vertex and standard form of $f(x) = 2x^2 - 8x + 3$.

Solution

$$f(x) = 2x^2 - 8x + 3 \qquad \bullet\ a = 2,\ b = -8,\ c = 3$$

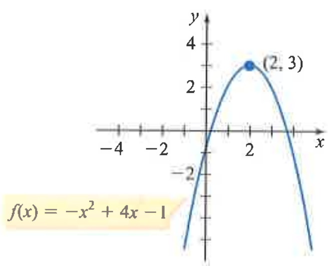
$$h = -\frac{b}{2a} = -\frac{-8}{2(2)} = 2 \qquad \bullet\ x\text{-coordinate of the vertex}$$

$$k = f\left(-\frac{b}{2a}\right) = 2(2)^2 - 8(2) + 3 = -5 \qquad \bullet\ y\text{-coordinate of the vertex}$$

The vertex is $(2, -5)$. Substituting into the standard form equation $f(x) = a(x - h)^2 + k$ yields the standard form $f(x) = 2(x - 2)^2 - 5$.

The graph of f is shown in Figure 2.54.

▶ **Try Exercise 20, page 207**



Maximum and Minimum of a Quadratic Function

Note from Example 2 that the graph of the parabola opens up and the vertex is the *lowest* point on the graph of the parabola. Therefore, the y-coordinate of the vertex is the *minimum* value of that function. This information can be used to determine the range of $f(x) = 2x^2 - 8x + 3$. The range is $\{y \mid y \geq -5\}$. Similarly, if the graph of a parabola opened down, the vertex would be the *highest* point on the graph and the y-coordinate of the vertex would be the *maximum* value of the function. For instance, the maximum value of $f(x) = -x^2 + 4x - 1$, graphed at the left, is 3, the y-coordinate of the vertex. The range of the function is $\{y \mid y \leq 3\}$. For the function in Example 2 and the function whose graph is shown at the left, the domain is the set of real numbers.

EXAMPLE 3 Find the Range of $f(x) = ax^2 + bx + c$

Find the range of $f(x) = -2x^2 - 6x - 1$.

Algebraic Solution

To find the range of f, determine the y-coordinate of the vertex of the graph of f.

$$f(x) = -2x^2 - 6x - 1 \qquad \bullet\ a = -2,\ b = -6,\ c = -1$$

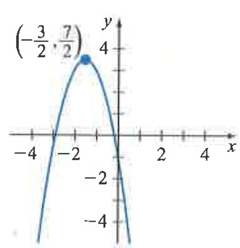
$$h = -\frac{b}{2a} = -\frac{-6}{2(-2)} = -\frac{3}{2} \qquad \bullet\ \text{Find the } x\text{-coordinate of the vertex.}$$

$$k = f\left(-\frac{3}{2}\right) = -2\left(-\frac{3}{2}\right)^2 - 6\left(-\frac{3}{2}\right) - 1 = \frac{7}{2} \qquad \bullet\ \text{Find the } y\text{-coordinate of the vertex.}$$

The vertex is $\left(-\frac{3}{2}, \frac{7}{2}\right)$. Because the parabola opens down, $\frac{7}{2}$ is the maximum value of f. Therefore, the range of f is $\left\{y \mid y \leq \frac{7}{2}\right\}$.

Visualize the Solution

The graph of f is shown below. The vertex of the graph is $\left(-\frac{3}{2}, \frac{7}{2}\right)$.



▶ **Try Exercise 32, page 207**

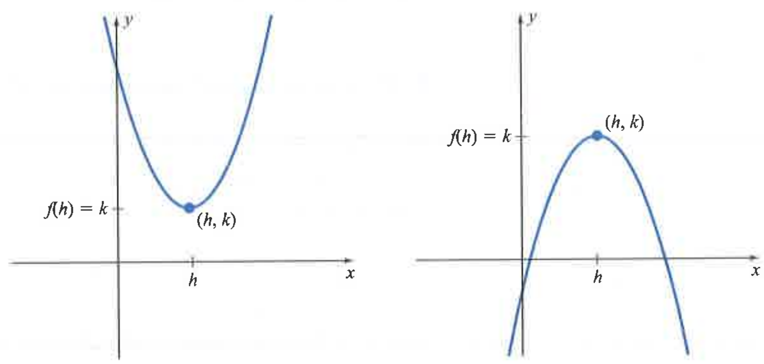
The following theorem can be used to determine the maximum value or the minimum value of a quadratic function.

Maximum or Minimum Value of a Quadratic Function

If $a > 0$, then the vertex (h, k) is the lowest point on the graph of $f(x) = a(x - h)^2 + k$ and the y-coordinate k of the vertex is the **minimum value** of the function f. See Figure 2.55a.

If $a < 0$, then the vertex (h, k) is the highest point on the graph of $f(x) = a(x - h)^2 + k$ and the y-coordinate k is the **maximum value** of the function f. See Figure 2.55b.

In either case, the maximum or minimum value is achieved when $x = h$.



a. k is the minimum value of f **b.** k is the maximum value of f

Figure 2.55

EXAMPLE 4 Find the Maximum or Minimum of a Quadratic Function

Find the maximum or minimum value of each quadratic function. State whether the value is a maximum or a minimum.

a. $F(x) = -2x^2 + 8x - 1$

b. $G(x) = x^2 - 3x + 1$

Solution

The maximum or minimum value of a quadratic function is the y-coordinate of the vertex of the graph of the function.

a. $h = -\dfrac{b}{2a} = -\dfrac{8}{2(-2)} = 2$ • x-coordinate of the vertex

$k = F\left(-\dfrac{b}{2a}\right) = -2(2)^2 + 8(2) - 1 = 7$ • y-coordinate of the vertex

Because $a < 0$, the function has a maximum value but no minimum value. The maximum value is 7. See Figure 2.56.

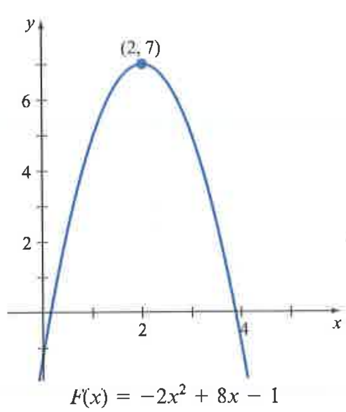


Figure 2.56

b. $h = -\frac{b}{2a} = -\frac{-3}{2(1)} = \frac{3}{2}$ • x-coordinate of the vertex

$$k = G\left(-\frac{b}{2a}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 1$$

$$= -\frac{5}{4}$$ • y-coordinate of the vertex

Because $a > 0$, the function has a minimum value but no maximum value. The minimum value is $-\frac{5}{4}$. See Figure 2.57.

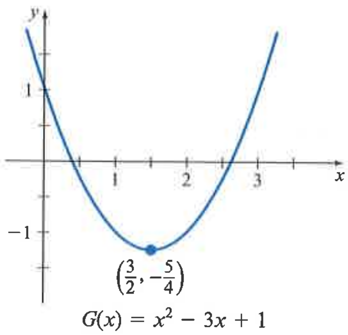


Figure 2.57

Try Exercise 38, page 207

Applications of Quadratic Functions

EXAMPLE 5 Calculate the Airtime for a Snowboarder's Jump

The height $h(t)$, in feet, of a snowboarder t seconds after beginning a certain jump can be approximated by $h(t) = -16t^2 + 22.9t + 9$. If the snowboarder lands at a point that is 3 feet below the base of the jump, determine the *airtime* (the time the snowboarder is in the air) for this jump. Round to the nearest tenth of a second.

Solution

Because $h(t)$ represents the height of the snowboarder t seconds after the beginning of the jump, the snowboarder lands when $h(t) = -3$, 3 feet *below* the base of the jump.

$$h(t) = -16t^2 + 22.9t + 9$$

$$-3 = -16t^2 + 22.9t + 9$$ • Replace $h(t)$ with -3.

$$0 = -16t^2 + 22.9t + 12$$

$$t = \frac{-22.9 \pm \sqrt{22.9^2 - 4(-16)(12)}}{2(-16)}$$ • Use the quadratic formula.

$$= \frac{-22.9 \pm \sqrt{1292.41}}{-32}$$

$$\approx -0.4 \quad \text{or} \quad 1.8$$ • Use a calculator.

Because a negative time is not possible, the airtime for this jump is approximately 1.8 seconds.

Try Exercise 48, page 207

EXAMPLE 6 Find the Maximum of a Quadratic Function

A long sheet of tin 20 inches wide is to be made into a trough by bending up two sides until they are perpendicular to the bottom. How many inches should be turned up so that the trough will achieve its maximum carrying capacity?

Solution

The trough is shown in Figure 2.58. If x is the number of inches to be turned up on each side, then the width of the base is $20 - 2x$ inches. The maximum carrying capacity of the trough will occur when the cross-sectional area is a maximum. The cross-sectional area $A(x)$ is given by

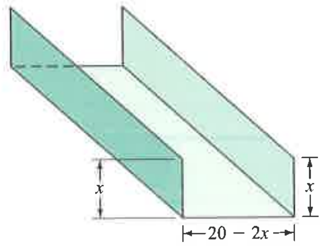


Figure 2.58

$$A(x) = x(20 - 2x) \quad \text{• Area = (length)(width)}$$
$$= -2x^2 + 20x$$

To find the point at which A obtains its maximum value, find the x-coordinate of the vertex of the graph of A. Using the vertex formula with $a = -2$ and $b = 20$, we have

$$x = -\frac{b}{2a} = -\frac{20}{2(-2)} = 5$$

Therefore, the maximum carrying capacity will be achieved when 5 inches are turned up. See Figure 2.59.

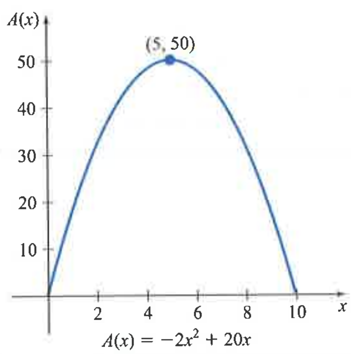


Figure 2.59

▸ Try Exercise 50, page 207

EXAMPLE 7 Solve a Business Application

A sporting goods store can sell 4 sets of golf clubs per week at a price of \$500 per set. The store manager estimates that for each \$25 price reduction per set, one more set of golf clubs can be sold each month. The golf clubs cost the store \$250 each. Find the price at which the store should sell a set of golf clubs to maximize the store's monthly profit on golf club sales.

Solution

Let x = the number of \$25 price reductions. Then the price per set, $p(x)$, is \$500 less x \$25 reductions.

$$p(x) = 500 - 25x \quad \text{• Price per set}$$

The number of sets sold each week, $q(x)$, is 4 plus x for each price reduction.

$$q(x) = 4 + x \quad \text{• Number of sets sold per week}$$

The revenue, $R(x)$, from the sale is the price per set times the number of sets sold.

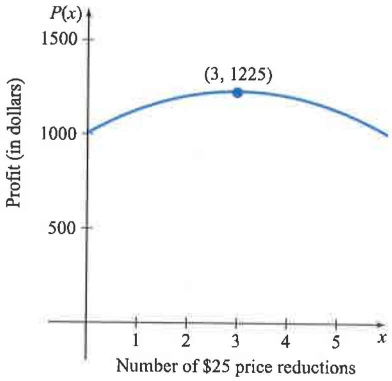
$$R(x) = (500 - 25x)(4 + x) = -25x^2 + 400x + 2000$$

The cost for the clubs, $C(x)$, is the cost to the store (\$250) times the number of sets sold.

$$C(x) = 250(4 + x) = 1000 + 250x$$

Profit, $P(x)$, equals revenue minus cost.

$$P(x) = (-25x^2 + 400x + 2000) - (1000 + 250x)$$
$$= -25x^2 + 150x + 1000$$



$P(x) = -25x^2 + 150x + 1000$

The graph of the profit function is a parabola that opens down. Thus the maximum profit occurs when

$$x = -\frac{b}{2a} = -\frac{150}{2(-25)} = 3$$

To find the price per set that maximizes monthly profit, evaluate $p(x)$ when $x = 3$.

$$p(x) = 500 - 25x$$
$$p(3) = 500 - 25(3) = 425$$

The store should sell a set of golf clubs for $425 to maximize profit.

Try Exercise 66, page 209.

EXAMPLE 8 Solve a Projectile Application

In Figure 2.60, a ball is thrown vertically upward with an initial velocity of 48 feet per second. If the ball started its flight at a height of 8 feet, then its height at time t can be determined by $s(t) = -16t^2 + 48t + 8$, where $s(t)$ is measured in feet above ground level and t is the number of seconds of flight.

a. Determine the time it takes the ball to attain its maximum height.

b. Determine the maximum height the ball attains.

c. Determine the time it takes the ball to hit the ground.

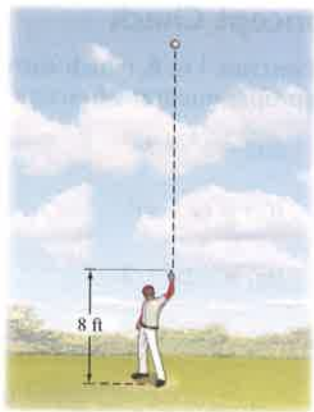


Figure 2.60

Solution

a. The graph of $s(t) = -16t^2 + 48t + 8$ is a parabola that opens downward. See Figure 2.61. Therefore, s will attain its maximum value at the vertex of its graph. Using the vertex formula with $a = -16$ and $b = 48$, we get

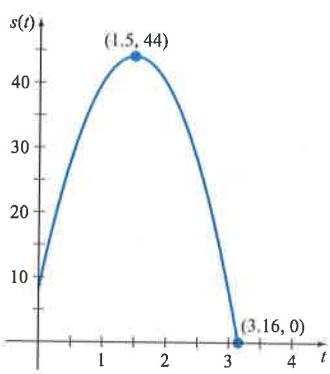
$$t = -\frac{b}{2a} = -\frac{48}{2(-16)} = \frac{3}{2}$$

Therefore, the ball attains its maximum height $1\frac{1}{2}$ seconds into its flight.

b. When $t = \frac{3}{2}$, the height of the ball is

$$s\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 8 = 44 \text{ feet}$$

c. The ball will hit the ground when its height $s(t) = 0$. Therefore, solve $-16t^2 + 48t + 8 = 0$ for t.



$s(t) = -16t^2 + 48t + 8$

Figure 2.61

(continued)

$$-16t^2 + 48t + 8 = 0$$

$$-2t^2 + 6t + 1 = 0 \qquad \text{• Divide each side by 8.}$$

$$t = \frac{-(6) \pm \sqrt{6^2 - 4(-2)(1)}}{2(-2)} \qquad \text{• Use the quadratic formula.}$$

$$= \frac{-6 \pm \sqrt{44}}{-4} = \frac{-3 \pm \sqrt{11}}{-2}$$

TO REVIEW

Quadratic Formula
See page 100.

Using a calculator to approximate the positive root, we find that the ball will hit the ground in $t \approx 3.16$ seconds. This is also the value of the t-coordinate of the t-intercept in Figure 2.61.

Try Exercise 70, page 209

EXERCISE SET 2.4

Concept Check

In Exercises 1 to 8, match each graph in *a.* through *h.* with the proper quadratic function.

1. $f(x) = x^2 - 3$

2. $f(x) = x^2 + 2$

3. $f(x) = (x - 4)^2$

4. $f(x) = (x + 3)^2$

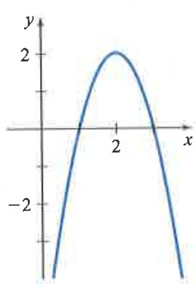
5. $f(x) = -2x^2 + 2$

6. $f(x) = -\frac{1}{2}x^2 + 3$

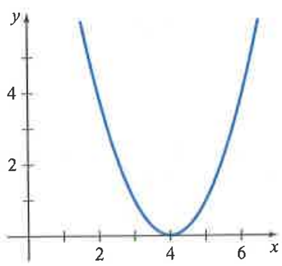
7. $f(x) = (x + 1)^2 + 3$

8. $f(x) = -2(x - 2)^2 + 2$

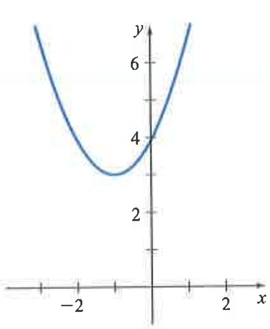
a.



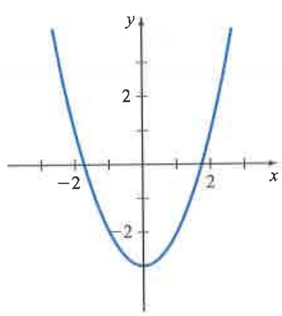
b.



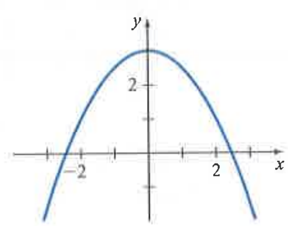
c.



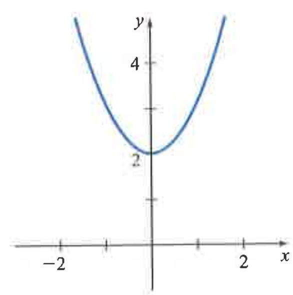
d.



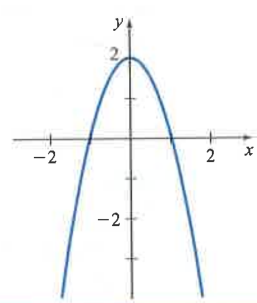
e.



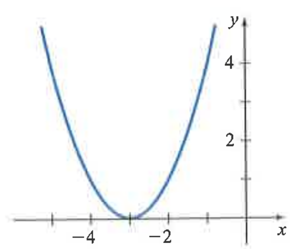
f.



g.



h.



In Exercises 9 to 18, use the method of completing the square to find the standard form of the quadratic function. State the vertex and axis of symmetry of the graph of the function and then sketch its graph.

9. $f(x) = x^2 + 4x + 1$

10. $f(x) = x^2 + 6x - 1$

11. $f(x) = x^2 - 8x + 5$

12. $f(x) = x^2 - 10x + 3$

13. $f(x) = x^2 + 3x + 1$

14. $f(x) = x^2 + 7x + 2$

15. $f(x) = -x^2 + 4x + 2$

16. $f(x) = -x^2 - 2x + 5$

17. $f(x) = -3x^2 + 3x + 7$

18. $f(x) = -2x^2 - 4x + 5$

Indicates Try It Exercises

In Exercises 19 to 28, use the vertex formula to determine the vertex of the graph of the function and write the function in standard form.

19. $f(x) = x^2 - 10x$

20. $f(x) = x^2 - 6x$

21. $f(x) = x^2 - 10$

22. $f(x) = x^2 - 4$

23. $f(x) = -x^2 + 6x + 1$

24. $f(x) = -x^2 + 4x + 1$

25. $f(x) = 2x^2 - 3x + 7$

26. $f(x) = 3x^2 - 10x + 2$

27. $f(x) = -4x^2 + x + 1$

28. $f(x) = -5x^2 - 6x + 3$

29. Find the range of $f(x) = x^2 - 2x - 1$.

30. Find the range of $f(x) = -x^2 - 6x - 2$.

31. Find the range of $f(x) = -2x^2 + 5x - 1$.

32. Find the range of $f(x) = 2x^2 + 6x - 5$.

In Exercises 33 to 36, find the real zeros of *f* and the *x*-intercepts of the graph of *f*.

33. $f(x) = x^2 + 2x - 24$

34. $f(x) = -x^2 + 6x + 7$

35. $f(x) = 2x^2 + 11x + 12$

36. $f(x) = 2x^2 - 9x + 10$

In Exercises 37 to 46, find the maximum or minimum value of the function. State whether this value is a maximum or a minimum.

37. $f(x) = x^2 + 8x$

38. $f(x) = -x^2 - 6x$

39. $f(x) = -x^2 + 6x + 2$

40. $f(x) = -x^2 + 10x - 3$

41. $f(x) = 2x^2 + 3x + 1$

42. $f(x) = 3x^2 + x - 1$

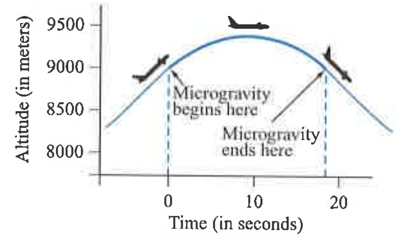
43. $f(x) = 5x^2 - 11$

44. $f(x) = 3x^2 - 41$

45. $f(x) = -\frac{1}{2}x^2 + 6x + 17$

46. $f(x) = -\frac{3}{4}x^2 - \frac{2}{5}x + 7$

47. Astronaut Training To prepare astronauts for the experience of zero gravity (technically, microgravity) in space, NASA uses a specially designed jet. A pilot accelerates the plane upward to an altitude of approximately 9000 meters and then reduces power. During the time of reduced power, the plane is in freefall and the astronauts experience microgravity. The altitude $A(t)$, in meters, of the plane t seconds after power was reduced can be approximated by $A(t) = -4.9t^2 + 90t + 9000$. The graph is shown as follows.



If the pilot increases power when the plane descends to 9000 meters, ending microgravity, find the time the astronauts experience microgravity during one of these maneuvers. Round to the nearest tenth of a second.

48. Soccer Ball Kick The height $h(t)$, in meters, above the ground of a certain soccer ball kick t seconds after the ball is kicked can be approximated by $h(t) = -4.9t^2 + 12.8t$. Determine the time for which the ball is in the air. Round to the nearest tenth of a second.

49. Height of an Arch The height of an arch is given by

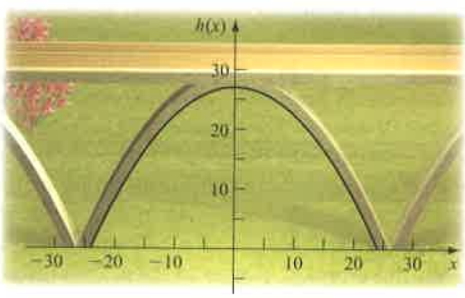
$$h(x) = -\frac{3}{64}x^2 + 27, \quad -24 \le x \le 24$$

where $|x|$ is the horizontal distance in feet from the center of the arch to the ground.

a. What is the maximum height of the arch?

b. What is the height of the arch 10 feet to the right of center?

c. How far from the center is the arch 8 feet tall?



50. Geometry The sum of the length l and the width w of a rectangular area is 240 meters.

a. Write w as a function of l.

b. Write the area A as a function of l.

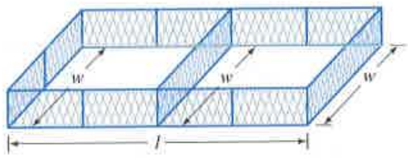
c. Find the dimensions that produce the greatest area.

51. Rectangular Enclosure A veterinarian uses 600 feet of chain-link fencing to enclose a rectangular region and to subdivide the region into two smaller rectangular regions by placing a fence parallel to one of the sides, as shown in the figure.

a. Write the width w as a function of the length l.

b. Write the total area A as a function of l.

c. Find the dimensions that produce the greatest enclosed area.



52. **Larvae Survival** Soon after insect larvae are hatched, they must begin to search for food. The survival rate of the larvae depends on many factors, but the temperature of the environment is one of the most important. For a certain species of insect, a model of the number of larvae, $N(T)$, that survive this searching period is given by $N(T) = -0.6T^2 + 32.1T - 350$ where T is the temperature in degrees Celsius.

   a. At what temperature will the maximum number of larvae survive? Round to the nearest degree.

   b. What is the maximum number of surviving larvae? Round to the nearest integer.

   c. Find the x-intercepts, to the nearest integer, for the graph of this function.

   d. Write a sentence that describes the meaning of the x-intercepts in the context of this problem.

53. **Temperature Fluctuations** The temperature $T(t)$, in degrees Fahrenheit, during the day can be modeled by the equation $T(t) = -0.7t^2 + 9.4t + 59.3$, where t is the number of hours after 6:00 A.M.

   a. At what time is the temperature a maximum? Round to the nearest minute.

   b. What is the maximum temperature? Round to the nearest degree.

54. **Geology** In June 2001, Mt. Etna in Sicily, Italy, erupted, sending volcanic bombs (masses of molten lava ejected from the volcano) into the air. A model of the height h, in meters, of a volcanic bomb above the crater of the volcano t seconds after the eruption is given by $h(t) = -9.8t^2 + 100t$. Find the maximum height of a volcanic bomb above the crater for this eruption. Round to the nearest meter.

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55. **Sports** When a softball player swings a bat, the amount of energy $E(t)$, in joules, that is transferred to the bat can be approximated by the function

$$E(t) = -279.67t^2 + 82.86t$$

where $0 \le t \le 0.3$ and t is measured in seconds. According to this model, what is the maximum energy of the bat? Round to the nearest tenth of a joule.

56. **Sports** A pitcher releases a baseball 6 feet above the ground at a speed of 132 feet per second (90 miles per hour) toward home plate, which is 60.5 feet away. The height $h(x)$, in feet, of the ball x feet from home plate can be approximated by $h(x) = -0.0009x^2 + 6$. To be considered a strike, the ball must cross home plate and be at least 2.5 feet high and less than 5.4 feet high. Assuming the ball crosses home plate, is this particular pitch a strike?

57. **Automotive Engineering** The fuel efficiency for a certain midsize car is given by

$$E(v) = -0.018v^2 + 1.476v + 3.4$$

where $E(v)$ is the fuel efficiency in miles per gallon for a car traveling v miles per hour.

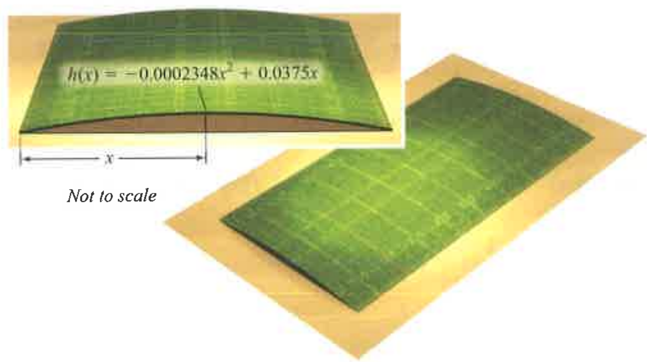
   a. What speed will yield the maximum fuel efficiency? Round to the nearest mile per hour.

   b. What is the maximum fuel efficiency for this car? Round to the nearest mile per gallon.

58. **Sports** Some football fields are built in a parabolic mound shape so that water will drain off the field. A model for the shape of a certain field is given by

$$h(x) = -0.0002348x^2 + 0.0375x$$

where $h(x)$ is the height, in feet, of the field at a distance of x feet from one sideline. Find the maximum height of the field. Round to the nearest tenth of a foot.



Business In Exercises 59 and 60, determine the number of units x that produce a maximum revenue, in dollars, for the given revenue function. Also determine the maximum revenue.

59. $R(x) = 296x - 0.2x^2$

60. $R(x) = 810x - 0.6x^2$

Business In Exercises 61 and 62, determine the number of units x that produce a maximum profit, in dollars, for the given profit function. Also determine the maximum profit.

61. $P(x) = -0.01x^2 + 1.7x - 48$

62. $P(x) = -\dfrac{x^2}{14{,}000} + 1.68x - 4000$

Business In Exercises 63 and 64, determine the profit function for the given revenue function and cost function. Also determine the break-even point or points.

63. $R(x) = x(102.50 - 0.1x)$; $C(x) = 52.50x + 1840$

64. $R(x) = x(210 - 0.25x)$; $C(x) = 78x + 6399$

65. **Tour Cost** A charter bus company has determined that the cost, in dollars, of providing x people with a tour is

$$C(x) = 180 + 2.50x$$

A full tour consists of 60 people. The ticket price per person is \$15 plus \$0.25 for each unsold ticket. Determine

a. The revenue function

b. The profit function

c. The company's maximum profit

d. The number of ticket sales that yields the maximum profit

66. **Delivery Cost** An air freight company has determined that the cost, in dollars, of delivering x parcels per flight is

$$C(x) = 2025 + 7x$$

The price per parcel, in dollars, the company charges to send x parcels is

$$p(x) = 22 - 0.01x$$

Determine

a. The revenue function

b. The profit function

c. The company's maximum profit

d. The price per parcel that yields the maximum profit

e. The minimum number of parcels the air freight company must ship to break even

67. **Gasoline Sales** A gasoline station can sell 10,000 gallons of gas per day at a price of \$3.95 per gallon. The station manager estimates that for each \$0.05 price reduction per gallon, 500 more gallons of gasoline can be sold each day. The cost per gallon of gas to the station is \$2.75. Find the price at which the station should sell a gallon of gasoline to maximize its daily profit on gasoline sales.

68. **Ticket Prices** Expand Your Wings Airlines finds that if it prices a Los Angeles to New York one-way ticket at \$390, it can sell 350 seats on its airplane. The airline estimates that for each \$10 price reduction per ticket, 25 more tickets can be sold each day. The cost to the airline for each ticket is \$150. Find the price at which the airline should sell a ticket to maximize its daily profit on ticket sales for the Los Angeles to New York flight.

69. **Projectile** If the initial velocity of a projectile is 128 feet per second, then its height, in feet, is a function of time t, in seconds, given by the equation $h(t) = -16t^2 + 128t$.

a. Find the time t when the projectile achieves its maximum height.

b. Find the maximum height of the projectile.

c. Find the time t when the projectile hits the ground.

70. **Projectile** The height in feet of a projectile with an initial velocity of 64 feet per second and an initial height of 80 feet is a function of time t in seconds given by

$$h(t) = -16t^2 + 64t + 80$$

a. Find the maximum height of the projectile.

b. Find the time t when the projectile achieves its maximum height.

c. Find the time t when the projectile has a height of 0 feet.

71. **Fire Management** The height of a stream of water from the nozzle of a fire hose can be modeled by

$$y(x) = -0.014x^2 + 1.19x + 5$$

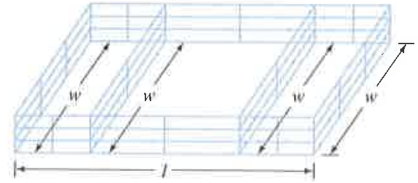
where $y(x)$ is the height, in feet, of the stream x feet from the firefighter. What is the maximum height that the stream of water from this nozzle can reach? Round to the nearest foot.

72. **Astronaut Training** A weightless environment can be created in an airplane by flying in a series of parabolic paths. This is one method that NASA uses to train astronauts for the experience of weightlessness. Suppose the height h, in feet, of NASA's airplane is modeled by $h(t) = -6.6t^2 + 430t + 28{,}000$, where t is the number of seconds after the plane enters its parabolic path. Find the maximum height of the plane to the nearest 1000 feet.

Enrichment Exercises

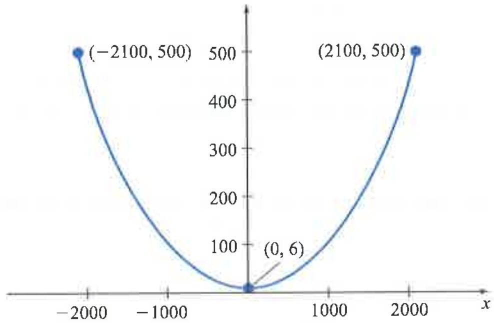
73. **Sports** For a serve to be legal in tennis, the ball must be at least 3 feet high when it is 39 feet from the server and it must land in a spot that is less than 60 feet from the server. Does the path of a ball given by $h(x) = -0.002x^2 - 0.03x + 8$, where $h(x)$ is the height of the ball (in feet) x feet from the server, satisfy the conditions of a legal serve?

74. **Rectangular Enclosure** A farmer uses 1200 feet of fence to enclose a rectangular region and to subdivide the region into three smaller rectangular regions by placing fences parallel to one of the sides. Find the dimensions that produce the greatest enclosed area.



75. **Norman Window** A Norman window has the shape of a rectangle surmounted by a semicircle. The exterior perimeter of the window shown in the figure is 48 feet. Find the height h and the radius r that will allow the maximum amount of light to enter the window. (*Hint:* Write the area of the window as a quadratic function of the radius r.)

76. **Golden Gate Bridge** The suspension cables of the main span of the Golden Gate Bridge are in the shape of a parabola. If a coordinate system is drawn as shown, find the quadratic function that models a suspension cable for the main span of the bridge.



SECTION 2.5

Properties of Graphs

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A11.

PS1. For the graph of the parabola whose equation is $f(x) = x^2 + 4x - 6$, what is the equation of the axis of symmetry? [2.4]

PS2. For $f(x) = \dfrac{3x^4}{x^2 + 1}$, show that $f(-3) = f(3)$. [2.2]

PS3. For $f(x) = 2x^3 - 5x$, show that $f(-2) = -f(2)$. [2.2]

PS4. Let $f(x) = x^2$ and $g(x) = x + 3$. Find $f(a) - g(a)$ for $a = -2, -1, 0, 1, 2$. [2.2]

PS5. What is the midpoint of the line segment between $P(-a, b)$ and $Q(a, b)$? [2.1]

PS6. What is the midpoint of the line segment between $P(-a, -b)$ and $Q(a, b)$? [2.1]

Symmetry

A graph is **symmetric with respect to the y-axis** if whenever the point given by (x, y) is on the graph, then $(-x, y)$ is also on the graph. The graph in Figure 2.62 is symmetric with respect to the y-axis. A graph is **symmetric with respect to the x-axis** if whenever the point given by (x, y) is on the graph, then $(x, -y)$ is also on the graph. The graph in Figure 2.63 is symmetric with respect to the x-axis.