Complete the following table by evaluating C (to the nearest dollar) for the indicated numbers of items.

x	100	200	500	750	1000
$C(x)$					

111. If $f(x) = x^2 - x - 5$ and $f(c) = 1$, find c.

112. If $g(x) = -2x^2 + 4x - 1$ and $g(c) = -4$, find c.

113. Determine whether 1 is in the range of $f(x) = \dfrac{x-1}{x+1}$.

114. Determine whether 0 is in the range of $g(x) = \dfrac{1}{x-3}$.

In Exercises 115 and 116, use a graphing calculator to graph each set of equations.

115. Graph $f(x) = x^2$, $g(x) = x^2 - 3$, and $h(x) = x^2 + 2$ on the same graphing calculator screen. How are the graphs of g and h related to the graph of f?

116. Graph $f(x) = x^2$, $g(x) = (x - 3)^2$, and $h(x) = (x + 2)^2$ on the same graphing calculator screen. How are the graphs of g and h related to the graph of f?

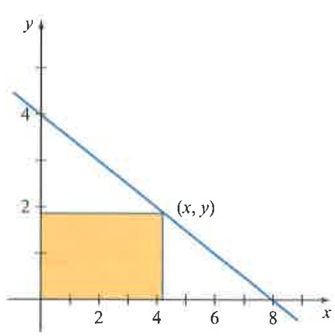
Enrichment Exercises

A *fixed point* of a function is a number a such that $f(a) = a$. In Exercises 117 and 118, find all fixed points for the given function.

117. $f(x) = x^2 + 3x - 3$

118. $g(x) = \dfrac{x}{x+5}$

119. **Area** A rectangle is bounded by the x- and y-axes and the graph of $y = -\frac{1}{2}x + 4$.



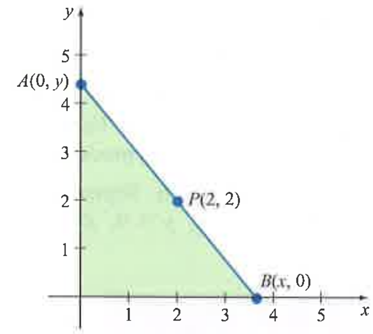
a. Find the area of the rectangle as a function of x.

b. Complete the following table.

x	Area
1	
2	
4	
6	
7	

c. What is the domain of this function?

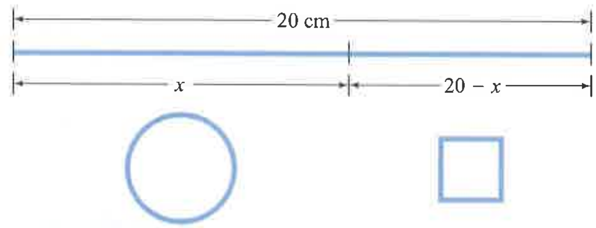
120. **Area** A triangle is bounded by the x- and y-axes and must pass through $P(2, 2)$, as shown below.



a. Find the area of the triangle as a function of x. (*Hint:* Let C be the point (0, 2) and D be the point (2, 0). Use the fact that ACP and PDB are similar triangles.)

b. What is the domain of the function you found in **a.**?

121. **Area** A piece of wire 20 centimeters long is cut at a point x centimeters from the left end, as shown in the following diagram. The left-hand piece is formed into a circle and the right-hand piece is formed into a square.



a. Find the area enclosed by the two figures as a function of x.

b. Complete the following table. Round the area to the nearest hundredth.

x	Total Area Enclosed (cm^2)
0	
4	
8	
12	
16	
20	

c. What is the domain of this function?

122. **Day of the Week** A formula known as Zeller's Congruence makes use of the greatest integer function $[\![x]\!]$ to determine the day of the week on which a given day fell or will fall. To use Zeller's Congruence, we first compute the integer z given by

$$z = \left[\!\!\left[\frac{13m-1}{5}\right]\!\!\right] + \left[\!\!\left[\frac{y}{4}\right]\!\!\right] + \left[\!\!\left[\frac{c}{4}\right]\!\!\right] + d + y - 2c$$

The variables c, y, d, and m are defined as follows:

c = int(the given year/100)

y = year of the century

d = day of the month

m = month, using 1 for March, 2 for April, . . . , 10 for December. January and February are assigned the values 11 and 12 of the previous year.

For example, for the date September 30, 2009, we use $c = \text{int}(2009/100) = 20$, $y = 9$, $d = 30$, and $m = 7$. The remainder of z divided by 7 gives the day of the week. A remainder of 0 represents a Sunday, a remainder of 1 a Monday, . . . , a remainder of 6 a Saturday.

a. Verify that December 7, 1941, was a Sunday.

b. Verify that January 1, 2020, will fall on a Wednesday.

c. Determine on what day of the week Independence Day (July 4, 1776) fell.

d. Determine on what day of the week you were born.

SECTION 2.3

Linear Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A9.

PS1. Find the distance on a real number line between the points whose coordinates are -2 and 5. [P.1]

PS2. Find the product of a nonzero number and its negative reciprocal. [P.5]

PS3. Given the points $P_1(-3, 4)$ and $P_2(2, -4)$, evaluate $\frac{y_2 - y_1}{x_2 - x_1}$. [P.1]

PS4. Solve $y - 3 = -2(x - 3)$ for y. [1.1]

PS5. Solve $3x - 5y = 15$ for y. [1.2]

PS6. Given $y = 3x - 2(5 - x)$, find the value of x for which $y = 0$. [1.1]

Slopes of Lines

A function that can be written in the form $f(x) = mx + b$ is called a *linear function* because its graph is a straight line. Consider the graph of a straight line in Figure 2.37. Observe that for each 1-unit increase in x, y increases by a constant amount of m units. In Figure 2.38, note that for each 1-unit increase in x, y decreases by a constant amount of m units.

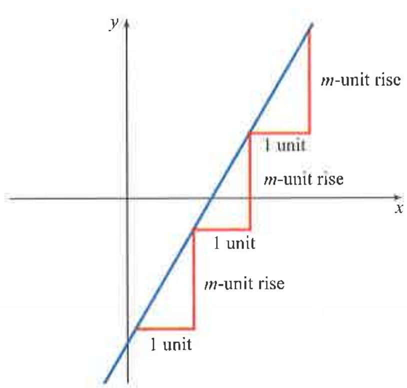


Figure 2.37

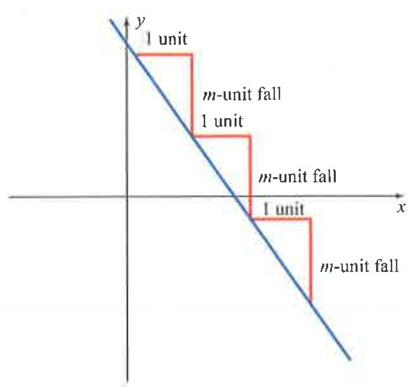


Figure 2.38

Graphs of linear functions are characterized by having a constant rise or fall. This rise or fall is called *slope*. The graph in Figure 2.37 has a *positive* slope; the y value is increasing as x increases. The graph in Figure 2.38 has a *negative* slope; the y value is decreasing as x increases.

The slope of a line can be calculated by finding the ratio of the change in y between two points to the change in x between the same two points. For instance,

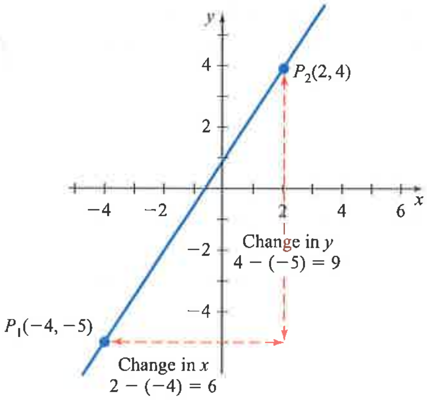


Figure 2.39

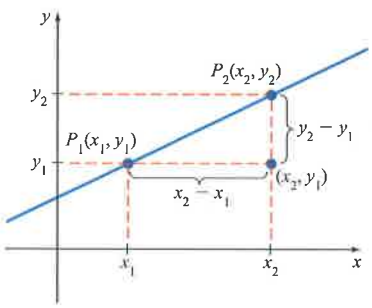


Figure 2.40

consider the graph of a straight line passing through the points $P_1(-4, -5)$ and $P_2(2, 4)$ shown in Figure 2.39. The slope of the line between the two points is

$$\text{Slope} = m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{9}{6} = \frac{3}{2}$$

The slope of the line is $\frac{3}{2}$.

Definition of the Slope of a Nonvertical Line

The **slope** m of the line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ with $x_1 \neq x_2$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

See Figure 2.40.

EXAMPLE

The slope of the line through $P_1(2, -3)$ and $P_2(-4, 1)$ is

$$m = \frac{1 - (-3)}{-4 - 2} = \frac{4}{-6} = -\frac{2}{3}$$

When computing the slope of a line, it does not matter which point we label P_1 and which we label P_2; the value of the slope will be the same. For instance, if we interchange the two points in the previous example so that we have $P_1(-4, 1)$ and $P_2(2, -3)$, then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{2 - (-4)} = \frac{-4}{6} = -\frac{2}{3}$$

Frequently, the Greek letter delta (Δ) is used to designate the change in a variable. Using this notation, $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$. Using Δ notation, the formula for slope is written $m = \frac{\Delta y}{\Delta x}$.

EXAMPLE 1 Find the Slope of a Line

Find the slope of the line passing through P_1 and P_2.

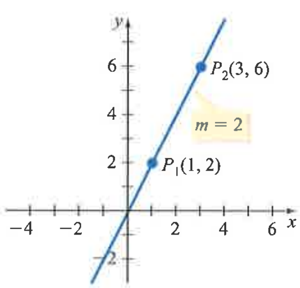
a. $P_1(1, 2)$, $P_2(3, 6)$ **b.** $P_1(-3, 4)$, $P_2(1, -2)$

Solution

a. The slope of the line passing through $P_1(1, 2)$ and $P_2(3, 6)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{3 - 1} = \frac{4}{2} = 2$$

Because $m > 0$, the line slants upward from left to right. The slope of the line is positive. See the graph at the left.

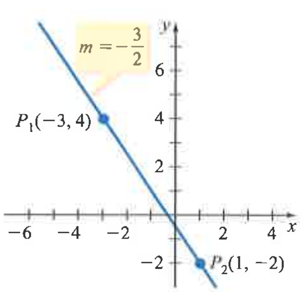


Positive slope

b. The slope of the line passing through $P_1(-3, 4)$ and $P_2(1, -2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{1 - (-3)} = -\frac{6}{4} = -\frac{3}{2}$$

Because $m < 0$, the line slants downward from left to right. The slope of the line is negative. See the graph at the left.



Negative slope

▸ Try Exercise 14, page 193

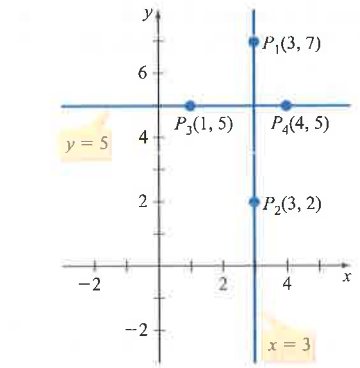


Figure 2.41

The definition of slope does not apply to vertical lines. Consider, for example, the points $P_1(3, 7)$ and $P_2(3, 2)$ on the vertical line in Figure 2.41. Applying the definition of slope to this line produces

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 7}{3 - 3} = \frac{-5}{0}$$

• Division by 0 is undefined.

Because division by 0 is undefined, we say that the slope of any vertical line is undefined.

The line through $P_3(1, 5)$ and $P_4(4, 5)$ in Figure 2.41 is a horizontal line. Its slope is given by

$$m = \frac{y_4 - y_3}{x_4 - x_3} = \frac{5 - 5}{4 - 1} = \frac{0}{3} = 0$$

The slope of every horizontal line is 0.

The results for vertical and horizontal lines are summarized below.

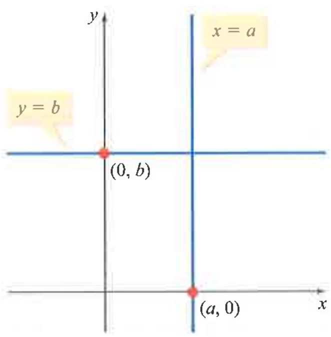


Figure 2.42

Horizontal and Vertical Lines

The graph of $x = a$ is a vertical line through $(a, 0)$. The slope of the line is undefined. See Figure 2.42.

The graph of $y = b$ is a horizonal line through $(0, b)$. The slope of the line is zero. See Figure 2.42.

EXAMPLE

The graph of $x = -2$ is a vertical line through $(-2, 0)$. The slope is undefined.

The graph of $y = 3$ is a horizontal line through $(0, 3)$. The slope is zero.

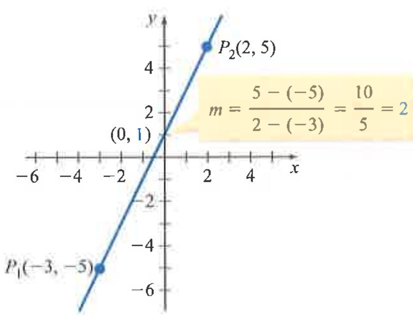


Figure 2.43

Slope–Intercept Form

The graph of $f(x) = 2x + 1$ is shown in Figure 2.43. Note that the slope of the line between the two points is 2, the coefficient of x in $f(x) = 2x + 1$. The y-coordinate of the y-intercept is 1, the constant term of $f(x) = 2x + 1$.

Slope–Intercept Form

The graph of $f(x) = mx + b$ is a line with slope m and y-intercept $(0, b)$.

EXAMPLE

The graph of $f(x) = -2x + 3$ is a line with slope -2 and y-intercept $(0, 3)$.

The graph of $f(x) = \frac{2}{3}x - 4$ is a line with slope $\frac{2}{3}$ and y-intercept $(0, -4)$.

Proof

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, $x_1 \neq x_2$, are two points on the line $f(x) = mx + b$, then $y_1 = f(x_1) = mx_1 + b$ and $y_2 = f(x_2) = mx_2 + b$. Using the formula for slope, we have

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1} = \frac{mx_2 - mx_1}{x_2 - x_1} = \frac{m(x_2 - x_1)}{x_2 - x_1} = m, \; x_1 \neq x_2$$

Thus the slope of the line between the two points is m.

To find the y-intercept, evaluate $f(x) = mx + b$ when $x = 0$.

$$f(x) = mx + b$$
$$f(0) = m(0) + b = b$$

The y-intercept is $(0, b)$. ■

If a function is written in the form $f(x) = mx + b$, then its graph can be drawn by first plotting the y-intercept $(0, b)$ and then using the slope m to determine another point on the line.

EXAMPLE 2 Graph a Linear Function

Graph: $f(x) = 2x - 1$

Solution

Replace $f(x)$ with y. The equation $y = 2x - 1$ is in slope–intercept form, with $b = -1$ and $m = 2$. Thus the y-intercept is $(0, -1)$ and the slope is 2. Write the slope as

$$m = \frac{2}{1} = \frac{\text{Change in } y}{\text{Change in } x}$$

To graph the equation, first plot the y-intercept and then use the slope to plot a second point. This second point is 2 units up (change in y) and 1 unit to the right (change in x) of the y-intercept. See Figure 2.44.

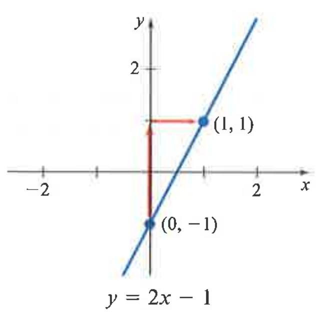


Figure 2.44

▶ Try Exercise 24, page 193

An equation of the form $Ax + By = C$, where A, B, and C are real numbers and both A and B are not zero, is called the **general form of a linear equation in two variables**. Examples of these equations are given below.

$$2x - 3y = 6 \qquad \bullet\ A = 2, B = -3, C = 6$$
$$-4x + 5y = 0 \qquad \bullet\ A = -4, B = 5, C = 0$$
$$x = 3 \qquad \bullet\ A = 1, B = 0, C = 3$$
$$y = -2 \qquad \bullet\ A = 0, B = 1, C = -2$$

To graph a linear equation given in general form, solve for y and then proceed as previously shown in Example 2.

EXAMPLE 3 Graph a Linear Equation Given in General Form

Graph: $3x + 2y = 4$

Solution

Solve for y.

$$3x + 2y = 4$$
$$2y = -3x + 4$$
$$y = -\frac{3}{2}x + 2$$

The slope is $-\frac{3}{2}$, and the y-intercept is $(0, 2)$.

$$m = -\frac{3}{2} = \frac{-3}{2} = \frac{\text{Change in } y}{\text{Change in } x}$$

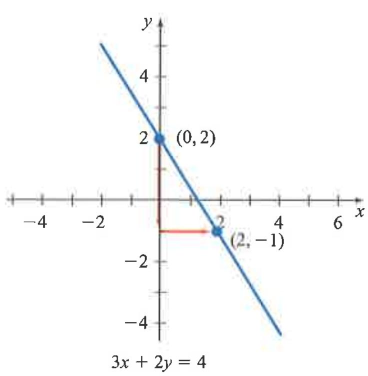


Figure 2.45

(continued)

To graph the equation, plot the y-intercept and then use the slope to plot a second point. The second point is 3 units down (change in y) and 2 units to the right (change in x) from the y-intercept. See Figure 2.45.

Try Exercise 40, page 193

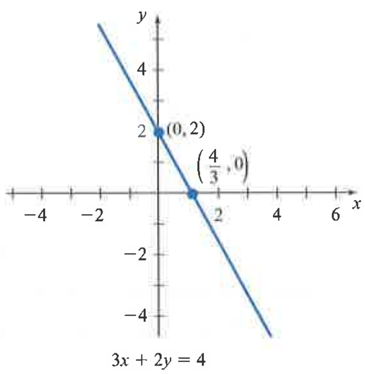


Figure 2.46

The graph in Example 3 can also be drawn by first finding the x- and y-intercepts and then drawing a straight line through those points.

To find the x-intercept, let $y = 0$ and solve for x.

$$3x + 2y = 4$$
$$3x + 2(0) = 4$$
$$3x = 4$$
$$x = \frac{4}{3}$$

The x-intercept is $\left(\frac{4}{3}, 0\right)$.

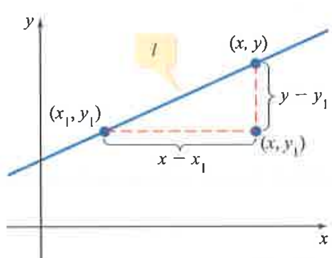
To find the y-intercept, let $x = 0$ and solve for y.

$$3x + 2y = 4$$
$$3(0) + 2y = 4$$
$$2y = 4$$
$$y = 2$$

The y-intercept is $(0, 2)$.

Plot the x- and y-intercepts and then draw a line through the points. See Figure 2.46.

Finding the Equation of a Line



The slope of line l is $m = \frac{y - y_1}{x - x_1}$.

Figure 2.47

We can find an equation of a line provided we know its slope and at least one point on the line. Figure 2.47 suggests that if (x_1, y_1) is a point on a line l of slope m and (x, y) is *any other* point on the line, then

$$\frac{y - y_1}{x - x_1} = m, \quad x \neq x_1$$

Multiplying each side of the previous equation by $x - x_1$ produces $y - y_1 = m(x - x_1)$. This equation is called the **point–slope form** of the equation of line l.

Point–Slope Form

The graph of

$$y - y_1 = m(x - x_1)$$

is a line that has slope m and passes through (x_1, y_1).

EXAMPLE 4 Use the Point–Slope Form

Find an equation of the line with slope -3 that passes through $(-1, 4)$.

Solution

Use the point–slope form with $m = -3$, $x_1 = -1$, and $y_1 = 4$.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -3[x - (-1)] \quad \text{• Substitute.}$$
$$y - 4 = -3x - 3 \quad \text{• Solve for } y.$$
$$y = -3x + 1 \quad \text{• Slope–intercept form}$$

Try Exercise 44, page 193

EXAMPLE 5 Find the Equation of a Line Given Two Points

Find the equation of the line that passes through $P_1(-2, 4)$ and $P_2(2, -1)$.

Solution

To find the equation of a line through two given points, first find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{2 - (-2)} = \frac{-5}{4} = -\frac{5}{4}$$

The slope is $-\frac{5}{4}$. Now use the point–slope form.

$$y - y_1 = m(x - x_1) \quad \text{• Use the point–slope form.}$$

$$y - 4 = -\frac{5}{4}(x - (-2)) \quad \text{• } x_1 = -2, y_1 = 4, m = -\frac{5}{4}.$$

$$y - 4 = -\frac{5}{4}x - \frac{5}{2} \quad \text{• Solve for } y.$$

$$y = -\frac{5}{4}x + \frac{3}{2}$$

Try Exercise 54, page 194

Parallel and Perpendicular Lines

Two nonintersecting lines in a plane are **parallel**. All vertical lines are parallel to one another. All horizontal lines are parallel to one another.

Two lines are **perpendicular** if and only if they intersect and form adjacent angles, each of which measures 90°. In a plane, vertical and horizontal lines are perpendicular to one another.

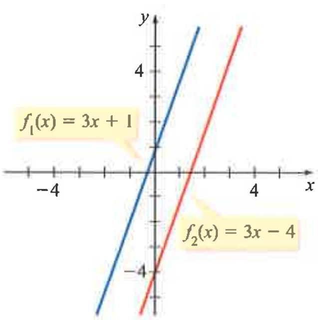


Figure 2.48

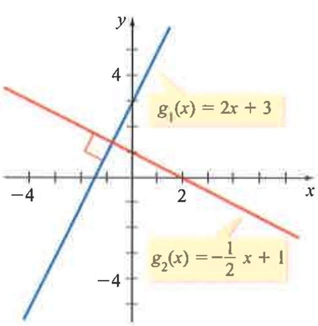


Figure 2.49

Parallel and Perpendicular Lines

Let l_1 be the graph of $f_1(x) = m_1x + b_1$ and l_2 be the graph of $f_2(x) = m_2x + b_2$. Then

- l_1 and l_2 are parallel if and only if $m_1 = m_2$.
- l_1 and l_2 are perpendicular if and only if $m_1 = -\frac{1}{m_2}$. In this case, the slope of l_1 is the *negative reciprocal* of the slope of l_2.

EXAMPLE

If $f_1(x) = 3x + 1$ and $f_2(x) = 3x - 4$, then the slopes are equal: $m_1 = m_2 = 3$. The lines are parallel. See Figure 2.48.

If $g_1(x) = 2x + 3$ and $g_2(x) = -\frac{1}{2}x + 1$, then $m_1 = 2$ and $m_2 = -\frac{1}{2} = -\frac{1}{m_1}$.

The lines are perpendicular. See Figure 2.49. The symbol ⌝ is used to denote an angle of 90°.

EXAMPLE 6 Find Equations of Parallel and Perpendicular Lines

a. Find the equation of the line whose graph is parallel to the graph of $2x - 3y = 7$ and passes through the point $P(-6, -2)$.

b. Find the equation of the line whose graph is perpendicular to the graph of $y = \frac{4}{3}x - 2$ and passes through the point $P(-4, 1)$.

Solution

a. Solving $2x - 3y = 7$ for y, we have $y = \frac{2}{3}x - \frac{7}{3}$. Therefore, the slope of a line parallel to the given line is $m = \frac{2}{3}$. Now use the point–slope form with $m = \frac{2}{3}$ and $P(-6, -2)$.

$$y - y_1 = m(x - x_1)$$ • Use the point–slope form.

$$y - (-2) = \frac{2}{3}(x - (-6))$$ • $x_1 = -6, y_1 = -2, m = \frac{2}{3}$

$$y + 2 = \frac{2}{3}x + 4$$ • Solve for y.

$$y = \frac{2}{3}x + 2$$

The equation of the line whose graph is parallel to the graph of $2x - 3y = 7$ and passes through the point $P(-6, -2)$ is $y = \frac{2}{3}x + 2$.

b. The slope of the given line is $\frac{4}{3}$. The slope of a line perpendicular to the given line is the negative reciprocal of $\frac{4}{3}$, or $-\frac{3}{4}$. Now use the point–slope form with $m = -\frac{3}{4}$ and $P(-4, 1)$.

$$y - y_1 = m(x - x_1)$$ • Use the point–slope form.

$$y - 1 = -\frac{3}{4}(x - (-4))$$ • $x_1 = -4, y_1 = 1, m = -\frac{3}{4}$

$$y - 1 = -\frac{3}{4}x - 3$$ • Solve for y.

$$y = -\frac{3}{4}x - 2$$

The equation of the line whose graph is perpendicular to the graph of $y = \frac{4}{3}x - 2$ and passes through the point $P(-4, 1)$ is $y = -\frac{3}{4}x - 2$.

Try Exercise 60, page 194

Applications of Linear Functions

EXAMPLE 7 Find a Linear Model of Data

The data in the table show the relationship between the swing speed of a golfer using a driver and the distance the ball will carry.

Swing speed, s (mph)	109	86	87	108	95	81	101	90	114	89
Carry distance, d (yards)	283	189	191	275	233	178	238	215	288	211

a. Find a linear function that models the distance d, in yards, a golf ball will carry in terms of the swing speed s, in miles per hour, of the golfer by using the data points (90, 215) and (109, 283). Round the slope and y-intercept to the nearest tenth.

b. Using the model you found in part **a**, how far would a golf ball carry that was hit with a swing speed of 120 mph?

Solution

a. First, calculate the slope of the line. Then use the point–slope formula to find the equation of the line.

$$m = \frac{d_2 - d_1}{s_2 - s_1} \qquad \text{• Find the slope.}$$

$$= \frac{215 - 283}{90 - 109} = \frac{-68}{-19}$$

$$\approx 3.6$$

$$d - d_1 = m(s - s_1) \qquad \text{• Use the point–slope formula.}$$

$$d - 215 = 3.6(s - 90) \qquad \text{• } d_1 = 215, s_1 = 90, m = 3.6$$

$$d = 3.6s - 109$$

In function notation, the linear model is $d(s) = 3.6s - 109$.

b. To find the distance a golf ball would carry when hit with a swing speed of 120 mph, evaluate d when $s = 120$.

$$d(s) = 3.6s - 109$$

$$d(120) = 3.6(120) - 109$$

$$= 323$$

The golf ball would carry 323 feet.

▶ **Try Exercise 74, page 194**

If a manufacturer produces x units of a product that sells for p dollars per unit, then the **cost function** C, the **revenue function** R, and the **profit function** P are defined as follows.

$$C(x) = \text{cost of producing and selling } x \text{ units}$$

$$R(x) = xp = \text{revenue from the sale of } x \text{ units at } p \text{ dollars each}$$

$$P(x) = \text{profit from selling } x \text{ units}$$

Because profit equals the revenue less the cost, we have

$$P(x) = R(x) - C(x)$$

The value of x for which $R(x) = C(x)$ is called the **break-even point**. At the break-even point, $P(x) = 0$.

EXAMPLE 8 Find the Profit Function and the Break-Even Point

A manufacturer finds that the costs incurred in the manufacture and sale of a particular type of calculator are $180,000 plus $27 per calculator.

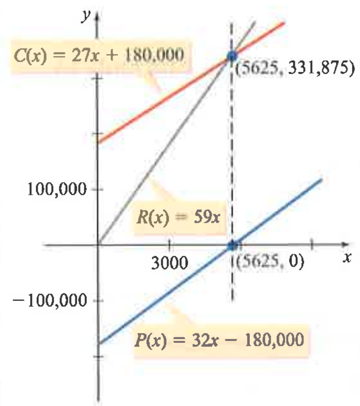
a. Determine the profit function P, given that x calculators are manufactured and sold at $59 each.

b. Determine the break-even point.

(continued)

Note

The graphs of C, R, and P are shown below. Observe that the graphs of C and R intersect at the break-even point, where $x = 5625$ and $P(5625) = 0$.



Solution

a. The cost function is $C(x) = 27x + 180{,}000$. The revenue function is $R(x) = 59x$. Thus the profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 59x - (27x + 180{,}000) \\ &= 32x - 180{,}000, \quad x \geq 0 \text{ and } x \text{ is an integer} \end{aligned}$$

b. At the break-even point, $R(x) = C(x)$.

$$\begin{aligned} 59x &= 27x + 180{,}000 \\ 32x &= 180{,}000 \\ x &= 5625 \end{aligned}$$

The manufacturer will break even when 5625 calculators are sold.

▶ Try Exercise 84, page 196

EXAMPLE 9 Determine a Point of Impact

A rock attached to a string is whirled horizontally about the origin in a circular counterclockwise path with radius 5 feet. When the string breaks, the rock travels on a linear path perpendicular to the radius OP and hits a wall located at

$$y = x + 12 \tag{1}$$

where x and y are measured in feet. If the string breaks when the rock is at $P(4, 3)$, determine the point at which the rock hits the wall. See Figure 2.50.

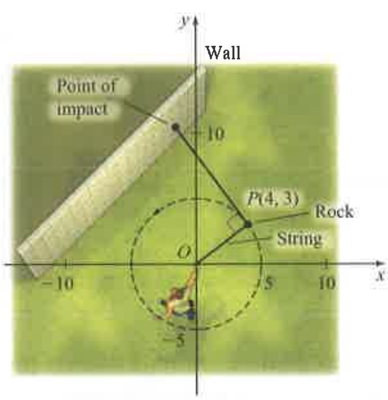


Figure 2.50

Solution

The slope of OP is $\frac{3}{4}$. The negative reciprocal of $\frac{3}{4}$ is $-\frac{4}{3}$. Therefore, the linear path of the rock is given by $y - 3 = -\frac{4}{3}(x - 4)$, or

$$y = -\frac{4}{3}x + \frac{25}{3} \tag{2}$$

To find the point at which the rock hits the wall, set the right side of Equation (1) equal to the right side of Equation (2) and solve for x.

$$\begin{aligned} -\frac{4}{3}x + \frac{25}{3} &= x + 12 \\ -4x + 25 &= 3x + 36 \qquad \text{• Multiply each side by 3.} \\ -7x &= 11 \\ x &= -\frac{11}{7} \end{aligned}$$

For every point on the wall, x and y are related by $y = x + 12$. Therefore, substituting $-\frac{11}{7}$ for x in $y = x + 12$ yields $y = -\frac{11}{7} + 12 = \frac{73}{7}$, and the rock hits the wall at $\left(-\frac{11}{7}, \frac{73}{7}\right)$.

▶ Try Exercise 92, page 197

EXERCISE SET 2.3

Concept Check

In Exercises 1 and 2, complete the sentence with *increases* or *decreases*.

1. If a line has a negative slope, then as the value of y increases, the value of x ________.

2. If a line has a positive slope, then as the value of y decreases, the value of x ________.

In Exercises 3 and 4, complete the sentence with *vertical* or *horizontal*.

3. The graph of a line with zero slope is ________.

4. The graph of a line whose slope is undefined is ______.

In Exercises 5 to 8, determine the slope of the line and the coordinates of the y-intercept.

5. $y = 4x - 5$

6. $y = 3 - 2x$

7. $f(x) = \frac{2x}{3}$

8. $f(x) = -1$

In Exercises 9 to 12, determine whether the graphs of the two equations are parallel, perpendicular, or neither parallel nor perpendicular.

9. $y = 3x - 4; y = -3x + 2$

10. $y = -\frac{2}{3}x + 1; y = 2 - \frac{2x}{3}$

11. $f(x) = 3x - 1; y = -\frac{x}{3} - 1$

12. $f(x) = \frac{4x}{3} + 2; f(x) = 2 - \frac{3}{4}x$

In Exercises 13 to 22, find the slope of the line that passes through the given points.

13. $(3, 4)$ and $(1, 7)$

14. $(-2, 4)$ and $(5, 1)$

15. $(4, 0)$ and $(0, 2)$

16. $(-3, 4)$ and $(2, 4)$

17. $(3, -7)$ and $(3, 2)$

18. $(0, 0)$ and $(3, 0)$

19. $(-3, 4)$ and $(-4, -2)$

20. $(-5, -1)$ and $(-3, 4)$

21. $\left(-4, \frac{1}{2}\right)$ and $\left(\frac{7}{3}, \frac{7}{2}\right)$

22. $\left(\frac{1}{2}, 4\right)$ and $\left(\frac{7}{4}, 2\right)$

Indicates Try It Exercises

In Exercises 23 to 34, graph y as a function of x by using the slope and y-intercept of each line.

23. $y = 2x - 4$

24. $y = -x + 1$

25. $y = \frac{3}{4}x + 1$

26. $y = -\frac{3}{2}x + 4$

27. $y = -2x + 3$

28. $y = 3x - 1$

29. $y = 3$

30. $y = -2$

31. $y = 2x$

32. $y = -3x$

33. $y = x$

34. $y = -x$

In Exercises 35 to 42, graph each equation by first writing the equation in slope–intercept form. Check the graph by finding the x- and y-intercepts.

35. $2x + y = 5$

36. $x - y = 4$

37. $4x + 3y - 12 = 0$

38. $2x + 3y + 6 = 0$

39. $2x - 5y = -15$

40. $3x - 4y = 8$

41. $x + 2y = 6$

42. $x - 3y = 9$

In Exercises 43 to 54, find the equation of the line satisfying the given conditions. Write the equation in the form $y = mx + b$.

43. y-intercept $(0, 3)$, slope 1

44. y-intercept $(0, 5)$, slope -2

45. y-intercept $\left(0, \frac{1}{2}\right)$, slope $\frac{3}{4}$

46. y-intercept $\left(0, \frac{3}{4}\right)$, slope $-\frac{2}{3}$

47. y-intercept $(0, 4)$, slope 0

48. y-intercept $(0, -1)$, slope $\frac{1}{2}$

49. Through $(-3, 2)$, slope -4

50. Through $(-5, -1)$, slope -3

51. Through $(3, 1)$ and $(-1, 4)$

52. Through $(5, -6)$ and $(2, -8)$

53. Through (7, 11) and (2, −1)

54. Through (−5, 6) and (−3, −4)

In Exercises 55 to 66, find the equation of the line, in slope–intercept form, that satisifies the given conditions.

55. The graph is parallel to the graph of $y = 2x + 3$ and passes through the point whose coordinates are (2, −4).

56. The graph is parallel to the graph of $y = -x + 1$ and passes through the point whose coordinates are (−2, 4).

57. The graph is parallel to the graph of $y = -\frac{3}{4}x + 3$ and passes through the point whose coordinates are (−4, 2).

58. The graph is parallel to the graph of $y = \frac{2}{3}x - 1$ and passes through the point whose coordinates are (−3, −5).

59. The graph is parallel to the graph of $2x - 5y = 2$ and passes through the point whose coordinates are (5, 2).

60. The graph is parallel to the graph of $x + 3y = 4$ and passes through the point whose coordinates are (−3, −1).

61. The graph is perpendicular to the graph of $y = 2x - 5$ and passes through the point whose coordinates are (3, −4).

62. The graph is perpendicular to the graph of $y = -x + 3$ and passes through the point whose coordinates are (−5, 2).

63. The graph is perpendicular to the graph of $y = -\frac{3}{4}x + 1$ and passes through the point whose coordinates are (−6, 0).

64. The graph is perpendicular to the graph of $3x - 2y = 5$ and passes through the point whose coordinates are (−3, 4).

65. The graph is perpendicular to the graph of $-x - 4y = 6$ and passes through the point whose coordinates are (5, 2).

66. The graph is perpendicular to the graph of $5x - y = 2$ and passes through the point whose coordinates are (10, −2).

In Exercises 67 to 70, find the zero of *f*. Verify that the solution of *f*(*x*) = 0 is the same as the *x*-coordinate of the *x*-intercept of the graph of *y* = *f*(*x*).

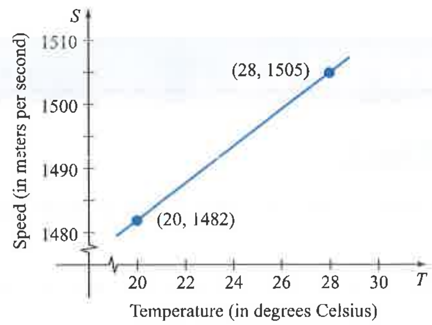
67. $f(x) = 3x - 12$

68. $f(x) = -2x - 4$

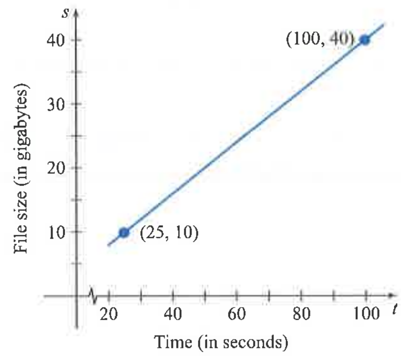
69. $f(x) = \frac{1}{4}x + 5$

70. $f(x) = -\frac{1}{3}x + 2$

71. **Oceanography** The graph at the top of the right side column shows the relationship between the speed of sound in water and the temperature of the water. Find the slope of this line, and write a sentence that explains the meaning of the slope in the context of this problem.



72. **Computer Science** The following graph shows the relationship between the time, in seconds, it takes to download a file and the size of the file, in gigabytes. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope in the context of this problem.



73. **Automotive Technology** The following table shows the U.S. Environmental Protection Agency (EPA) fuel economy values for selected two-seater cars for the 2009 model year. (*Source:* http://www.fueleconomy.gov.)

EPA Fuel Economy Values for Selected Two-Seater Cars

Car	City, *c* (mpg)	Highway, *H* (mpg)
Audi, TT Roadster	23	31
BMW, M3	14	20
Ferrari, 458 Italia	12	18
Lamborghini, Gallarado	12	20
Chevrolet, Corvette	16	26
Maserati, Gran Turismo	13	21

a. Using the data for the Lamborghini and the Audi, find a linear function that predicts highway miles per gallon in terms of city miles per gallon.

b. Using your model, predict the highway miles per gallon for a Porsche Cayman, whose city fuel efficiency is 19 miles per gallon.

74. **Consumer Credit** The amount of revolving consumer credit (such as credit cards) gradually decreased between

2007 and 2011, as given in the table below. (*Source:* http://www.federalreserve.gov.)

Year, t	Consumer Credit, C (billions of \$)
2007	1008.1
2008	1010.3
2009	921.9
2010	857.4
2011	864.9

a. Using the data for 2007 and 2011, find a linear model that predicts the amount of revolving consumer credit (in billions of dollars) for year t.

b. Using this model, in what year would consumer credit first fall below \$750 billion?

75. **Labor Market** According to the Bureau of Labor Statistics (BLS), there were 279,200 graphic designer jobs in the United States in 2010. The BLS projects that there will be 316,500 graphic designer jobs in 2020.

a. Using the BLS data, find the number of graphic designer jobs as a linear function of the year.

b. Using your model, in what year will the number of graphic designer jobs first exceed 300,000?

76. **Pottery** A piece of pottery is removed from a kiln and allowed to cool in a controlled environment. The temperature (in degrees Fahrenheit) of the pottery after it is removed from the kiln is shown for various times (in minutes) in the following table.

Time, t (min)	Temperature, T (°F)
15	2200
20	2150
30	2050
60	1750

a. Find a linear model for the temperature of the pottery after t minutes.

b. Explain the meaning of the slope of this line in the context of the problem.

c. Assuming that the temperature continues to decrease at the same rate, what will be the temperature of the pottery in 3 hours?

77. **Lumber Industry** The number of board-feet (bf) B that can be obtained from a log depends on the diameter d, in inches, of the log and its length. The following table shows the number of board-feet of lumber that can be obtained from a log that is 32 feet long.

Diameter (in.)	bf
16	180
18	240
20	300
22	360

a. Find a linear model for the number of board-feet as a function of log diameter.

b. Explain the meaning of the slope of this line in the context of the problem.

c. Using this model, how many board-feet of lumber can be obtained from a log 32 feet long with a diameter of 19 inches?

78. **Ecology** The rate at which water evaporates from a certain reservoir depends on the air temperature. The table below shows the number of acre-feet (af) of water per day that evaporates from the reservoir for various temperatures in degrees Fahrenheit.

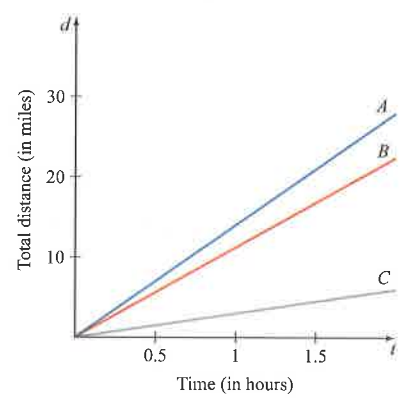
Temperature (°F)	(af)
40	800
60	1640
70	2060
85	2690

a. Find a linear model for the number of acre-feet of water that evaporates E as a function of temperature T.

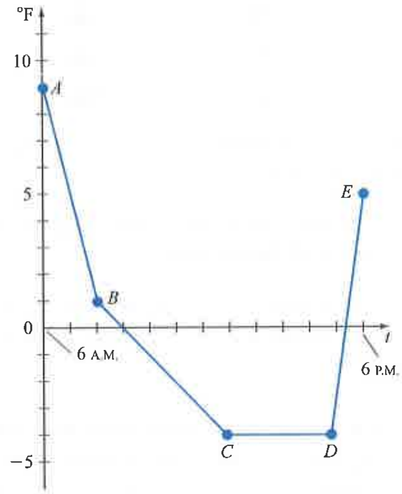
b. Explain the meaning of the slope of this line in the context of this problem.

c. Assuming that water continues to evaporate at the same rate, how many acre-feet of water will evaporate per day when the temperature is 75°F?

79. **Cycling Speeds** Michelle and Amanda start from the same place on a cycling course. Michelle is riding at 15 miles per hour, and Amanda is cycling at 12 miles per hour. The graph below shows the total distance traveled by each cyclist and the total distance between Michelle and Amanda after t hours. Which distance does each line represent?



80. **Temperature** The following graph shows the temperature, in degrees Fahrenheit, over a 12-hour period at a weather station.



a. By how many degrees per hour did the temperature change between A and B?

b. Between which two points did the temperature change most rapidly?

c. Between which two points was the temperature constant?

81. **Health** The following table shows the average remaining lifetime, by age, of women in the United States in 2008. (*Source*: Social Security Administration.)

Current Age, x	Remaining Years, y
0	80.5
20	61.2
30	51.5
50	32.7
65	19.9

a. Find a linear model of these data using the points (0, 80.5) and (65, 19.9). Round the slope to the nearest ten-thousandth.

b. Based on your model, what is the average remaining lifetime of a woman whose age in 2008 was 25? Round to the nearest year.

82. **Health** The following table shows the average remaining lifetime, by age, of men in the United States in 2008. (*Source*: Social Security Administration.)

Current Age, x	Remaining Years, y
0	75.5
20	56.5
30	47.2
50	29.0
65	17.2

a. Find a linear model of these data using the points (0, 75.5) and (65, 17.2). Round the slope to the nearest ten-thousandth.

b. Based on your model, what is the average remaining lifetime of a man whose age in 2008 was 25? Round to the nearest year.

Business In Exercises 83 to 86, determine the profit function for the given revenue function and cost function. Also determine the break-even point.

83. $R(x) = 92.50x$; $C(x) = 52x + 1782$

84. $R(x) = 124x$; $C(x) = 78.5x + 5005$

85. $R(x) = 259x$; $C(x) = 180x + 10{,}270$

86. $R(x) = 14{,}220x$; $C(x) = 8010x + 1{,}602{,}180$

87. **Marginal Cost** In business, *marginal cost* is a phrase used to represent the rate of change, or slope, of a cost function that relates the cost C to the number of units x produced. If a cost function is given by $C(x) = 8x + 275$, find

a. $C(0)$ **b.** $C(1)$ **c.** $C(10)$ **d.** Marginal cost

88. **Marginal Revenue** In business, *marginal revenue* is a phrase used to represent the rate of change, or slope, of a revenue function that relates the revenue R to the number of units x sold. If a revenue function is given by the function $R(x) = 210x$, find

a. $R(0)$ **b.** $R(1)$ **c.** $R(10)$ **d.** Marginal revenue

89. **Break-Even Point for a Rental Truck** A rental company purchases a truck for \$19,500. The truck requires an average cost of \$6.75 per day in maintenance.

a. Find a linear function that expresses the total cost C of owning the truck after t days.

b. The truck rents for \$55.00 a day. Find a linear function that expresses the revenue R when the truck has been rented for t days.

c. The profit after t days, $P(t)$, is given by $P(t) = R(t) - C(t)$. Find the linear function $P(t)$.

d. Use the function $P(t)$ that you obtained in **c.** to determine how many days it will take the company to break even on the purchase of the truck. Assume that the truck is in use every day.

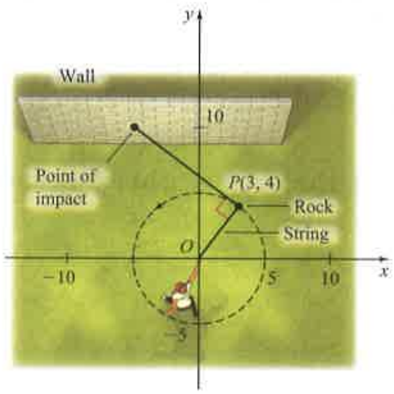
90. **Break-Even Point for a Publisher** A magazine company had a profit of \$98,000 per year when it had 32,000 subscribers. When it obtained 35,000 subscribers, it had a profit of \$117,500. Assume that the profit P is a linear function of the number of subscribers s.

a. Find the function P.

b. What will the profit be if the company has a total of 50,000 subscribers?

c. What is the number of subscribers needed to break even?

91. **Point of Impact** A rock attached to a string is whirled horizontally about the origin in a counterclockwise circular path with radius 5 feet. When the string breaks, the rock travels on a linear path perpendicular to the radius OP and hits a wall located at $y = 10$ feet.



If the string breaks when the rock is at $P(3, 4)$, find the x-coordinate of the point at which the rock hits the wall.

92. **Point of Impact** A rock attached to a string is whirled horizontally about the origin in a counterclockwise circular path with radius 4 feet. When the string breaks, the rock travels on a linear path perpendicular to the radius OP and hits a wall located at $y = 14$ feet. If the string breaks when the rock is at $P(\sqrt{15}, 1)$, find the x-coordinate of the point at which the rock hits the wall.

Enrichment Exercises

93. Find the value of a in the domain of $f(x) = 2x + 3$ for which $f(a) = -1$.

94. Find the value of a in the domain of $f(x) = 4 - 3x$ for which $f(a) = 7$.

95. Find the value of a in the domain of $f(x) = 1 - 4x$ for which $f(a) = 3$.

96. Find the value of a in the domain of $f(x) = \dfrac{2x}{3} + 2$ for which $f(a) = 4$.

97. **Slope of a Secant Line** The graph of $f(x) = x^2 + 1$ is shown in the next column along with points P and Q. The secant line PQ is also shown.

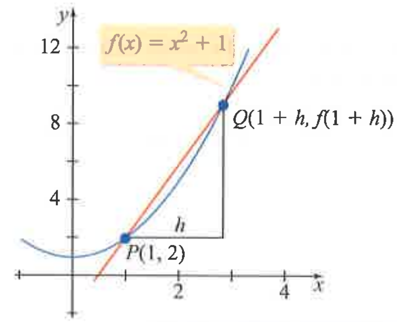
a. If $h = 1$, determine the coordinates of Q and the slope of the line PQ.

b. If $h = 0.1$, determine the coordinates of Q and the slope of the line PQ.

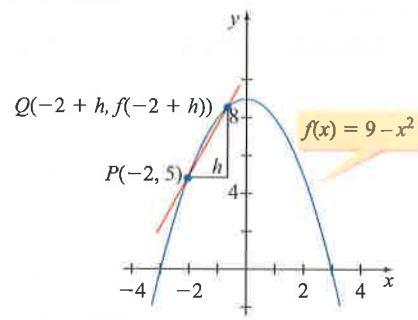
c. If $h = 0.01$, determine the coordinates of Q and the slope of the line PQ.

d. As h approaches 0, what value does the slope of the line PQ seem to be approaching?

e. Show that the slope of the line PQ is $2 + h$.



98. **Slope of a Secant Line** The graph of $f(x) = 9 - x^2$ is shown below along with points P and Q. The secant line PQ is also shown.



a. If $h = 1$, determine the coordinates of Q and the slope of the line PQ.

b. If $h = 0.1$, determine the coordinates of Q and the slope of the line PQ.

c. If $h = 0.01$, determine the coordinates of Q and the slope of the line PQ.

d. As h approaches 0, what value does the slope of the line PQ seem to be approaching?

e. Show that the slope of the line PQ is $4 - h$.

99. Determine the point $P(x, y)$ on the graph of the equation $y = x^2$ such that the slope of the line through the point $(3, 9)$ and P is $\dfrac{15}{2}$.

100. Determine the point $P(x, y)$ on the graph of the equation $y = \sqrt{x + 1}$ such that the slope of the line through the point $(3, 2)$ and P is $\dfrac{3}{8}$.

MID-CHAPTER 2 QUIZ

1. Find the coordinates of the midpoint and the length of the line segment between $P_1(-3, 4)$ and $P_2(1, -2)$.

2. Find the coordinates of the center and the radius of the circle whose equation is $x^2 + y^2 - 6x + 4y - 2 = 0$.

3. Evaluate $f(x) = x^2 - 6x + 1$ when $x = -3$.

4. Write the domain of $f(x) = \sqrt{2 - x}$ in interval notation.

5. Find the zeros of $f(x) = x^2 - x - 12$.

6. Find the slope of the line between the points $P_1(8, -2)$ and $P_2(-2, 3)$.

7. Find the equation of the line parallel to the graph of $2x + 3y = 5$ and passing through $P_1(3, -1)$.

8. Graph $f(x) = -\frac{2}{3}x + 1$ by using the slope and y-intercept.

SECTION 2.4

Standard Form of a Quadratic Function

Maximum and Minimum of a Quadratic Function

Applications of Quadratic Functions

Quadratic Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A10.

PS1. Factor: $3x^2 + 10x - 8$ [P.4]

PS2. Complete the square of $x^2 - 8x$. Write the resulting trinomial as the square of a binomial. [1.3]

PS3. Find $f(-3)$ for $f(x) = 2x^2 - 5x - 7$. [2.2]

PS4. Solve for x: $2x^2 - x = 1$ [1.3]

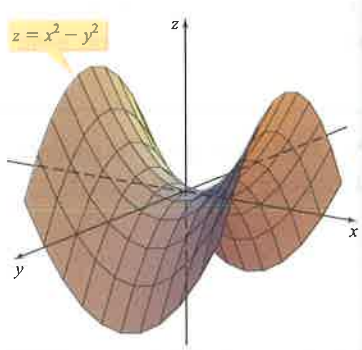
PS5. Solve for x: $x^2 + 3x - 2 = 0$ [1.3]

PS6. Suppose that $h = -16t^2 + 64t + 5$. Find two values of t for which $h = 53$. [1.3]

Some applications can be modeled by a *quadratic function.*

Note

The equation $z = x^2 - y^2$ defines z as a quadratic function of x and y. The graph of $z = x^2 - y^2$ is the *saddle* shown in the figure below. You will study quadratic functions involving two or more independent variables in calculus.



Definition of a Quadratic Function

A **quadratic function** of x is a function that can be represented by an equation of the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$.

EXAMPLE

$f(x) = 2x^2 - 3x + 1$ • $a = 2, b = -3, c = 1$

$g(x) = -x^2 - 5$ • $a = -1, b = 0, c = -5$

$h(x) = x^2 + 5x$ • $a = 1, b = 5, c = 0$

The graph of $f(x) = ax^2 + bx + c, a \neq 0$, is a *parabola*. The graph opens up when $a > 0$, as in Figure 2.51a, and opens down when $a < 0$, as in Figure 2.51b. The **vertex of a parabola** is the lowest point on a parabola that opens up or the highest point on a parabola that opens down. The graph of a parabola has an **axis of symmetry**, a vertical line through the vertex such that if the parabola were folded along that line, the two parts of the graph would match up.