33. $y = 0.25x^2$

34. $3x^2 + 2y = -4$

35. $y = -2|x - 3|$

36. $y = |x + 3| - 2$

37. $y = x^2 - 3$

38. $y = x^2 + 1$

39. $y = \frac{1}{2}(x - 1)^2$

40. $y = 2(x + 2)^2$

41. $y = x^2 + 2x - 8$

42. $y = x^2 - 2x - 8$

43. $y = -x^2 + 2$

44. $y = -x^2 - 1$

In Exercises 45 to 52, find the *x*- and *y*-intercepts of the graph of each equation. Use the intercepts and additional points as needed to draw the graph of the equation.

45. $2x + 5y = 12$

46. $3x - 4y = 15$

47. $x = -y^2 + 5$

48. $x = y^2 - 6$

49. $x = |y| - 4$

50. $x = y^3 - 2$

51. $x^2 + y^2 = 4$

52. $x^2 = y^2$

In Exercises 53 to 60, determine the center and radius of the circle with the given equation.

53. $x^2 + y^2 = 36$

54. $x^2 + y^2 = 49$

55. $(x - 1)^2 + (y - 3)^2 = 49$

56. $(x - 2)^2 + (y - 4)^2 = 25$

57. $(x + 2)^2 + (y + 5)^2 = 25$

58. $(x + 3)^2 + (y + 5)^2 = 121$

59. $(x - 8)^2 + y^2 = \frac{1}{4}$

60. $x^2 + (y - 12)^2 = 1$

In Exercises 61 to 68, find an equation of the circle that satisfies the given conditions. Write your answer in standard form.

61. Center (4, 1), radius 2

62. Center (5, −3), radius 4

63. Center $\left(\frac{1}{2}, \frac{1}{4}\right)$, radius $\sqrt{5}$

64. Center $\left(0, \frac{2}{3}\right)$, radius $\sqrt{11}$

65. Center (0, 0), passing through (−3, 4)

66. Center (0, 0), passing through (5, 12)

67. Center (1, 3), passing through (4, −1)

68. Center (−2, 5), passing through (1, 7)

In Exercises 69 to 76, find the equation of the circle described. Write your answers in standard form.

69. The coordinates of the center are (−2, 5) and the length of the diameter is 10.

70. The coordinates of the center are (0, −1) and the length of the diameter is 8.

71. The circle has a diameter with endpoints whose coordinates are (2, 3) and (−4, 11).

72. The circle has a diameter with endpoints whose coordinates are (7, −2) and (−3, 5).

73. The circle has a diameter with endpoints whose coordinates are (5, −3) and (−1, −5).

74. The circle has a diameter with endpoints whose coordinates are (4, −6) and (0, −2).

75. The circle has center with coordinates (7, 11) and is tangent to the *x*-axis.

76. The circle has center with coordinates (−2, 3) and is tangent to the *y*-axis.

In Exercises 77 to 84, find the center and radius of the graph of the circle. The equations of the circles are written in general form.

77. $x^2 + y^2 - 6x + 5 = 0$

78. $x^2 + y^2 - 6x - 4y + 12 = 0$

79. $x^2 + y^2 - 14x + 8y + 53 = 0$

80. $x^2 + y^2 - 10x + 2y + 18 = 0$

81. $x^2 + y^2 - x + 3y - \frac{15}{4} = 0$

82. $x^2 + y^2 + 3x - 5y + \frac{25}{4} = 0$

83. $x^2 + y^2 + 3x - 6y + 2 = 0$

84. $x^2 + y^2 - 5x - y - 4 = 0$

Enrichment Exercises

85. Find all points on the x-axis that are 10 units from the point (4, 6). (*Hint*: First write the distance formula with (4, 6) as one of the points and $(x, 0)$ as the other point.)

86. Find all points on the y-axis that are 12 units from the point (5, −3).

In Exercises 87 and 88, find the *x*- and *y*-intercepts of the graph of each equation. Use the intercepts and additional points as needed to draw the graph of the equation.

87. $|x| + |y| = 4$

88. $|x - 4y| = 8$

89. Find a formula for the set of all points (x, y) for which the distance from (x, y) to (3, 4) is 5.

90. Find a formula for the set of all points (x, y) for which the distance from (x, y) to (−5, 12) is 13.

SECTION 2.2 Introduction to Functions

- Relations
- Functions
- Function Notation
- Graphs of Functions
- Greatest Integer Function (Floor Function)
- Applications of Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A8.

PS1. Evaluate $x^2 + 3x - 4$ when $x = -3$. [P.3]

PS2. From the set of ordered pairs $A = \{(-3, 2), (-2, 4), (-1, 1), (0, 4), (2, 5)\}$, create two new sets, D and R, where D is the set of the first coordinates of the ordered pairs of A and R is the set of the second coordinates of the ordered pairs of A. [P.1/2.1]

PS3. Find the length of the line segment connecting $P_1(-4, 1)$ and $P_2(3, -2)$. [2.1]

PS4. For what values of x is $\sqrt{2x - 6}$ a real number? [P.6/1.5]

PS5. For what values of x is $\dfrac{x + 3}{x^2 - x - 6}$ not a real number? [P.5]

PS6. If $a = 3x + 4$ and $a = 6x - 5$, find the values of a. [1.1]

Relations

In many situations in science, business, and mathematics, a correspondence exists between the two sets. The correspondence is often defined by a *table*, an *equation*, or a *graph*, each of which can be viewed from a mathematical perspective as a set of ordered pairs. In mathematics, any set of ordered pairs is called a **relation.**

Table 2.1 defines a correspondence between a set of percent scores and a set of letter grades. For each score from 0 to 100, there corresponds only one letter grade. The score 94% corresponds to the letter grade of A. Using ordered-pair notation, we record this correspondence as (94, A).

Table 2.1

Score	Grade
[90, 100]	A
[80, 90)	B
[70, 80)	C
[60, 70)	D
[0, 60)	F

The equation $d = 16t^2$ indicates that the distance d that a rock falls (neglecting air resistance) corresponds to the time t that it has been falling. For each nonnegative value t, the equation assigns only one value for the distance d. According to this equation, in 3 seconds a rock will fall 144 feet, which we record as (3, 144). Some of the other ordered pairs determined by $d = 16t^2$ are (0, 0), (1, 16), (2, 64), and (2.5, 100).

Equation: $d = 16t^2$

If $t = 3$, then $d = 16(3)^2 = 144$

The graph in Figure 2.19, on page 165, defines a correspondence between the length of a pendulum and the time it takes the pendulum to complete one oscillation. For each nonnegative pendulum length, the graph yields only one time. According to the graph, a pendulum length of 2 feet yields an oscillation time of 1.6 seconds and a pendulum length of 4 feet yields an oscillation time of 2.2 seconds, where the time is measured to the nearest tenth of a second. These results can be recorded as the ordered pairs (2, 1.6) and (4, 2.2).

Functions

The preceding table and equation and the following graph each determine a special type of relation called a *function.*

Graph: A pendulum's oscillation time

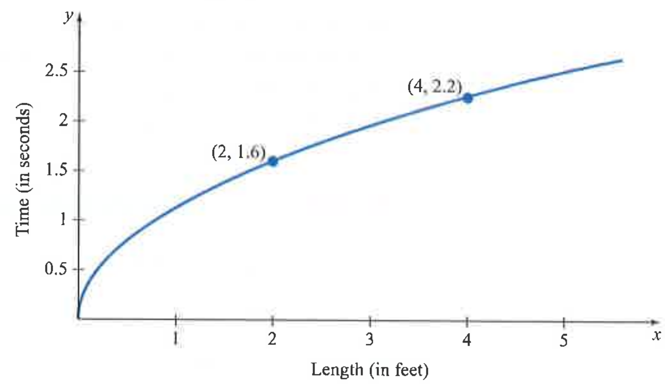


Figure 2.19

Math Matters

Historians generally agree that Leonhard Euler (1707–1783) was the first person to use the word *function*. His definition of function occurs in his book *Introduction to Analysis of the Infinite*, published in 1748. Euler contributed to many areas of mathematics and was one of the most prolific expositors of mathematics.

Definition of a Function

A **function** is a set of ordered pairs in which no two ordered pairs have the same first coordinate and different second coordinates.

Although every function is a relation, not every relation is a function. For instance, consider (94, A) from the grading correspondence. The first coordinate, 94, is paired with a second coordinate, A. It would not make sense to have 94 paired with A, (94, A), and 94 paired with B, (94, B). The same first coordinate would be paired with two different second coordinates. This would mean that two students with the same score received different grades, one student an A and the other a B!

Functions may have ordered pairs with the same second coordinate. For instance, (94, A) and (95, A) are both ordered pairs that belong to the function defined by Table 2.1. Thus a function may have different first coordinates and the same second coordinate.

The equation $d = 16t^2$ represents a function because for each value of t there is only one value of d. However, not every equation represents a function. For instance, $y^2 = 25 - x^2$ does not represent a function. The ordered pairs $(-3, 4)$ and $(-3, -4)$ are both solutions of the equation. However, these ordered pairs do not satisfy the definition of a function: There are two ordered pairs with the same first coordinate but *different* second coordinates.

Question • Does the set $\{(0, 0), (1, 0), (2, 0), (3, 0), (4, 0)\}$ define a function?

The **domain** of a function (or relation) is the set of all the first coordinates of the ordered pairs. The **range** of a function (or relation) is the set of all the second coordinates. In the function determined by the grading correspondence in Table 2.1, the domain is the interval [0, 100]. The range is {A, B, C, D, F}. In a function, each domain element is paired with one and only one range element.

Answer • Yes. There are no two ordered pairs with the same first coordinate but different second coordinates.

If a function is defined by an equation, the variable that represents elements of the domain is the **independent variable**. The variable that represents elements of the range is the **dependent variable**. For the situation involving the free fall of a rock, we used the equation $d = 16t^2$. The elements of the domain represented the time the rock fell, and the elements of the range represented the distance the rock fell. Thus, in $d = 16t^2$, the independent variable is t and the dependent variable is d.

The specific letters used for the independent and dependent variables are not important. For example, $y = 16x^2$ represents the same function as $d = 16t^2$. Traditionally, x is used for the independent variable and y for the dependent variable. Anytime we use the phrase "y is a function of x" or a similar phrase with different letters, the variable that follows "function of" is the independent variable.

EXAMPLE 1 Identify Functions

State whether the relation defines y as a function of x.

a. $\{(2, 3), (4, 1), (4, 5)\}$ **b.** $3x + y = 1$ **c.** $-4x^2 + y^2 = 9$

d. The correspondence between the x values and the y values in Figure 2.20.

Solution

a. There are two ordered pairs, (4, 1) and (4, 5), with the same first coordinate and different second coordinates. This set does not define y as a function of x.

b. Solving $3x + y = 1$ for y yields $y = -3x + 1$. Because $-3x + 1$ is a unique real number for each x, this equation defines y as a function of x.

c. Solving $-4x^2 + y^2 = 9$ for y yields $y = \pm\sqrt{4x^2 + 9}$. The right side $\pm\sqrt{4x^2 + 9}$ produces two values of y for each value of x. For example, when $x = 0$, $y = 3$ or $y = -3$. Thus $-4x^2 + y^2 = 9$ does not define y as a function of x.

d. Each x is paired with one and only one y. The correspondence in Figure 2.20 defines y as a function of x.

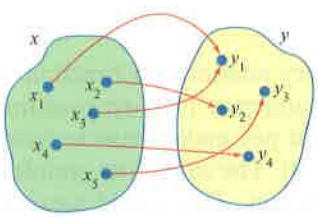


Figure 2.20

▶ Try Exercise 14, page 178

Function Notation

Functions can be named by using a letter or a combination of letters, such as f, g, A, log, or tan. If x is an element of the domain of f, then $f(x)$, which is read "f of x" or "the value of f at x," is the element in the range of f that corresponds to the domain element x. The notation "f" and the notation "$f(x)$" mean different things. "f" is the name of the function, whereas "$f(x)$" is the value of the function at x. Finding the value of $f(x)$ is referred to as *evaluating f* at x. To evaluate $f(x)$ at $x = a$, substitute a for x and simplify.

EXAMPLE 2 Evaluate Functions

Let $f(x) = x^2 - 1$, and evaluate.

a. $f(-5)$ **b.** $f(3b)$ **c.** $3f(b)$ **d.** $f(a + 3)$ **e.** $f(a) + f(3)$

Solution

a. $f(-5) = (-5)^2 - 1 = 25 - 1 = 24$ • Substitute -5 for x, and simplify.

b. $f(3b) = (3b)^2 - 1 = 9b^2 - 1$ • Substitute $3b$ for x, and simplify.

c. $3f(b) = 3(b^2 - 1) = 3b^2 - 3$ • Substitute b for x, and simplify.

d. $f(a + 3) = (a + 3)^2 - 1$ • Substitute $a + 3$ for x.
$= a^2 + 6a + 8$ • Simplify.

e. $f(a) + f(3) = (a^2 - 1) + (3^2 - 1)$ • Substitute a for x; substitute 3 for x.
$= a^2 + 7$ • Simplify.

▶ **Try Exercise 26, page 178**

Sometimes the domain of a function is stated explicitly. For example, each of f, g, and h below is given by an equation followed by a statement that indicates the domain.

$$f(x) = x^2,\ x > 0 \qquad g(t) = \frac{1}{t^2 + 4},\ 0 \le t \le 5 \qquad h(x) = x^2,\ x = 1, 2, 3$$

Although f and h have the same equation, they are different functions because they have different domains. If the domain of a function is not explicitly stated, then its domain is determined by the following convention.

Domain of a Function

Unless otherwise stated, the domain of a function is the set of all real numbers for which the function makes sense and yields real numbers.

EXAMPLE 3 Determine the Domain of a Function

Determine the domain of each function.

a. $G(t) = \dfrac{t - 2}{t^2 - 3t - 10}$ b. $f(x) = \sqrt{4 - x^2}$

c. $A(s) = s^2$, where $A(s)$ is the area of a square whose side is s units

Solution

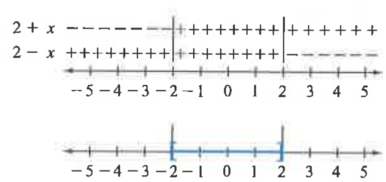
a. Any value of t for which the denominator of G is 0 is not in the domain of G because division by 0 is not defined. To find these values, solve $t^2 - 3t - 10 = 0$ for t.

$$t^2 - 3t - 10 = 0 \qquad\qquad t + 2 = 0 \qquad t - 5 = 0$$
$$(t + 2)(t - 5) = 0 \quad \Rightarrow \quad t = -2 \qquad t = 5$$

The domain is all real numbers except −2 and 5.

b. Any value of x for which $4 - x^2 < 0$ is not in the domain of f because these values would produce a complex number value of the function. Therefore, the domain of f is all values of x for which $4 - x^2 \ge 0$. Solve the inequality for x using the critical-value method discussed in Section 1.5.

$$4 - x^2 \ge 0$$
$$(2 + x)(2 - x) \ge 0$$



(continued)

The sign diagram above shows that $4 - x^2 \geq 0$ when $-2 \leq x \leq 2$. In interval notation, the domain of f is $[-2, 2]$.

c. For any value of s, $A(s)$ is a real number. However, s is given as the length of the side of a square. Therefore $s > 0$. In interval notation, the domain of A is $(0, \infty)$.

Try Exercise 36, page 179

Graphs of Functions

If a is an element of the domain of a function f, then $(a, f(a))$ is an ordered pair that belongs to that function.

Definition of the Graph of a Function

The **graph of a function** is the graph of all ordered pairs that belong to the function.

EXAMPLE 4 Graph a Function by Plotting Points

Graph each of the following. State the domain of each function.

a. $f(x) = 2x - 3$ **b.** $g(x) = 2x^2 - 3$ **c.** $h(x) = 2\sqrt{x} - 3$

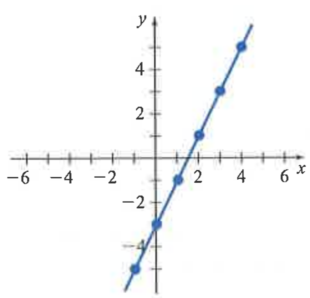
Solution

For each part, we create a table of ordered pairs for the function, plot the ordered pairs, and then draw a graph through them. Although each of the functions has a similar look, their graphs are quite different. In some cases, it takes a bit of effort to produce, by just plotting points, an accurate graph of a function.

a. Because $2x - 3$ is a real number for all values of x, the domain of f is all real numbers. This can be written $(-\infty, \infty)$.

x	-1	0	1	2	3	4
$y = f(x) = 2x - 3$	-5	-3	-1	1	3	5

Plot each ordered pair, and then draw a smooth graph through the points. The graph is shown in Figure 2.21.



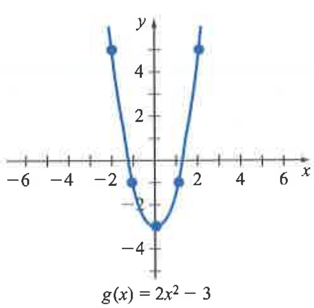
$f(x) = 2x - 3$

Figure 2.21

b. Because $2x^2 - 3$ is a real number for all values of x, the domain of g is all real numbers. This can be written $(-\infty, \infty)$.

x	-2	-1	0	1	2
$y = g(x) = 2x^2 - 3$	5	-1	-3	-1	5

Plot each ordered pair, and then draw a smooth graph through the points. The graph is shown in Figure 2.22.



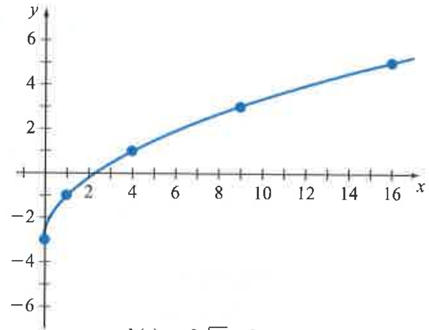
$g(x) = 2x^2 - 3$

Figure 2.22

c. Because $2\sqrt{x} - 3$ is *not* a real number when $x < 0$, the domain of h is all real numbers greater than or equal to zero. This can be written $[0, \infty)$.

x	0	1	4	9	16
$y = h(x) = 2\sqrt{x} - 3$	-3	-1	1	3	5

Plot each ordered pair, and then draw a smooth graph through the points. The graph is shown in Figure 2.23.



$h(x) = 2\sqrt{x} - 3$

Figure 2.23

▶ **Try Exercise 52, page 179**

Integrating Technology

A graphing utility can be used to draw the graph of a function. The graphs of $g(x) = 2x^2 - 3$ and $h(x) = 2\sqrt{x} - 3$ are shown in Figures 2.24 and 2.25. To graph g, enter the equation Y1=2x²−3. To graph h, enter the equation Y1=2√x−3.

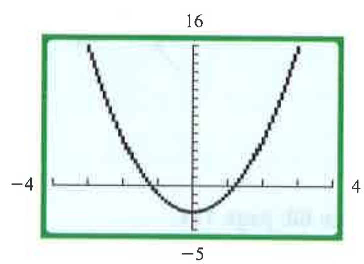


Figure 2.24

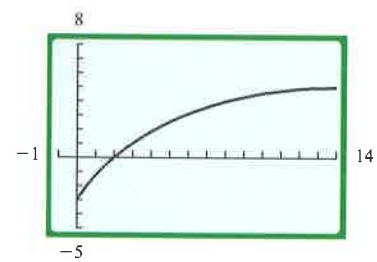


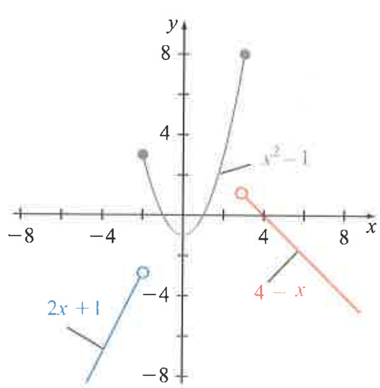
Figure 2.25

Piecewise-defined functions are functions that are represented by more than one expression. For instance, the function f defined below consists of three pieces, $2x + 1$, $x^2 - 1$, and $4 - x$.

$$f(x) = \begin{cases} 2x + 1, & x < -2 \\ x^2 - 1, & -2 \le x \le 3 \\ 4 - x, & x > 3 \end{cases}$$

To evaluate this function at x, determine the interval in which x lies and then use the expression that corresponds to that interval to evaluate the function. The following examples show how to evaluate $f(-4)$, $f(5)$, and $f(2)$.

Since $-4 < -2$, use $2x + 1$.	$f(-4) = 2(-4) + 1 = -7$
Since $5 > 3$, use $4 - x$.	$f(5) = 4 - 5 = -1$
Since $-2 \le 2 \le 3$, use $x^2 - 1$.	$f(2) = 2^2 - 1 = 3$



The graph is shown at the left. Note the use of solid and open circles on the graph. For instance, the solid circle at $(-2, 3)$ indicates that when $x = -2$, $x^2 - 1$ is used because f is defined to be $x^2 - 1$ for $-2 \le x \le 3$ and $(-2)^2 - 1 = 3$. The open circle indicates that a point is not included. For instance, the open circle at $(3, 1)$ is used to indicate it is not part of the graph of $4 - x$ because that graph is defined for $x > 3$, not when $x = 3$.

EXAMPLE 5 Evaluate and Graph a Piecewise-Defined Function

Let $f(x) = \begin{cases} x + 6, & x \le -2 \\ x^2, & -2 < x \le 3 \\ -2x + 15, & x > 3 \end{cases}$

Find: **a.** $f(4)$ **b.** $f(0)$ **c.** $f(-3)$ **d.** Graph f.

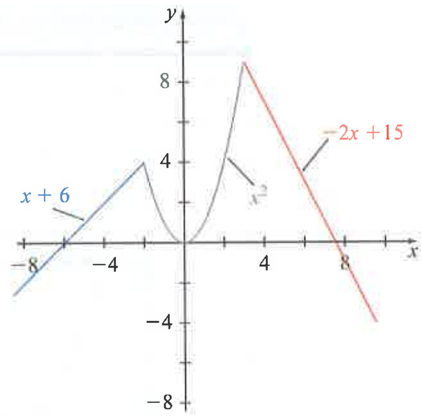
Solution

a. Because $4 > 3$, use $-2x + 15$ $\qquad f(4) = -2(4) + 15 = 7$

b. Because $-2 < 0 \le 3$, use x^2 $\qquad f(0) = 0^2 = 0$

c. Because $-3 \le -2$, use $x + 6$ $\qquad f(-3) = -3 + 6 = 3$

d. To graph f, graph $x + 6$ for $x \le -2$, graph x^2 for $-2 < x \le 3$, and graph $-2 + 15$ for $x > 3$. The graph is shown below.



▶ Try Exercise 60, page 179.

The following example shows how to determine the specific value or values of the independent variable for a given value of the dependent variable.

EXAMPLE 6 Determine a Domain Value Given a Range Value

Find the values of a in the domain of $f(x) = x^2 - x - 4$ for which $f(a) = 2$.

Algebraic Solution

$$f(a) = 2$$

$$a^2 - a - 4 = 2 \qquad \text{• Replace } f(a) \text{ with } a^2 - a - 4.$$

$$a^2 - a - 6 = 0 \qquad \text{• Solve for } a.$$

$$(a + 2)(a - 3) = 0$$

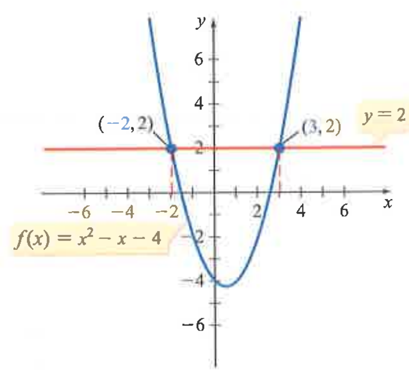
$$a + 2 = 0 \quad \text{or} \quad a - 3 = 0$$

$$a = -2 \quad \text{or} \quad a = 3$$

The values of a in the domain of f for which $f(a) = 2$ are -2 and 3.

Visualize the Solution

By graphing $y = 2$ and $f(x) = x^2 - x - 4$, we can see that $f(x) = 2$ when $x = -2$ and $x = 3$.



▶ Try Exercise 66, page 179

It may be that, for a given number b, there is no number in the domain of f for which $f(a) = b$. For instance, suppose $f(x) = x^2 + 3$ and we are asked to find a value in the domain of f for which $f(a) = 2$. If we attempt to solve as we did in Example 6, we get the result shown here.

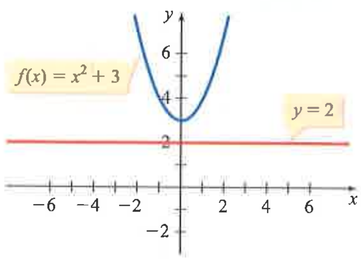


Figure 2.26

$$f(a) = 3$$

$$a^2 + 3 = 2 \qquad \text{• Replace } f(a) \text{ with } a^2 + 3.$$

$$a^2 = -1 \qquad \text{• Solve for } a.$$

$$\sqrt{a^2} = \sqrt{-1}$$

$$a = \pm i$$

The values of a are complex numbers and not in the domain of f. Note from the graph in Figure 2.26 that the horizontal line through (0, 2) does not intersect the graph as it did in Example 6.

A problem of special interest is determining the values in the domain of a function f for which $f(a) = 0$.

Zero of a Function

A value a in the domain of a function f for which $f(a) = 0$ is called a **zero** of f.

EXAMPLES

- Let $f(x) = 2x - 4$. When $x = 2$, we have

  $$f(x) = 2x - 4$$
  $$f(2) = 2(2) - 4$$
  $$= 0$$

  Because $f(2) = 0$, 2 is a zero of f.

- Let $g(x) = x^2 + 2x - 15$. When $x = -5$ and $x = 3$, we have

  $$g(x) = x^2 + 2x - 15 \qquad g(x) = x^2 + 2x - 15$$
  $$g(-5) = (-5)^2 + 2(-5) - 15 \qquad g(3) = (3)^2 + 2(3) - 15$$
  $$= 25 - 10 - 15 \qquad = 9 + 6 - 15$$
  $$= 0 \qquad = 0$$

  In this case, $g(-5) = 0$ and $g(3) = 0$, so there are two zeros of g, -5 and 3.

- Let $h(x) = x^2 + 1$. When $x = 0$, we have

  $$h(x) = x^2 + 1$$
  $$h(0) = 0^2 + 1$$
  $$= 1$$

  In this case, $h(0) = 1 \neq 0$, so 0 is not a zero of the function.

EXAMPLE 7 Determine a Zero of a Function

Find the zeros of $f(x) = x^2 - 2x - 3$.

Algebraic Solution

To find the zeros of f, solve the equation $f(x) = 0$ for x.

$$f(x) = 0 \quad \bullet \text{ To find the zeros, let } f(x) = 0.$$

$$x^2 - 2x - 3 = 0 \quad \bullet \text{ Replace } f(x) \text{ with } x^2 - 2x - 3.$$

$$(x + 1)(x - 3) = 0 \quad \bullet \text{ Solve for } x.$$

$$x + 1 = 0 \qquad x - 3 = 0$$

$$x = -1 \qquad x = 3$$

The zeros of $f(x) = x^2 - 2x - 3$ are -1 and 3.

Visualize the Solution

By graphing $f(x) = x^2 - 2x - 3$, we can see that $f(x) = 0$ when $x = -1$ and $x = 3$. The zeros of f are -1 and 3, as shown in Figure 2.27.

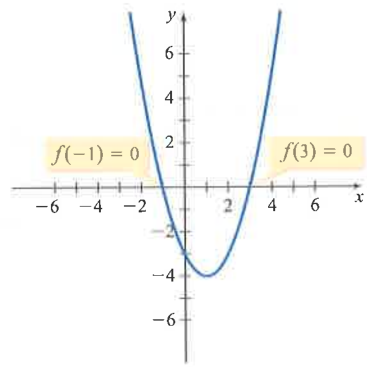


Figure 2.27

▸ Try Exercise 78, page 180

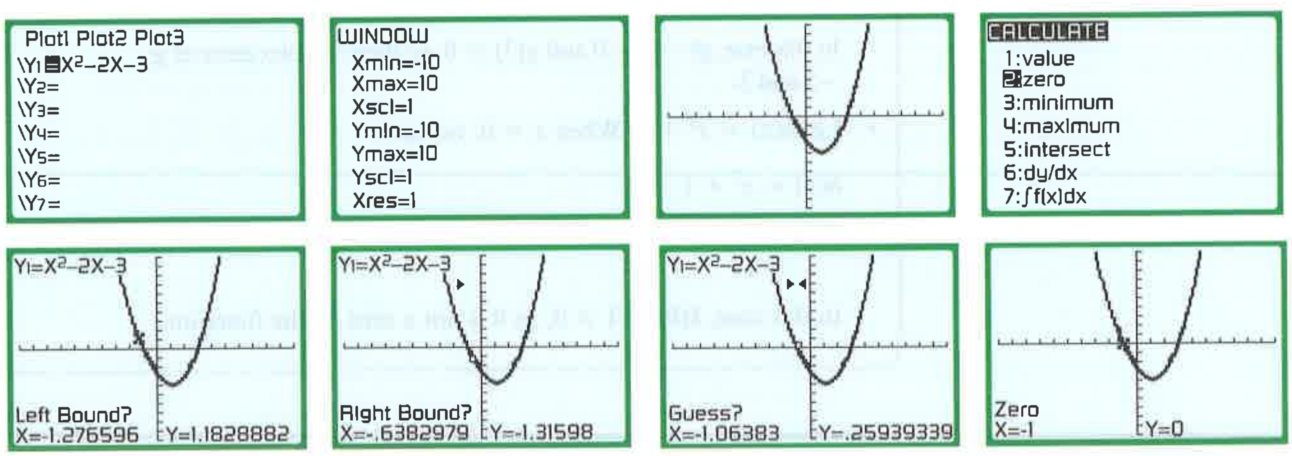
In Example 7, note that the zeros of f are the x-coordinates of the x-intercepts of the graph of f. This fact can be used to find solutions of some equations with a graphing calculator.

Real Zeros and x-Intercepts Theorem

The real number c is a zero of f if and only if $(c, 0)$ is an x-intercept of the graph of $y = f(x)$.

Integrating Technology

The graphs below show a sequence of steps on a TI-83 Plus/TI-84 Plus graphing calculator that was used to find the zero of -1 in Example 7. Note the fourth screen, which shows the calculating of the zero, and observe that the zero is the x-coordinate of the x-intercept of the graph of f. In Chapter 3, we explore in more depth the zeros of a function.



The definition of a function as a set of ordered pairs in which no two ordered pairs that have the same first coordinate have different second coordinates implies that any vertical line intersects the graph of a function at no more than one point. This is known as the *vertical line test.*

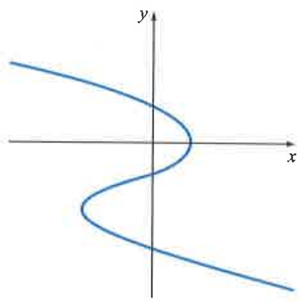
The Vertical Line Test for Functions

A graph is the graph of a function if and only if no vertical line intersects the graph at more than one point.

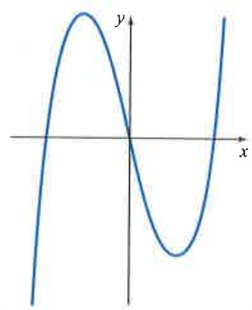
EXAMPLE 8 Apply the Vertical Line Test

State whether the graph is the graph of a function.

a.



b.



Solution

a. This graph *is not* the graph of a function because some vertical lines intersect the graph in more than one point.

b. This graph *is* the graph of a function because every vertical line intersects the graph in at most one point.

▶ **Try Exercise 80, page 180**

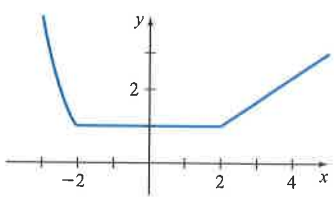


Figure 2.28

Consider the graph in Figure 2.28. As a point on the graph moves from left to right, this graph falls for values of $x \le -2$, remains the same height from $x = -2$ to $x = 2$, and rises for $x \ge 2$. The function represented by the graph is said to be *decreasing* on the interval $(-\infty, -2]$, *constant* on the interval $[-2, 2]$, and *increasing* on the interval $[2, \infty)$.

Definition of Increasing, Decreasing, and Constant Functions

If a and b are elements of an interval I that is a subset of the domain of a function f, then

- f is **increasing** on I if $f(a) < f(b)$ whenever $a < b$.
- f is **decreasing** on I if $f(a) > f(b)$ whenever $a < b$.
- f is **constant** on I if $f(a) = f(b)$ for all a and b.

Recall that a function is a relation in which no two ordered pairs that have the same first coordinate have different second coordinates. This means that, given any x, there is only one y that can be paired with that x. A function f that satisfies the additional condition that given any element b in the range of f there is *exactly one* element a in the domain of f such that $f(a) = b$ is called a *one-to-one function*.

Definition of a One-to-One Function

A function f is a **one-to-one** function if and only if $f(a) = f(b)$ implies $a = b$.

EXAMPLES

- Let $f(x) = 2x + 3$ and suppose $f(a) = f(b)$. Then

$$f(a) = f(b)$$
$$2a + 3 = 2b + 3 \quad \text{• Evaluate } f \text{ at } a \text{ and } b.$$
$$2a = 2b \quad \text{• Subtract 3 from each side.}$$
$$a = b \quad \text{• Divide both sides by 2.}$$

Because $a = b$, $f(x) = 2x + 3$ is a one-to-one function.

- Let $g(x) = x^2$ and suppose $g(a) = g(b)$. Then

$$g(a) = g(b)$$
$$a^2 = b^2 \quad \text{• Evaluate } g \text{ at } a \text{ and } b.$$
$$\sqrt{a^2} = \sqrt{b^2} \quad \text{• Take the square root of each side.}$$
$$|a| = |b| \quad \text{• Simplify.}$$
$$a = \pm b$$

In this case, a could equal $-b$ or b, so g is not a one-to-one function. For instance, $g(-3) = 9$ and $g(3) = 9$ but $-3 \neq 3$.

- All increasing functions and all decreasing functions are one-to-one functions.

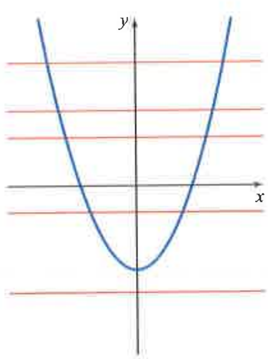


Figure 2.29
Some horizontal lines intersect this graph at more than one point. This is *not* the graph of a one-to-one function.

In a manner similar to applying the vertical line test, we can apply a horizontal line test to identify one-to-one functions.

Horizontal Line Test for a One-to-One Function

If every horizontal line intersects the graph of a function at most once, then the graph is the graph of a one-to-one function.

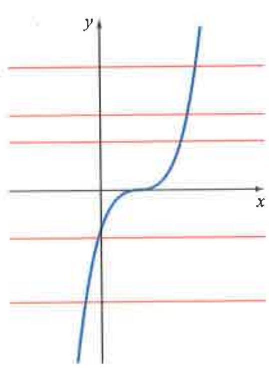


Figure 2.30
Every horizontal line intersects this graph at most once. This is the graph of a one-to-one function.

For example, some horizontal lines intersect the graph in Figure 2.29 at more than one point. This is *not* the graph of a one-to-one function. Every horizontal line intersects the graph in Figure 2.30 at most once. This is the graph of a one-to-one function.

Greatest Integer Function (Floor Function)

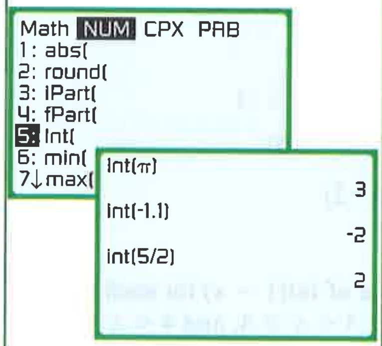
The graphs of some functions do not have any breaks or gaps. These functions, whose graphs can be drawn without lifting a pencil off the paper, are called *continuous functions*. The graphs of other functions do have breaks or *discontinuities*. One such function is the **greatest integer function** or **floor function**. This function is denoted by symbols such as $[\![x]\!]$, $\lfloor x \rfloor$, and int(x).

The value of the greatest integer function at x is the greatest integer that is less than or equal to x. For instance,

$$\lfloor -1.1 \rfloor = -2 \qquad [\![-3]\!] = -3 \qquad \text{int}\left(\frac{5}{2}\right) = 2 \qquad \lfloor 5 \rfloor = 5 \qquad [\![\pi]\!] = 3$$

Integrating Technology

Many graphing calculators use the notation int(x) for the greatest integer function. Here are screens from a TI-83 Plus/TI-84 Plus.



Question • Evaluate. **a.** $\text{int}\left(-\frac{3}{2}\right)$ **b.** $\lfloor 2 \rfloor$

To graph the floor function, first observe that the value of the floor function is constant between any two consecutive integers. For instance, between the integers 1 and 2, we have

$$\text{int}(1.1) = 1 \qquad \text{int}(1.35) = 1 \qquad \text{int}(1.872) = 1 \qquad \text{int}(1.999) = 1$$

Between -3 and -2, we have

$$\text{int}(-2.98) = -3 \qquad \text{int}(-2.4) = -3 \qquad \text{int}(-2.35) = -3 \qquad \text{int}(-2.01) = -3$$

Using this property of the floor function, we can create a table of values and then graph the floor function. See Figure 2.31.

x	$y = \text{int}(x)$
$-5 \le x < -4$	-5
$-4 \le x < -3$	-4
$-3 \le x < -2$	-3
$-2 \le x < -1$	-2
$-1 \le x < 0$	-1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2
$3 \le x < 4$	3
$4 \le x < 5$	4

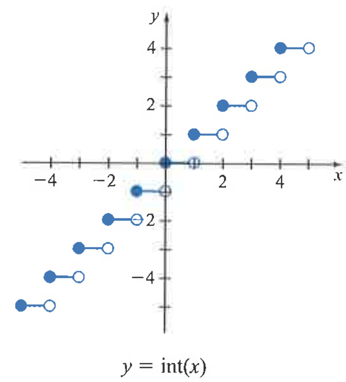


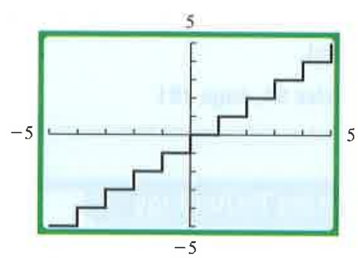
Figure 2.31

The graph of the floor function has discontinuities (breaks) whenever x is an integer. The domain of the floor function is the set of real numbers; the range is the set of integers. Because the graph appears to be a series of steps, sometimes the floor function is referred to as a **step function**.

Integrating Technology

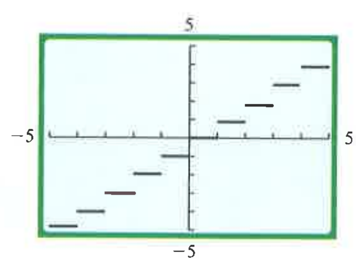
Many graphing calculators use the notation int(x) for the floor function. The screens shown are from a TI-83 Plus/TI-84 Plus graphing calculator. The graph in Figure 2.32 was drawn in "connected" mode. This graph does not show the discontinuities that occur whenever x is an integer.

The graph in Figure 2.33 was constructed by graphing the floor function in "dot" mode. In this case, the discontinuities at the integers are apparent.



$y = \text{int}(x)$
connected mode

Figure 2.32



$y = \text{int}(x)$
dot mode

Figure 2.33

Answer • **a.** Because -2 is the greatest integer less than or equal to $-\frac{3}{2}$, $\text{int}\left(-\frac{3}{2}\right) = -2$.

b. Because 2 is the greatest integer less than or equal to 2, $\lfloor 2 \rfloor = 2$.

EXAMPLE 9 Use the Greatest Integer Function to Model Expenses

The cost of parking in a garage is \$3 for the first hour or any part of the first hour and \$2 for each additional hour or any part of an hour thereafter. If x is the time in hours that you park your car, then the cost is given by

$$C(x) = 3 - 2\,\text{int}(1 - x), \quad x > 0$$

a. Evaluate $C(2)$ and $C(2.5)$. **b.** Graph $y = C(x)$ for $0 < x \le 5$.

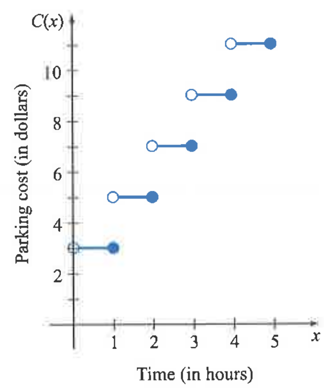
Solution

a. $C(2) = 3 - 2\,\text{int}(1 - 2)$
$= 3 - 2\,\text{int}(-1)$
$= 3 - 2(-1)$
$= \$5$

$C(2.5) = 3 - 2\,\text{int}(1 - 2.5)$
$= 3 - 2\,\text{int}(-1.5)$
$= 3 - 2(-2)$
$= \$7$

b. To graph $C(x)$ for $0 < x \le 5$, consider the value of $\text{int}(1 - x)$ for each of the intervals $0 < x \le 1$, $1 < x \le 2$, $2 < x \le 3$, $3 < x \le 4$, and $4 < x \le 5$. For instance, when $0 < x \le 1$, $0 \le 1 - x < 1$. Thus $\text{int}(1 - x) = 0$ when $0 < x \le 1$. Now consider $1 < x \le 2$. When $1 < x \le 2$, $-1 \le 1 - x < 0$. Thus $\text{int}(1 - x) = -1$ when $1 < x \le 2$. Applying the same reasoning to each of the other intervals gives the following table of values and the graph of C shown in Figure 2.34.

x	$C(x) = 3 - 2\,\text{int}(1 - x)$
$0 < x \le 1$	3
$1 < x \le 2$	5
$2 < x \le 3$	7
$3 < x \le 4$	9
$4 < x \le 5$	11



$C(x) = 3 - 2\,\text{int}(1 - x)$

Figure 2.34

Because $C(1) = 3$, $C(2) = 5$, $C(3) = 7$, $C(4) = 9$, and $C(5) = 11$, we can use a solid circle at the right endpoint of each "step" and an open circle at each left endpoint.

Try Exercise 94, page 181

Integrating Technology

The function graphed in Example 9 is an example of a function for which a graphing calculator may not produce a graph that is a good representation of the function. You may be required to *make adjustments* in the MODE, SET UP, or WINDOW of the graphing calculator so that it will produce a better representation of the function. A graph may also require some *fine tuning,* such as open or solid circles at particular points, to accurately represent the function.

Applications of Functions

EXAMPLE 10 Solve an Application

A car was purchased for \$16,500. Assuming that the car depreciates at a constant rate of \$2200 per year (*straight-line depreciation*) for the first 7 years, write the value v of the car as a function of time, and calculate the value of the car 3 years after purchase.

Solution

Let t represent the number of years that have passed since the car was purchased. Then $2200t$ is the amount by which the value of the car has depreciated after t years. The value of the car at time t is given by

$$v(t) = 16{,}500 - 2200t, \; 0 \le t \le 7$$

When $t = 3$, the value of the car is

$$v(3) = 16{,}500 - 2200(3) = 16{,}500 - 6600 = \$9900$$

Try Exercise 98, page 181

Often in applied mathematics, formulas are used to determine the functional relationship that exists between two variables.

EXAMPLE 11 Solve an Application

A lighthouse is 2 miles south of a port. A ship leaves port and sails east at a rate of 7 miles per hour. Express the distance d between the ship and the lighthouse as a function of time, given that the ship has been sailing for t hours.

Solution

Draw a diagram and label it as shown in Figure 2.35. Because Distance = (Rate)(Time) and the rate is 7, in t hours the ship has sailed a distance of $7t$.

Figure 2.35

$$[d(t)]^2 = (7t)^2 + 2^2$$ • Use the Pythagorean Theorem.

$$[d(t)]^2 = 49t^2 + 4$$

$$d(t) = \sqrt{49t^2 + 4}$$ • The ± sign is not used because the distance $d(t)$ must be nonnegative.

Try Exercise 104, page 182

EXAMPLE 12 Solve an Application

An open box is to be made from a square piece of cardboard that measures 40 inches on each side. To construct the box, squares that measure x inches on each side are cut from each corner of the cardboard as shown in Figure 2.36.

a. Express the volume V of the box as a function of x.

b. Determine the domain of V.

(continued)

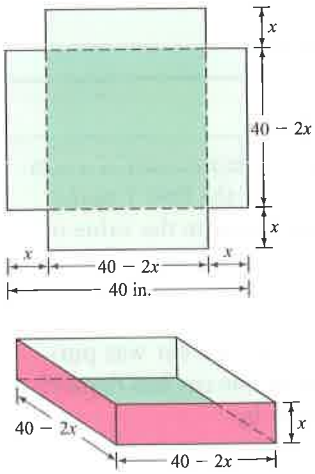


Figure 2.36

Solution

a. The length l of the box is $40 - 2x$. The width w is also $40 - 2x$. The height of the box is x. The volume V of a box is the product of its length, its width, and its height. Thus

$$V = (40 - 2x)^2 x$$

b. The squares that are cut from each corner require x to be larger than 0 inches but less than 20 inches. Thus the domain is $\{x \mid 0 < x < 20\}$.

Try Exercise 100, page 181

EXERCISE SET 2.2

Concept Check

In Exercises 1 to 4, write the domain and range for each relation. Then state whether the relation is a function.

1. $\{(2, 3), (5, 1), (-4, 3), (7, 11)\}$
2. $\{(5, 10), (3, -2), (4, 7), (5, 8)\}$
3. $\{(4, 4), (6, 1), (5, -3), (4, 5)\}$
4. $\{(1, 0), (2, 0), (3, 0)\}$

In Exercises 5 to 10, determine whether the given value of *x* is in the domain of the function.

5. $f(x) = \dfrac{3x}{x + 4}, x = 0$
6. $g(x) = 1 - x^2, x = -1$
7. $F(x) = \dfrac{x - 1}{x + 1}, x = -1$
8. $y(x) = \sqrt{2x - 8}, x = 2$
9. $g(t) = \dfrac{5t - 1}{t^2 + 1}, t = -1$
10. $F(t) = \dfrac{1}{t^3 + 8}, t = -2$

In Exercises 11 to 18, determine whether the equation defines *y* as a function of *x*.

11. $2x + 3y = 7$
12. $5x + y = 8$
13. $-x + y^2 = 2$
14. $x^2 - 2y = 2$
15. $x^2 + y^2 = 9$
16. $y = \sqrt[3]{x}$
17. $y = |x| + 5$
18. $y = \sqrt{x^2 + 4}$

In Exercises 19 to 24, determine whether the given value of the variable is a zero of the function.

19. $f(x) = 3x + 6, x = -2$
20. $f(x) = 2x^3 - 4x^2 + 5x, x = 0$
21. $G(t) = 3t^2 + 2t - 1, t = -\dfrac{1}{3}$
22. $s(t) = \dfrac{2t + 6}{t + 1}, t = -1$
23. $y(s) = 5s^2 - 2s - 2, s = 1$
24. $g(x) = \dfrac{3x + 9}{x^2 - 4}, x = -3$

In Exercises 25 to 32, evaluate each function.

25. Given $f(x) = 3x - 1$, find
 a. $f(2)$ **b.** $f(-1)$ **c.** $f(0)$
 d. $f\left(\dfrac{2}{3}\right)$ **e.** $f(k)$ **f.** $f(k + 2)$
26. Given $g(x) = 2x^2 + 3$, find
 a. $g(3)$ **b.** $g(-1)$ **c.** $g(0)$
 d. $g\left(\dfrac{1}{2}\right)$ **e.** $g(c)$ **f.** $g(c + 5)$
27. Given $A(w) = \sqrt{w^2 + 5}$, find
 a. $A(0)$ **b.** $A(2)$ **c.** $A(-2)$
 d. $A(4)$ **e.** $A(r + 1)$ **f.** $A(-c)$
28. Given $J(t) = 3t^2 - t$, find
 a. $J(-4)$ **b.** $J(0)$ **c.** $J\left(\dfrac{1}{3}\right)$

Indicates Try It Exercises

d. $J(-c)$ e. $J(x+1)$ f. $J(x+h)$

29. Given $f(x) = \dfrac{1}{|x|}$, find

a. $f(2)$ b. $f(-2)$ c. $f\left(\dfrac{-3}{5}\right)$

d. $f(2) + f(-2)$ e. $f(c^2+4)$ f. $f(2+h)$

30. Given $T(x) = 5$, find

a. $T(-3)$ b. $T(0)$ c. $T\left(\dfrac{2}{7}\right)$

d. $T(3) + T(1)$ e. $T(x+h)$ f. $T(3k+5)$

31. Given $s(x) = \dfrac{x}{|x|}$, find

a. $s(4)$ b. $s(5)$ c. $s(-2)$

d. $s(-3)$ e. $s(t),\ t > 0$ f. $s(t),\ t < 0$

32. Given $r(x) = \dfrac{x}{x+4}$, find

a. $r(0)$ b. $r(-1)$ c. $r(-3)$

d. $r\left(\dfrac{1}{2}\right)$ e. $r(0.1)$ f. $r(10{,}000)$

In Exercises 33 and 34, evaluate each piecewise-defined function for the indicated values.

33. $P(x) = \begin{cases} 3x+1, & \text{if } x < 2 \\ -x^2+11, & \text{if } x \ge 2 \end{cases}$

a. $P(-4)$ b. $P(\sqrt{5})$

c. $P(c),\quad c < 2$ d. $P(k+1),\quad k \ge 1$

34. $Q(t) = \begin{cases} 4, & \text{if } 0 \le t \le 5 \\ -t+9, & \text{if } 5 < t \le 8 \\ \sqrt{t-7}, & \text{if } 8 < t \le 11 \end{cases}$

a. $Q(0)$ b. $Q(e),\quad 6 < e < 7$

c. $Q(n),\quad 1 < n < 2$ d. $Q(m^2+7),\quad 1 < m \le 2$

In Exercises 35 to 46, determine the domain of the function represented by the given equation.

35. $f(x) = 3x - 4$ **36.** $f(x) = -2x + 1$

37. $f(x) = x^2 + 2$ **38.** $f(x) = 3x^2 + 1$

39. $f(x) = \dfrac{4}{x+2}$ **40.** $f(x) = \dfrac{6}{x-5}$

41. $f(x) = \sqrt{7+x}$ **42.** $f(x) = \sqrt{4-x}$

43. $f(x) = \sqrt{4-x^2}$ **44.** $f(x) = \sqrt{12-x^2}$

45. $f(x) = \dfrac{1}{\sqrt{x+4}}$ **46.** $f(x) = \dfrac{1}{\sqrt{5-x}}$

In Exercises 47 to 62, graph each function.

47. $f(x) = 3x - 4$ **48.** $f(x) = 2 - \dfrac{1}{2}x$

49. $g(x) = x^2 - 1$ **50.** $g(x) = 3 - x^2$

51. $f(x) = \sqrt{x+4}$ **52.** $h(x) = \sqrt{5-x}$

53. $f(x) = |x-2|$ **54.** $h(x) = 3 - |x|$

55. $L(x) = \left[\!\left[\dfrac{1}{3}x\right]\!\right]$ for $-6 \le x \le 6$

56. $M(x) = [\![x]\!] + 2$ for $0 \le x \le 4$

57. $N(x) = \text{int}(-x)$ for $-3 \le x \le 3$

58. $P(x) = \text{int}(x) + x$ for $0 \le x \le 4$

59. $f(x) = \begin{cases} 1-x, & x < 2 \\ 2x, & x \ge 2 \end{cases}$ **60.** $g(x) = \begin{cases} 2x, & x \le -1 \\ \dfrac{x}{2}, & x > -1 \end{cases}$

61. $r(x) = \begin{cases} -x^2+4, & x < -1 \\ -x+2, & -1 \le x \le 1 \\ 3x-2, & x > 1 \end{cases}$

62. $A(x) = \begin{cases} |x|, & x < 1 \\ x^2, & 1 \le x < 3 \\ -x+2, & x \ge 3 \end{cases}$

In Exercises 63 to 70, find the value or values of *a* in the domain of *f* for which *f*(*a*) equals the given number.

63. $f(x) = 3x - 2;\ f(a) = 10$

64. $f(x) = 2 - 5x;\ f(a) = 7$

65. $f(x) = x^2 + 2x - 2;\ f(a) = 1$

66. $f(x) = x^2 - 5x - 16;\ f(a) = -2$

67. $f(x) = |x|;\ f(a) = 4$

68. $f(x) = |x+2|;\ f(a) = 6$

69. $f(x) = x^2 + 2,\ f(a) = 1$

70. $f(x) = |x| - 2;\ f(a) = -3$

In Exercises 71 to 78, find the zeros of *f*.

71. $f(x) = 3x - 6$ **72.** $f(x) = 6 + 2x$

73. $f(x) = 5x + 2$ **74.** $f(x) = 8 - 6x$

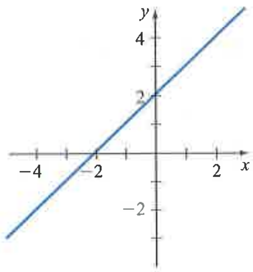
75. $f(x) = x^2 - 4$ **76.** $f(x) = x^2 + 4x - 21$

77. $f(x) = x^2 - 5x - 24$

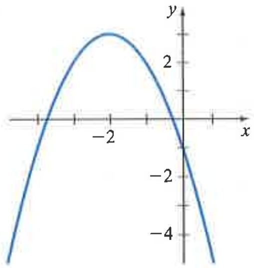
78. $f(x) = 2x^2 + 3x - 5$

79. **Vertical Line Test** Use the vertical line test to determine which of the following graphs are graphs of functions.

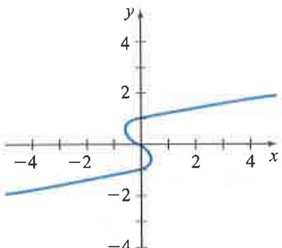
a.



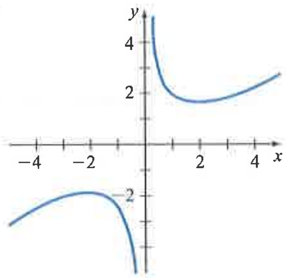
b.



c.

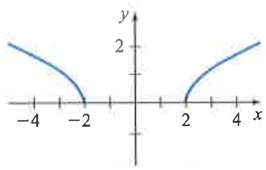


d.

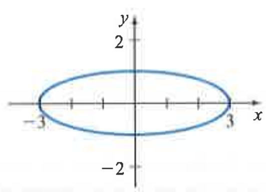


80. **Vertical Line Test** Use the vertical line test to determine which of the following graphs are graphs of functions.

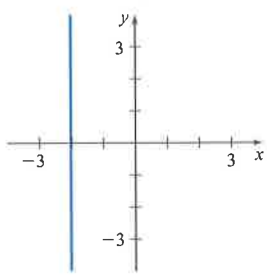
a.



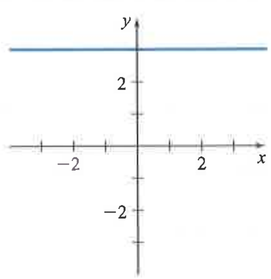
b.



c.

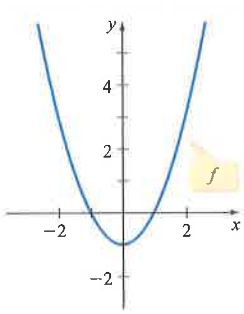


d.

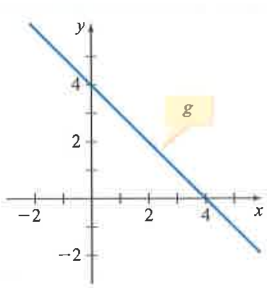


In Exercises 81–90, use the indicated graph to identify the intervals over which the function is increasing, constant, or decreasing.

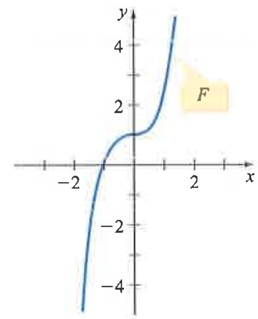
81.



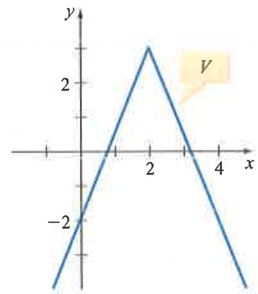
82.



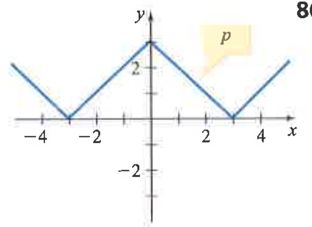
83.



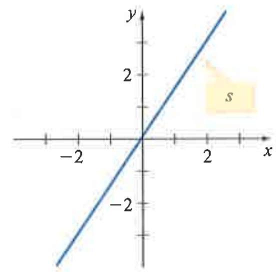
84.



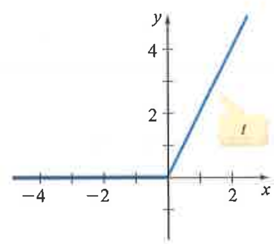
85.



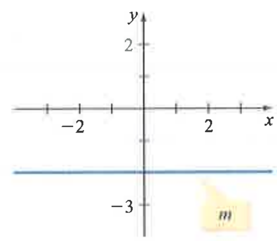
86.



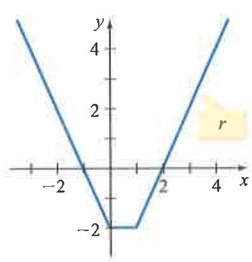
87.



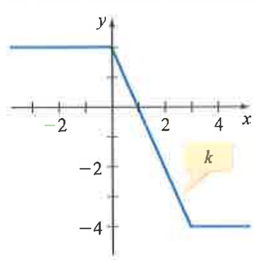
88.



89.



90.



91. **Horizontal Line Test** Use the horizontal line test to determine which of the following functions are one-to-one.

f as shown in Exercise 81
g as shown in Exercise 82
F as shown in Exercise 83
V as shown in Exercise 84
p as shown in Exercise 85

92. **Horizontal Line Test** Use the horizontal line test to determine which of the following functions are one-to-one.

s as shown in Exercise 86
t as shown in Exercise 87
m as shown in Exercise 88
r as shown in Exercise 89
k as shown in Exercise 90

93. Postage Rates In 2012, the cost to mail contents in a large envelope was given by

$$C(w) = 0.90 - 0.20\,\text{int}(1 - w), \quad w > 0$$

where C is in dollars and w is the weight of the envelope in ounces.

a. What was the cost (in 2012) to mail a large envelope that weighed 2.8 ounces?

b. Graph C for $0 < w \le 4$.

94. Income Tax The amount of federal income tax $T(x)$ a single person owed in 2012 is given by

$$T(x) = \begin{cases} 0.10x & 0 \le x < 8700 \\ 0.15(x - 8700) + 870 & 8700 \le x < 35{,}350 \\ 0.25(x - 35{,}350) + 4867.50 & 35{,}350 \le x < 85{,}650 \\ 0.28(x - 85{,}650) + 17{,}442.50 & 85{,}650 \le x < 178{,}650 \\ 0.33(x - 178{,}650) + 43{,}482.50 & 178{,}650 \le x < 388{,}350 \\ 0.35(x - 388{,}350) + 112{,}683.50 & x \ge 388{,}350 \end{cases}$$

where x is the adjusted gross income of the taxpayer.

a. What is the domain of this function?

b. Find the income tax of a person whose adjusted gross income was \$50,020, which was the approximate median income in the United States in 2012.

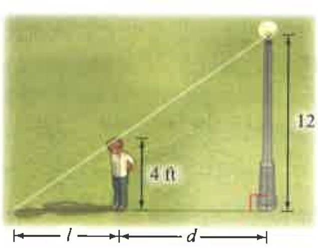
c. Find the income tax of a chemical engineer whose adjusted gross income was \$123,500.

95. Geometry A rectangle has a length of l feet and a perimeter of 50 feet.

a. Write the width w of the rectangle as a function of its length.

b. Write the area A of the rectangle as a function of its length.

96. Length of a Shadow A child 4 feet tall is standing near a street lamp that is 12 feet high. The light from the lamp casts a shadow as shown in the diagram.



a. Find the length l of the shadow as a function of the distance d of the child from the lamppost. (*Hint*: Use the fact that the ratios of corresponding sides of similar triangles are equal.)

b. What is the domain of the function?

c. What is the length of the shadow when the child is 8 feet from the base of the lamppost?

97. Depreciation A bus was purchased for \$80,000. Assuming that the bus depreciates at a rate of \$6500 per year (straight-line depreciation) for the first 10 years, write the value v of the bus as a function of the time t (measured in years) for $0 \le t \le 10$.

98. Depreciation A boat was purchased for \$44,000. Assuming that the boat depreciates at a rate of \$4200 per year (straight-line depreciation) for the first 8 years, write the value v of the boat as a function of the time t (measured in years) for $0 \le t \le 8$.

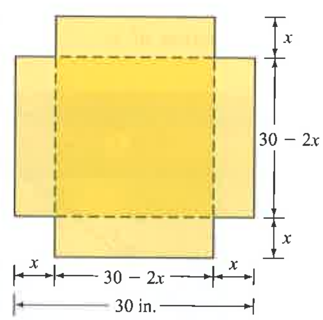
99. Cost, Revenue, and Profit A manufacturer produces a product at a cost of \$22.80 per unit. The manufacturer has a fixed cost of \$400.00 per day. Each unit retails for \$37.00. Let x represent the number of units produced in a 5-day period.

a. Write the total cost C as a function of x.

b. Write the revenue R as a function of x.

c. Write the profit P as a function of x. (*Hint*: The profit function is given by $P(x) = R(x) - C(x)$.)

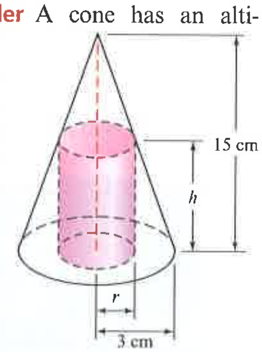
100. Volume of a Box An open box is to be made from a square piece of cardboard with the dimensions 30 inches by 30 inches by cutting out squares of area x^2 from each corner, as shown in the figure.



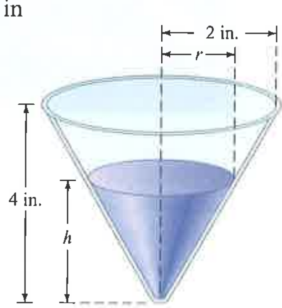
a. Express the volume V of the box as a function of x.

b. State the domain of V.

101. Height of an Inscribed Cylinder A cone has an altitude of 15 centimeters and a radius of 3 centimeters. A right circular cylinder of radius r and height h is inscribed in the cone as shown in the figure. Use similar triangles to write h as a function of r.



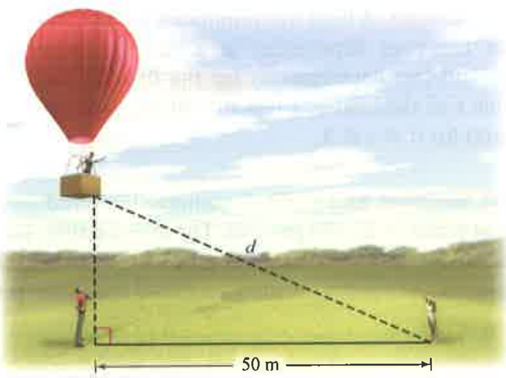
102. Volume of Water Water is flowing into a conical drinking cup with an altitude of 4 inches and a radius of 2 inches, as shown in the figure.



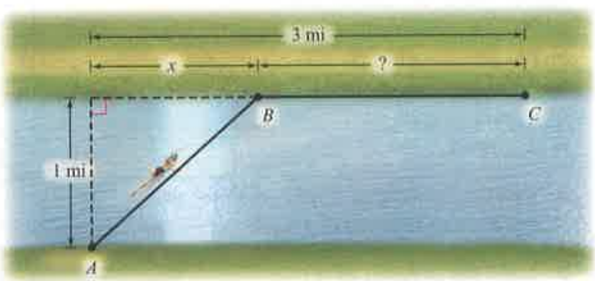
a. Write the radius r of the surface of the water as a function of its depth h.

b. Write the volume V of the water as a function of its depth h.

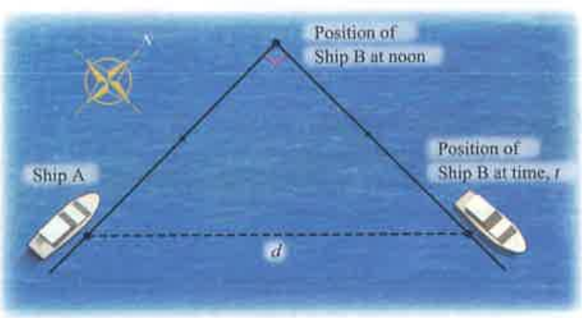
103. Distance from a Balloon For the first minute of flight, a hot air balloon rises vertically at a rate of 3 meters per second. If t is the time in seconds that the balloon has been airborne, write the distance d between the balloon and a point on the ground 50 meters from the point of liftoff as a function of t.



104. Time for an Athlete An athlete swims from point A to point B at a rate of 2 miles per hour and runs from point B to point C at a rate of 8 miles per hour. Use the dimensions in the figure to write the time t required to reach point C as a function of x.

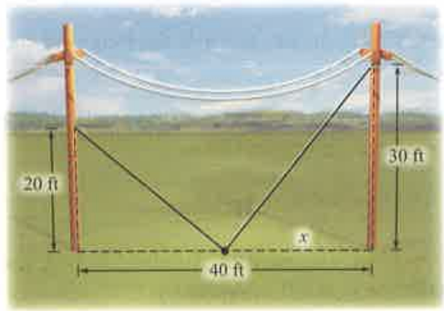


105. Distance Between Ships At noon, Ship A is 45 miles due south of Ship B and is sailing north at a rate of 8 miles per hour. Ship B is sailing east at a rate of 6 miles per hour. Write the distance d between the ships as a function of the time t, where $t = 0$ represents noon.



106. Distance Between Ships Use the diagram for Exercise 105. At noon, Ship B is 60 miles due north of Ship A and is sailing east at a rate of 10 miles per hour. Ship A is sailing north at a rate of 7 miles per hour. Write the distance d between the ships as a function of the time t, where $t = 0$ represents noon.

107. Length Two guy wires are attached to utility poles that are 40 feet apart, as shown in the following diagram.



a. Find the total length of the two guy wires as a function of x.

b. Complete the following table. Round the length to the nearest hundredth of a foot.

x	Total Length of Wires (ft)
0	
10	
20	
30	
40	

c. What is the domain of this function?

108. Sales vs. Price A business finds that the number f of feet of pipe it can sell per week is a function of the price p in cents per foot as given by

$$f(p) = \frac{320{,}000}{p + 25}, \ 40 \le p \le 90$$

Complete the following table by evaluating f (to the nearest hundred feet) for the indicated values of p.

p	40	50	60	75	90
$f(p)$					

109. Model Yield The yield Y of apples per tree is related to the amount x of a particular type of fertilizer applied (in pounds per year) as given by the function

$$Y(x) = 400[1 - 5(x - 1)^{-2}], \ 5 \le x \le 20$$

Complete the following table by evaluating Y (to the nearest apple) for the indicated amounts of fertilizer.

x	5	10	12.5	15	20
$Y(x)$					

110. Model Cost A manufacturer finds that the cost C in dollars of producing x items of a product is given by

$$C(x) = (225 + 1.4\sqrt{x})^2, \ 100 \le x \le 1000$$

Complete the following table by evaluating C (to the nearest dollar) for the indicated numbers of items.

x	100	200	500	750	1000
$C(x)$					

111. If $f(x) = x^2 - x - 5$ and $f(c) = 1$, find c.

112. If $g(x) = -2x^2 + 4x - 1$ and $g(c) = -4$, find c.

113. Determine whether 1 is in the range of $f(x) = \dfrac{x-1}{x+1}$.

114. Determine whether 0 is in the range of $g(x) = \dfrac{1}{x-3}$.

In Exercises 115 and 116, use a graphing calculator to graph each set of equations.

115. Graph $f(x) = x^2$, $g(x) = x^2 - 3$, and $h(x) = x^2 + 2$ on the same graphing calculator screen. How are the graphs of g and h related to the graph of f?

116. Graph $f(x) = x^2$, $g(x) = (x-3)^2$, and $h(x) = (x+2)^2$ on the same graphing calculator screen. How are the graphs of g and h related to the graph of f?

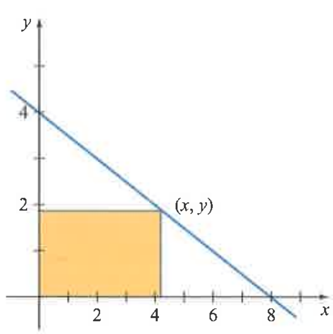
Enrichment Exercises

A *fixed point* of a function is a number a such that $f(a) = a$. In Exercises 117 and 118, find all fixed points for the given function.

117. $f(x) = x^2 + 3x - 3$

118. $g(x) = \dfrac{x}{x+5}$

119. **Area** A rectangle is bounded by the x- and y-axes and the graph of $y = -\frac{1}{2}x + 4$.



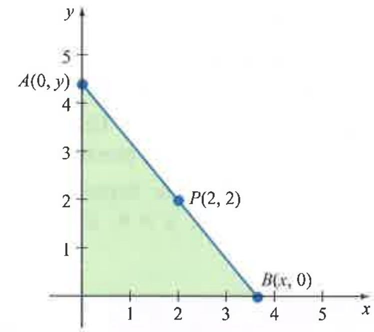
a. Find the area of the rectangle as a function of x.

b. Complete the following table.

x	Area
1	
2	
4	
6	
7	

c. What is the domain of this function?

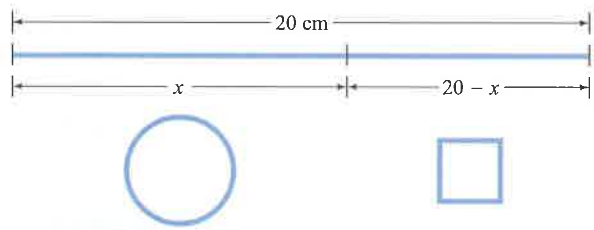
120. **Area** A triangle is bounded by the x- and y-axes and must pass through $P(2, 2)$, as shown below.



a. Find the area of the triangle as a function of x. (*Hint:* Let C be the point (0, 2) and D be the point (2, 0). Use the fact that ACP and PDB are similar triangles.)

b. What is the domain of the function you found in **a.**?

121. **Area** A piece of wire 20 centimeters long is cut at a point x centimeters from the left end, as shown in the following diagram. The left-hand piece is formed into a circle and the right-hand piece is formed into a square.



a. Find the area enclosed by the two figures as a function of x.

b. Complete the following table. Round the area to the nearest hundredth.

x	Total Area Enclosed (cm^2)
0	
4	
8	
12	
16	
20	

c. What is the domain of this function?

122. **Day of the Week** A formula known as Zeller's Congruence makes use of the greatest integer function $[\![x]\!]$ to determine the day of the week on which a given day fell or will fall. To use Zeller's Congruence, we first compute the integer z given by

$$z = \left[\!\!\left[\frac{13m-1}{5}\right]\!\!\right] + \left[\!\!\left[\frac{y}{4}\right]\!\!\right] + \left[\!\!\left[\frac{c}{4}\right]\!\!\right] + d + y - 2c$$

The variables c, y, d, and m are defined as follows:

c = int(the given year/100)

y = year of the century

d = day of the month

m = month, using 1 for March, 2 for April, . . . , 10 for December. January and February are assigned the values 11 and 12 of the previous year.

For example, for the date September 30, 2009, we use $c = \text{int}(2009/100) = 20$, $y = 9$, $d = 30$, and $m = 7$. The remainder of z divided by 7 gives the day of the week. A remainder of 0 represents a Sunday, a remainder of 1 a Monday, . . . , a remainder of 6 a Saturday.

a. Verify that December 7, 1941, was a Sunday.

b. Verify that January 1, 2020, will fall on a Wednesday.

c. Determine on what day of the week Independence Day (July 4, 1776) fell.

d. Determine on what day of the week you were born.

SECTION 2.3

Slopes of Lines
Slope–Intercept Form
Finding the Equation of a Line
Parallel and Perpendicular Lines
Applications of Linear Functions

Linear Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A9.

PS1. Find the distance on a real number line between the points whose coordinates are -2 and 5. [P.1]

PS2. Find the product of a nonzero number and its negative reciprocal. [P.5]

PS3. Given the points $P_1(-3, 4)$ and $P_2(2, -4)$, evaluate $\dfrac{y_2 - y_1}{x_2 - x_1}$. [P.1]

PS4. Solve $y - 3 = -2(x - 3)$ for y. [1.1]

PS5. Solve $3x - 5y = 15$ for y. [1.2]

PS6. Given $y = 3x - 2(5 - x)$, find the value of x for which $y = 0$. [1.1]

Slopes of Lines

A function that can be written in the form $f(x) = mx + b$ is called a *linear function* because its graph is a straight line. Consider the graph of a straight line in Figure 2.37. Observe that for each 1-unit increase in x, y increases by a constant amount of m units. In Figure 2.38, note that for each 1-unit increase in x, y decreases by a constant amount of m units.

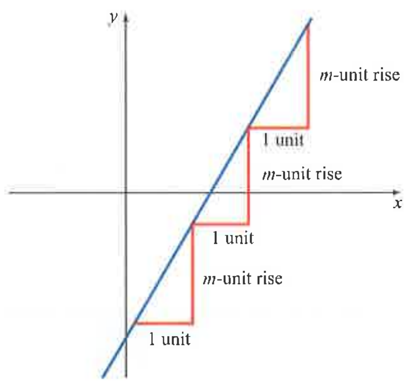


Figure 2.37

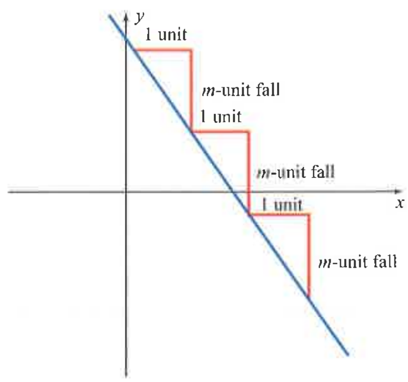


Figure 2.38

Graphs of linear functions are characterized by having a constant rise or fall. This rise or fall is called *slope*. The graph in Figure 2.37 has a *positive* slope; the y value is increasing as x increases. The graph in Figure 2.38 has a *negative* slope; the y value is decreasing as x increases.

The slope of a line can be calculated by finding the ratio of the change in y between two points to the change in x between the same two points. For instance,