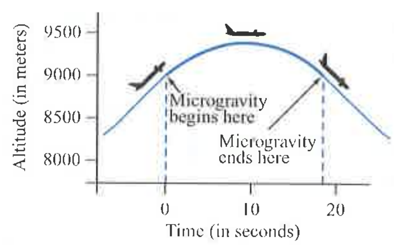
The image at the right shows the crew of STS-135, the last U.S. manned space shuttle mission, along with the other crew members of Expedition 28, floating in the International Space Station. The American flag shown in the picture was flown on STS-1, the first shuttle mission.

CHAPTER 2 Functions and Graphs

Functions as Models

To prepare astronauts for the experience of zero gravity (technically, microgravity) in space, the National Aeronautics and Space Administration (NASA) uses a specially designed jet. A pilot accelerates the jet upward to an altitude of approximately 9000 meters and then reduces power. At that time, the plane continues upward, noses over, and begins to descend until the pilot increases power. The maneuver is then repeated. The figure below shows one maneuver.



During the climb and the point at which the pilot increases power, the force on the astronauts is approximately twice what they experience on Earth. During the time of reduced power (about 15 to 20 seconds), the plane is in free fall and the astronauts experience microgravity. The sudden changes in gravity effects have a tendency to make astronauts sick. Because of this, the plane has been dubbed the Vomit Comet.

A parabola, one of the topics of this chapter, can approximate the height of the jet. The time during which the astronauts experience microgravity in one maneuver can be determined using an equation of the parabola. See Exercise 47, page 207.

SECTION 2.1

Cartesian Coordinate Systems
Distance and Midpoint Formulas
Graph of an Equation
Intercepts
Circles, Their Equations, and Their Graphs

Two-Dimensional Coordinate System and Graphs

Cartesian Coordinate Systems

Each point on a coordinate axis is associated with a number called its **coordinate**. Each point on a flat, two-dimensional surface, called a **coordinate plane** or *xy*-plane, is associated with an **ordered pair** of numbers called **coordinates** of the point. Ordered pairs are denoted by (a, b), where the real number a is the ***x*-coordinate** or **abscissa** and the real number b is the ***y*-coordinate** or **ordinate**.

The coordinates of a point are determined by the point's position relative to a horizontal coordinate axis called the ***x*-axis** and a vertical coordinate axis called the ***y*-axis**. The axes intersect at the point $(0, 0)$, called the **origin**. In Figure 2.1, the axes are labeled such that positive numbers appear to the right of the origin on the *x*-axis and above the origin on the *y*-axis. The four regions formed by the axes are called **quadrants** and are numbered counterclockwise. This two-dimensional coordinate system is referred to as a **Cartesian coordinate system** in honor of René Descartes.

Note

Abscissa comes from the same root word as *scissors*. An open pair of scissors looks like an *x*.

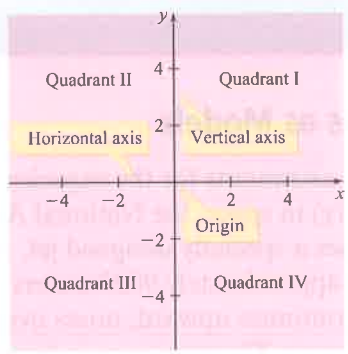


Figure 2.1

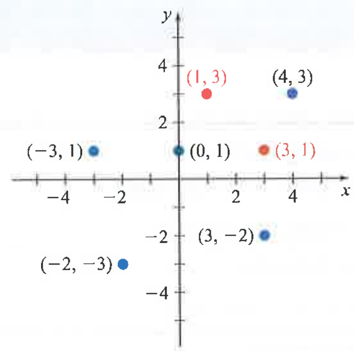


Figure 2.2

To **plot a point** $P(a, b)$ means to draw a dot at its location in the coordinate plane. In Figure 2.2, we have plotted the points $(4, 3)$, $(-3, 1)$, $(-2, -3)$, $(3, -2)$, $(0, 1)$, $(1, 3)$, and $(3, 1)$. The order in which the coordinates of an ordered pair are listed is important. Figure 2.2 shows that $(1, 3)$ and $(3, 1)$ do not denote the same point.

Data often are displayed in visual form as a set of points called a *scatter plot*. For instance, the scatter plot in Figure 2.3 shows the growth in text messaging during the years 2005 to 2011, with each point representing the data from a 12-month period ending in June. For example, the point with coordinates (2009, 1360) means that during the 12 months ending in June 2009, 1360 billion text messages were sent. The line segments that connect the points in Figure 2.3 help illustrate trends.

Math Matters

The concepts of *analytic geometry* developed over an extended period, culminating in 1637 with the publication of two works: *Discourse on the Method for Rightly Directing One's Reason and Searching for Truth in the Sciences* by René Descartes (1596–1650) and *Introduction to Plane and Solid Loci* by Pierre de Fermat. Each of these works was an attempt to integrate the study of geometry with the study of algebra. Of the two mathematicians, Descartes is usually given most of the credit for developing analytic geometry. In fact, Descartes became so famous in La Haye, the city in which he was born, that it was renamed La Haye-Descartes.

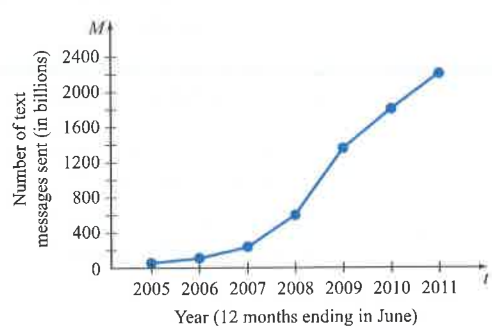


Figure 2.3

Source: CTIA.

Note

The notation (a, b) was used earlier to denote an interval on a one-dimensional number line. In this section, (a, b) denotes an ordered pair in a two-dimensional plane. This should not cause confusion in future sections because as each mathematical topic is introduced, it will be clear whether a one-dimensional or a two-dimensional coordinate system is involved.

Question • According to the data in Figure 2.3, was the number of text messages sent during the 12 months ending June 2010 more or less than twice the number of text messages sent during the 12 months ending June 2008?

In some instances, it is important to know when two ordered pairs are equal.

Definition of the Equality of Ordered Pairs

The ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.

EXAMPLE

If $(3, y) = (x, -2)$, then $x = 3$ and $y = -2$.

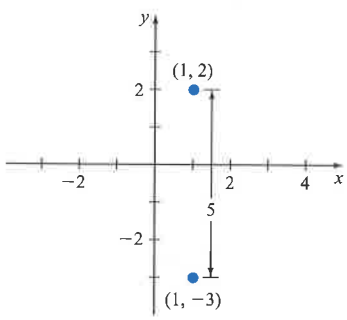


Figure 2.4

TO REVIEW

Pythagorean Theorem
See pages 102–103.

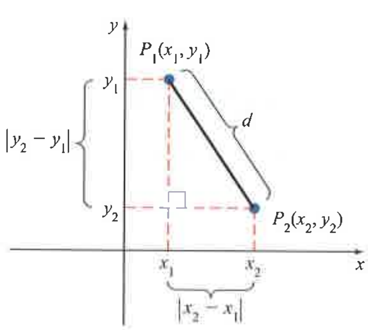


Figure 2.5

Distance and Midpoint Formulas

The Cartesian coordinate system makes it possible to combine the concepts of algebra and geometry into a branch of mathematics called *analytic geometry*.

The distance between two points on a horizontal line is the absolute value of the difference between the x-coordinates of the two points. The distance between two points on a vertical line is the absolute value of the difference between the y-coordinates of the two points. For example, as shown in Figure 2.4, the distance d between the points with coordinates $(1, 2)$ and $(1, -3)$ is $d = |2 - (-3)| = 5$.

If two points are not on a horizontal or vertical line, then a *distance formula* for the distance between the two points can be developed as follows.

The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in Figure 2.5 is the length of the hypotenuse of a right triangle whose sides are horizontal and vertical line segments that measure $|x_2 - x_1|$ and $|y_2 - y_1|$, respectively. Applying the Pythagorean Theorem to this triangle produces

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

• Use the square root procedure. Because d is nonnegative, the negative root is not listed.

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

• $|x_2 - x_1|^2 = (x_2 - x_1)^2$ and $|y_2 - y_1|^2 = (y_2 - y_1)^2$

Thus we have established the following theorem.

Distance Formula

The distance $d(P_1, P_2)$ between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

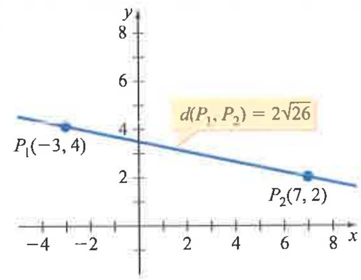
(continued)

Answer • More. The number of text messages sent during the 12 months ending June 2008 was approximately 600 billion. The number of text messages sent during the 12 months ending June 2010 was approximately 1800 billion.

EXAMPLE

The distance between $P_1(-3, 4)$ and $P_2(7, 2)$ is given by

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[7 - (-3)]^2 + (2 - 4)^2}$$
$$= \sqrt{10^2 + (-2)^2}$$
$$= \sqrt{104} = 2\sqrt{26} \approx 10.2$$



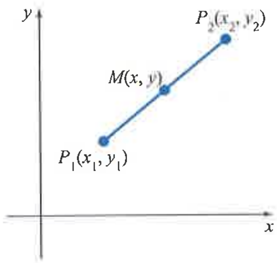


Figure 2.6

The **midpoint** M of a line segment is the point on the line segment that is equidistant from the endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ of the segment. See Figure 2.6.

Midpoint Formula

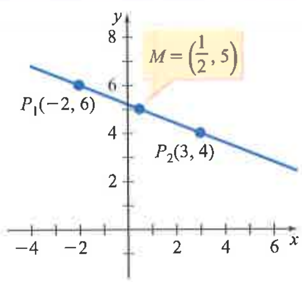
The midpoint M of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

EXAMPLE

The midpoint of the line segment between $P_1(-2, 6)$ and $P_2(3, 4)$ is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{(-2) + 3}{2}, \frac{6 + 4}{2}\right) = \left(\frac{1}{2}, 5\right)$$



The midpoint formula states that the x-coordinate of the midpoint of a line segment is the *average* of the x-coordinates of the endpoints of the line segment and that the y-coordinate of the midpoint of a line segment is the *average* of the y-coordinates of the endpoints of the line segment.

EXAMPLE 1 Find the Midpoint and Length of a Line Segment

Find the midpoint and the length of the line segment connecting the points whose coordinates are $P_1(-4, 3)$ and $P_2(4, -2)$.

Solution

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-4 + 4}{2}, \frac{3 + (-2)}{2}\right) = \left(0, \frac{1}{2}\right)$$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - (-4))^2 + (-2 - 3)^2} = \sqrt{(8)^2 + (-5)^2}$$
$$= \sqrt{64 + 25} = \sqrt{89}$$

▸ Try Exercise 10, page 162

Graph of an Equation

The equations below are equations in two variables.

$$y = 3x^3 - 4x + 2 \qquad x^2 + y^2 = 25 \qquad y = \frac{x}{x+1}$$

The solution of an equation in two variables is an ordered pair (x, y) whose coordinates satisfy the equation. For instance, the ordered pairs (3, 4), (4, −3), and (0, 5) are some of the solutions of $x^2 + y^2 = 25$. Generally, there are an infinite number of solutions of an equation in two variables. These solutions can be displayed in a *graph*.

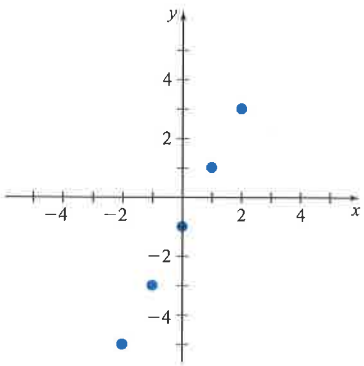


Figure 2.7

Definition of the Graph of an Equation

The **graph of an equation** in the two variables x and y is the set of all points (x, y) whose coordinates satisfy the equation.

Consider $y = 2x - 1$. Substituting various values of x into the equation and solving for y produces some of the ordered pairs that satisfy the equation. It is convenient to record the results in a table similar to the one shown below. The graph of the ordered pairs is shown in Figure 2.7.

x	$y = 2x - 1$	y	(x, y)
−2	2(−2) − 1	−5	(−2, −5)
−1	2(−1) − 1	−3	(−1, −3)
0	2(0) − 1	−1	(0, −1)
1	2(1) − 1	1	(1, 1)
2	2(2) − 1	3	(2, 3)

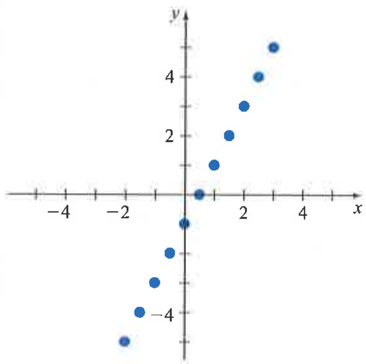


Figure 2.8

Choosing some noninteger values of x produces more ordered pairs to graph, such as $\left(-\frac{3}{2}, -4\right)$ and $\left(\frac{5}{2}, 4\right)$, as shown in Figure 2.8. Using still other values of x would add even more ordered pairs to graph. The result would be so many dots that the graph would appear as the straight line shown in Figure 2.9, which is the graph of $y = 2x - 1$.

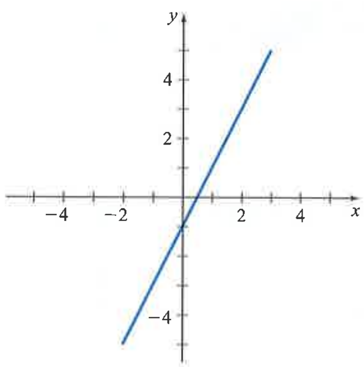


Figure 2.9

EXAMPLE 2 Draw a Graph by Plotting Points

Graph: $-x^2 + y = 1$

Solution

Solve the equation for y.

$$y = x^2 + 1$$

Select values of x and use the equation to calculate y. Choose enough values of x so that an accurate graph can be drawn. Plot the points and draw a curve through them. See Figure 2.10.

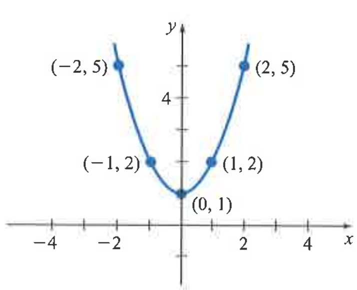


Figure 2.10

(continued)

x	$y = x^2 + 1$	y	(x, y)
-2	$(-2)^2 + 1$	5	$(-2, 5)$
-1	$(-1)^2 + 1$	2	$(-1, 2)$
0	$(0)^2 + 1$	1	$(0, 1)$
1	$(1)^2 + 1$	2	$(1, 2)$
2	$(2)^2 + 1$	5	$(2, 5)$

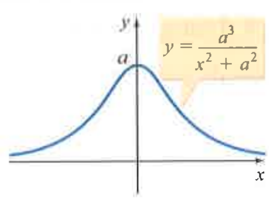
▶ **Try Exercise 32, page 162**

Math Matters

Maria Agnesi (1718–1799) wrote *Foundations of Analysis for the Use of Italian Youth*, one of the most successful textbooks of the eighteenth century. The French Academy authorized a translation into French in 1749, noting that "there is no other book, in any language, which would enable a reader to penetrate as deeply, or as rapidly, into the fundamental concepts of analysis." A curve that Agnesi discusses in her text is given by the equation

$$y = \frac{a^3}{x^2 + a^2}$$

Unfortunately, due to a translation error from Italian to English, the curve became known as the "witch of Agnesi."



Integrating Technology

Some graphing calculators, such as the TI-83/TI-83 Plus/TI-84 Plus, have a TABLE feature that allows you to create a table similar to the one shown in Example 2. Enter the equation to be graphed, the first value for x, and the increment (the difference between successive values of x). For instance, entering $y_1 = x^2 + 1$, an initial value of x of -2, and an increment of 1 yields a display similar to the one in Figure 2.11. Changing the initial value to -6 and the increment to 2 gives the table in Figure 2.12.

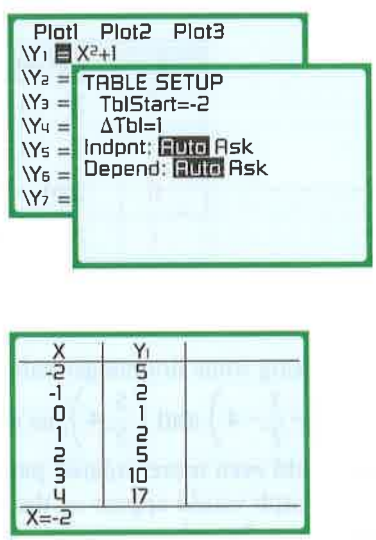


Figure 2.11

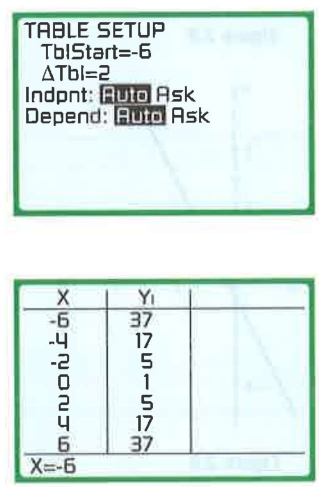


Figure 2.12

With some calculators, you can scroll through the table by using the up- or down-arrow keys. In this way, you can determine many more ordered pairs of the graph.

EXAMPLE 3 Graph by Plotting Points

Graph: $y = |x - 2|$

Solution

This equation is already solved for y, so start by choosing an x value and using the equation to determine the corresponding y value. For example, if

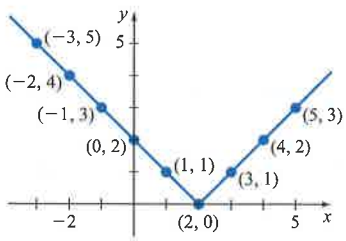


Figure 2.13

$x = -3$, then $y = |(-3) - 2| = |-5| = 5$. Continuing in this manner produces the following table.

When x is	−3	−2	−1	0	1	2	3	4	5
y is	5	4	3	2	1	0	1	2	3

Now plot the points listed in the table. Connecting the points forms a V shape, as shown in Figure 2.13.

▶ Try Exercise 36, page 163

EXAMPLE 4 Graph by Plotting Points

Graph: $y^2 = x$

Solution

Solve the equation for y.

$$y^2 = x$$
$$y = \pm\sqrt{x}$$

Choose several x values, and use the equation to determine the corresponding y values.

When x is	0	1	4	9	16
y is	0	±1	±2	±3	±4

Plot the points as shown in Figure 2.14. The graph is a *parabola*.

▶ Try Exercise 38, page 163

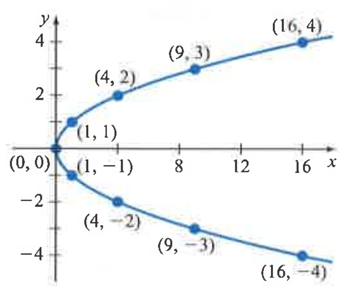


Figure 2.14

Integrating Technology

A graphing calculator or computer graphing software can be used to draw the graphs in Examples 3 and 4. These graphing utilities graph a curve in much the same way as you would, by selecting values of x and calculating the corresponding values of y. A curve is then drawn through the points.

If you use a graphing utility to graph $y = |x - 2|$, you will need to use the *absolute value* function that is built into the utility. The equation you enter will look similar to $Y_1=|X-2|$.

To graph the equation in Example 4, you will enter two equations. The equations you enter will be similar to

$$Y_1 = \sqrt{X}$$
$$Y_2 = -\sqrt{X}$$

The graph of the first equation will be the top half of the parabola; the graph of the second equation will be the bottom half.

▶ Intercepts

On a graph, any point that has an x- or a y-coordinate of zero is called an **intercept** of the graph, because it is at this point that the graph intersects the x- or the y-axis.

Definitions of x-Intercepts and y-Intercepts

If $(x_1, 0)$ satisfies an equation in two variables, then the point whose coordinates are $(x_1, 0)$ is called an **x-intercept** of the graph of the equation.

If $(0, y_1)$ satisfies an equation in two variables, then the point whose coordinates are $(0, y_1)$ is called a **y-intercept** of the graph of the equation.

To find the x-intercepts of the graph of an equation, let $y = 0$ and solve the equation for x. To find the y-intercepts of the graph of an equation, let $x = 0$ and solve the equation for y.

EXAMPLE 5 Find x- and y-Intercepts

Find the x- and y-intercepts of the graph of $y = x^2 - 2x - 3$.

Algebraic Solution

To find the y-intercept, let $x = 0$ and solve for y.

$$y = 0^2 - 2(0) - 3 = -3$$

To find the x-intercepts, let $y = 0$ and solve for x.

$$0 = x^2 - 2x - 3$$
$$0 = (x - 3)(x + 1)$$
$$(x - 3) = 0 \quad \text{or} \quad (x + 1) = 0$$
$$x = 3 \quad \text{or} \quad x = -1$$

Because $y = -3$ when $x = 0$, $(0, -3)$ is a y-intercept. Because $x = 3$ or -1 when $y = 0$, $(3, 0)$ and $(-1, 0)$ are x-intercepts. Figure 2.15 confirms that these three points are intercepts.

Visualize the Solution

The graph of $y = x^2 - 2x - 3$ is shown below. Observe that the graph intersects the x-axis at $(-1, 0)$ and $(3, 0)$, the x-intercepts. The graph also intersects the y-axis at $(0, -3)$, the y-intercept.

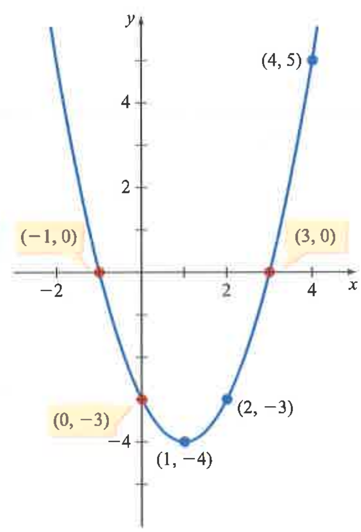


Figure 2.15

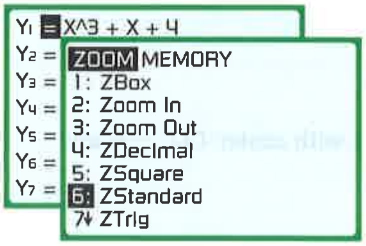
▶ Try Exercise 46, page 163

Integrating Technology

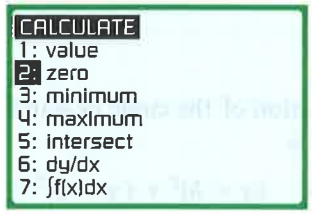
In Example 5, it was possible to find the x-intercepts by solving a quadratic equation. In some instances, however, solving an equation to find the intercepts may be very difficult. In these cases, a graphing calculator can be used to estimate the x-intercepts.

The x-intercepts of the graph of $y = x^3 + x + 4$ can be estimated using the ZERO feature of a TI-83/TI-83 Plus/TI-84 Plus calculator. The keystrokes and some sample screens for this procedure are shown on the next page.

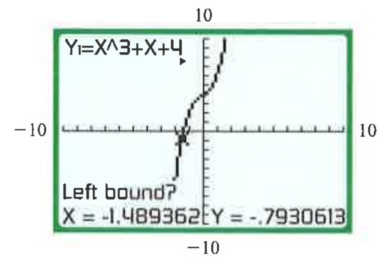
Press [Y=]. Now enter X^3+X+4. Press [ZOOM] and select the standard viewing window. Press [ENTER].



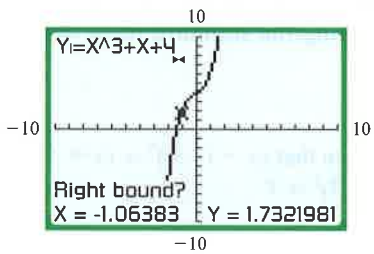
Press [2nd] CALC to access the CALCULATE menu. The y-coordinate of an x-intercept is zero. Therefore, select 2: zero. Press [ENTER].



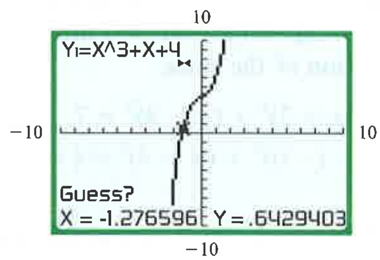
The Left bound? shown at the bottom of the screen means to move the cursor until it is to the left of the desired x-intercept. Press [ENTER].



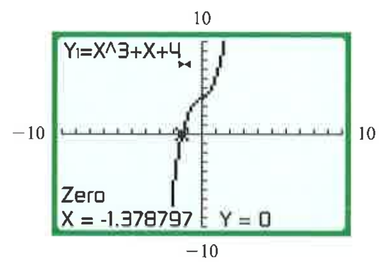
The Right bound? shown at the bottom of the screen means to move the cursor until it is to the right of the desired x-intercept. Press [ENTER].



Guess? is shown at the bottom of the screen. Move the cursor until it is approximately on the x-intercept. Press [ENTER].



The Zero shown at the bottom of the screen means that the value of y is 0 when $x = -1.378797$. The x-intercept is about $(-1.378797, 0)$.



Circles, Their Equations, and Their Graphs

Frequently you will sketch graphs by plotting points. However, some graphs can be sketched merely by recognizing the form of the equation. A *circle* is an example of a curve whose graph you can sketch after you have inspected its equation.

Definition of a Circle

A **circle** is the set of points in a plane that are a fixed distance from a specified point. The fixed distance is the **radius** of the circle, and the specified point is the **center** of the circle.

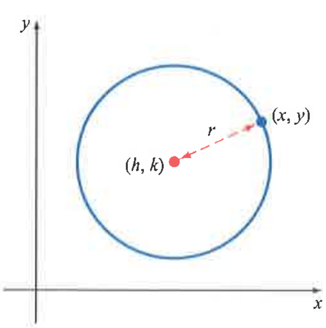


Figure 2.16

The standard form of the equation of a circle is derived by using the definition of a circle. To derive the standard form, we use the distance formula. Figure 2.16 is a circle with center (h, k) and radius r. The point (x, y) is on the circle if and only if it is a distance of r units from the center (h, k). Thus (x, y) is on the circle if and only if

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2 \qquad \text{• Square each side.}$$

Standard Form of the Equation of a Circle

Let $C(h, k)$ be the coordinates of the center of a circle of radius r. Then **the standard form of the equation of a circle** is given by

$$(x - h)^2 + (y - k)^2 = r^2$$

EXAMPLES

- The equation of the circle in standard form with center $C(2, -5)$ and radius 4 is

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y - (-5))^2 = 4^2 \qquad \bullet\ h = 2, k = -5, r = 4$$

$$(x - 2)^2 + (y + 5)^2 = 16 \qquad \bullet\ \text{Simplify.}$$

Although $(x - 2)^2 + (y - (-5))^2 = 4^2$ is actually standard form, we usually write the equation in simplest form as $(x - 2)^2 + (y + 5)^2 = 16$.

- The coordinates of the center and the radius of the circle whose equation is $(x + 3)^2 + (y - 4)^2 = 7$ are found by writing the standard form of the equation of the circle.

$$(x + 3)^2 + (y - 4)^2 = 7$$

$$(x - (-3))^2 + (y - 4)^2 = (\sqrt{7})^2 \qquad \bullet\ \text{Note that } (x - (-3))^2 = (x + 3)^2;\ (\sqrt{7})^2 = 7$$

The center is $C(-3, 4)$ and the radius is $r = \sqrt{7}$.

If a circle is centered at the origin (0, 0), then $h = 0$ and $k = 0$ and the standard form of the equation of the circle simplifies to

$$x^2 + y^2 = r^2 \qquad \bullet\ \text{Equation of a circle with center at the origin and radius } r$$

For instance, $x^2 + y^2 = 9$ is the equation of the circle with center at the origin and radius of $\sqrt{9} = 3$.

Question • What are the radius and the coordinates of the center of the circle with equation $x^2 + (y - 2)^2 = 30$?

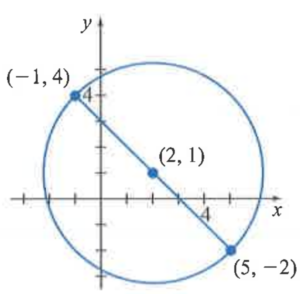


Figure 2.17

EXAMPLE 6 Find the Standard Form of the Equation of a Circle

Find the standard form of the equation of a circle that has a diameter with endpoints whose coordinates are (−1, 4) and (5, −2), as shown in Figure 2.17.

Solution

Find the midpoint of the diameter. The midpoint gives the coordinates for the center of the circle.

$$\text{Coordinates of center: } \left(\frac{-1 + 5}{2}, \frac{4 + (-2)}{2}\right) = (2, 1)$$

Answer • The radius is $\sqrt{30}$, and the coordinates of the center are (0, 2).

Find the radius of the circle by finding the length of the line segment from the center to one of the endpoints of the diameter.

$$r = \sqrt{(2-5)^2 + [1-(-2)]^2} = \sqrt{(-3)^2 + 3^2}$$
$$= \sqrt{9+9} = \sqrt{18}$$

Write the equation of the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y-1)^2 = 18 \qquad \bullet\ r^2 = (\sqrt{18})^2 = 18$$

▶ **Try Exercise 72, page 163**

If we rewrite $(x+4)^2 + (y+2)^2 = 25$ by squaring and combining like terms, we produce

$$x^2 + 8x + 16 + y^2 + 4y + 4 = 25$$
$$x^2 + y^2 + 8x + 4y - 5 = 0$$

This form of the equation is known as the **general form of the equation of a circle.** By completing the square, it is always possible to rewrite an equation in the general form $x^2 + y^2 + Ax + By + C = 0$ in the standard form

$$(x-h)^2 + (y-k)^2 = s$$

for some number s. If $s > 0$, the graph is a circle with radius $r = \sqrt{s}$. If $s = 0$, the graph is the point (h, k). If $s < 0$, the equation has no real solutions and there is no graph.

EXAMPLE 7 Find the Center and Radius of a Circle by Completing the Square

Find the center and radius of the circle given by

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

Solution

First, rearrange and group the terms as shown.

$$(x^2 - 6x) + (y^2 + 4y) = 3$$

Now complete the squares of both $(x^2 - 6x)$ and $(y^2 + 4y)$.

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4 \qquad \bullet\ \text{Add 9 and 4 to each side of the equation.}$$
$$(x-3)^2 + (y+2)^2 = 16$$
$$(x-3)^2 + (y-(-2))^2 = 4^2$$

This equation is the standard form of the equation of a circle and indicates that the circle is centered at $(3, -2)$ with radius 4. See Figure 2.18.

▶ **Try Exercise 78, page 163**

TO REVIEW

Completing the Square
See page 98.

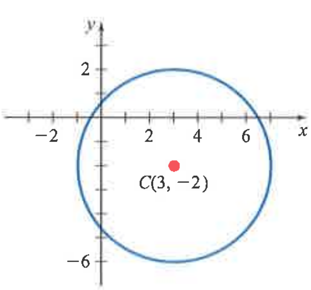


Figure 2.18

EXERCISE SET 2.1

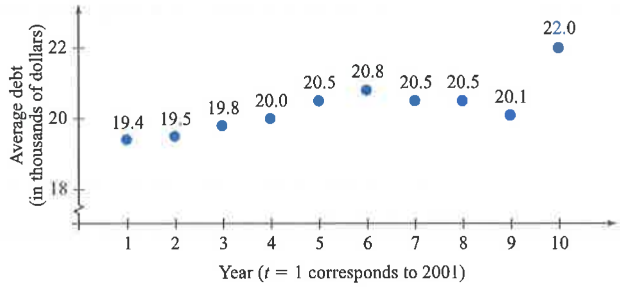
Concept Check

In Exercises 1 and 2, plot the points whose coordinates are given on a Cartesian coordinate system.

1. $(2, 4), (0, -3), (-2, 1), (-5, -3)$

2. $(-3, -5), (-4, 3), (0, 2), (-2, 0)$

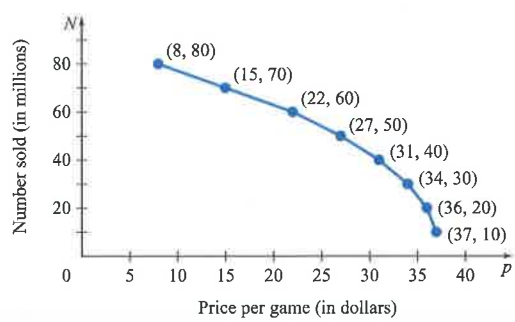
3. **College Debt** The following graph shows the average debt, in constant 2010 dollars per borrower, of college students upon graduation.



a. According to the graph, between which consecutive years did the average debt decrease?

b. If the increase between 2010 and 2011 was the same as the increase between 2009 and 2010, what would be the average debt in 2011?

4. **Computer Games** The graph below shows the results of market research conducted by a developer of computer games. It shows the projected numbers of sales N, in millions, of a game for selected selling prices p in dollars per game.



a. Explain the meaning of the ordered pair (22, 60) in the context of this problem.

b. Based on the graph, does the projected number of sales increase or decrease as the price of this game increases?

c. The product of the coordinates of the ordered pairs, $R = p \cdot N$, indicates the revenue R to the company generated by the sale of N games at p dollars per game. Create a scatter plot of (p, R).

d. Based on the scatter plot in **c.**, what happens to the revenue as the price of the game increases?

In Exercises 5 to 8, determine whether the given ordered pair is a solution of the equation.

5. $2x + 5y = 16; (-2, 4)$

6. $2x^2 - 3y = 4; (1, -1)$

7. $y = 3x^2 - 4x + 2; (-3, 17)$

8. $x^2 + y^2 = 169; (-2, 12)$

In Exercises 9 to 18, find the distance between the points whose coordinates are given.

9. $(6, 4), (-8, 11)$

10. $(-5, 8), (-10, 14)$

11. $(-4, -20), (-10, 15)$

12. $(40, 32), (36, 20)$

13. $(5, -8), (0, 0)$

14. $(0, 0), (5, 13)$

15. $(\sqrt{3}, \sqrt{8}), (\sqrt{12}, \sqrt{27})$

16. $(\sqrt{125}, \sqrt{20}), (6, 2\sqrt{5})$

17. $(a, b), (-a, -b)$

18. $(a - b, b), (a, a + b)$

19. $(x, 4x), (-2x, 3x)$, given that $x < 0$

20. $(x, 4x), (-2x, 3x)$, given that $x > 0$

In Exercises 21 to 26, find the midpoint of the line segment with the given endpoints.

21. $(1, -1), (5, 5)$

22. $(-5, -2), (6, 10)$

23. $(6, -3), (6, 11)$

24. $(4, 7), (-10, 7)$

25. $(1.75, 2.25), (-3.5, 5.57)$

26. $(-8.2, 10.1), (-2.4, -5.7)$

In Exercises 27 to 30, find the other endpoint of the line segment that has the given endpoint and midpoint.

27. Endpoint $(5, 1)$, midpoint $(9, 3)$

28. Endpoint $(4, -6)$, midpoint $(-2, 11)$

29. Endpoint $(-3, -8)$, midpoint $(2, -7)$

30. Endpoint $(5, -4)$, midpoint $(0, 0)$

In Exercises 31 to 44, graph each equation by plotting points that satisfy the equation.

31. $x - y = 4$

32. $2x + y = -1$

Indicates Try It Exercises

33. $y = 0.25x^2$

34. $3x^2 + 2y = -4$

35. $y = -2|x - 3|$

36. $y = |x + 3| - 2$

37. $y = x^2 - 3$

38. $y = x^2 + 1$

39. $y = \frac{1}{2}(x - 1)^2$

40. $y = 2(x + 2)^2$

41. $y = x^2 + 2x - 8$

42. $y = x^2 - 2x - 8$

43. $y = -x^2 + 2$

44. $y = -x^2 - 1$

In Exercises 45 to 52, find the *x*- and *y*-intercepts of the graph of each equation. Use the intercepts and additional points as needed to draw the graph of the equation.

45. $2x + 5y = 12$

46. $3x - 4y = 15$

47. $x = -y^2 + 5$

48. $x = y^2 - 6$

49. $x = |y| - 4$

50. $x = y^3 - 2$

51. $x^2 + y^2 = 4$

52. $x^2 = y^2$

In Exercises 53 to 60, determine the center and radius of the circle with the given equation.

53. $x^2 + y^2 = 36$

54. $x^2 + y^2 = 49$

55. $(x - 1)^2 + (y - 3)^2 = 49$

56. $(x - 2)^2 + (y - 4)^2 = 25$

57. $(x + 2)^2 + (y + 5)^2 = 25$

58. $(x + 3)^2 + (y + 5)^2 = 121$

59. $(x - 8)^2 + y^2 = \frac{1}{4}$

60. $x^2 + (y - 12)^2 = 1$

In Exercises 61 to 68, find an equation of the circle that satisfies the given conditions. Write your answer in standard form.

61. Center (4, 1), radius 2

62. Center (5, −3), radius 4

63. Center $\left(\frac{1}{2}, \frac{1}{4}\right)$, radius $\sqrt{5}$

64. Center $\left(0, \frac{2}{3}\right)$, radius $\sqrt{11}$

65. Center (0, 0), passing through (−3, 4)

66. Center (0, 0), passing through (5, 12)

67. Center (1, 3), passing through (4, −1)

68. Center (−2, 5), passing through (1, 7)

In Exercises 69 to 76, find the equation of the circle described. Write your answers in standard form.

69. The coordinates of the center are (−2, 5) and the length of the diameter is 10.

70. The coordinates of the center are (0, −1) and the length of the diameter is 8.

71. The circle has a diameter with endpoints whose coordinates are (2, 3) and (−4, 11).

72. The circle has a diameter with endpoints whose coordinates are (7, −2) and (−3, 5).

73. The circle has a diameter with endpoints whose coordinates are (5, −3) and (−1, −5).

74. The circle has a diameter with endpoints whose coordinates are (4, −6) and (0, −2).

75. The circle has center with coordinates (7, 11) and is tangent to the *x*-axis.

76. The circle has center with coordinates (−2, 3) and is tangent to the *y*-axis.

In Exercises 77 to 84, find the center and radius of the graph of the circle. The equations of the circles are written in general form.

77. $x^2 + y^2 - 6x + 5 = 0$

78. $x^2 + y^2 - 6x - 4y + 12 = 0$

79. $x^2 + y^2 - 14x + 8y + 53 = 0$

80. $x^2 + y^2 - 10x + 2y + 18 = 0$

81. $x^2 + y^2 - x + 3y - \frac{15}{4} = 0$

82. $x^2 + y^2 + 3x - 5y + \frac{25}{4} = 0$

83. $x^2 + y^2 + 3x - 6y + 2 = 0$

84. $x^2 + y^2 - 5x - y - 4 = 0$

Enrichment Exercises

85. Find all points on the x-axis that are 10 units from the point (4, 6). (*Hint*: First write the distance formula with (4, 6) as one of the points and $(x, 0)$ as the other point.)

86. Find all points on the y-axis that are 12 units from the point (5, −3).

In Exercises 87 and 88, find the x- and y-intercepts of the graph of each equation. Use the intercepts and additional points as needed to draw the graph of the equation.

87. $|x| + |y| = 4$

88. $|x - 4y| = 8$

89. Find a formula for the set of all points (x, y) for which the distance from (x, y) to (3, 4) is 5.

90. Find a formula for the set of all points (x, y) for which the distance from (x, y) to (−5, 12) is 13.

SECTION 2.2 Introduction to Functions

- Relations
- Functions
- Function Notation
- Graphs of Functions
- Greatest Integer Function (Floor Function)
- Applications of Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A8.

PS1. Evaluate $x^2 + 3x - 4$ when $x = -3$. [P.3]

PS2. From the set of ordered pairs $A = \{(-3, 2), (-2, 4), (-1, 1), (0, 4), (2, 5)\}$, create two new sets, D and R, where D is the set of the first coordinates of the ordered pairs of A and R is the set of the second coordinates of the ordered pairs of A. [P.1/2.1]

PS3. Find the length of the line segment connecting $P_1(-4, 1)$ and $P_2(3, -2)$. [2.1]

PS4. For what values of x is $\sqrt{2x - 6}$ a real number? [P.6/1.5]

PS5. For what values of x is $\dfrac{x + 3}{x^2 - x - 6}$ not a real number? [P.5]

PS6. If $a = 3x + 4$ and $a = 6x - 5$, find the values of a. [1.1]

Relations

In many situations in science, business, and mathematics, a correspondence exists between the two sets. The correspondence is often defined by a *table*, an *equation*, or a *graph*, each of which can be viewed from a mathematical perspective as a set of ordered pairs. In mathematics, any set of ordered pairs is called a **relation**.

Table 2.1 defines a correspondence between a set of percent scores and a set of letter grades. For each score from 0 to 100, there corresponds only one letter grade. The score 94% corresponds to the letter grade of A. Using ordered-pair notation, we record this correspondence as (94, A).

Table 2.1

Score	Grade
[90, 100]	A
[80, 90)	B
[70, 80)	C
[60, 70)	D
[0, 60)	F

The equation $d = 16t^2$ indicates that the distance d that a rock falls (neglecting air resistance) corresponds to the time t that it has been falling. For each nonnegative value t, the equation assigns only one value for the distance d. According to this equation, in 3 seconds a rock will fall 144 feet, which we record as (3, 144). Some of the other ordered pairs determined by $d = 16t^2$ are (0, 0), (1, 16), (2, 64), and (2.5, 100).

Equation: $d = 16t^2$

If $t = 3$, then $d = 16(3)^2 = 144$

The graph in Figure 2.19, on page 165, defines a correspondence between the length of a pendulum and the time it takes the pendulum to complete one oscillation. For each nonnegative pendulum length, the graph yields only one time. According to the graph, a pendulum length of 2 feet yields an oscillation time of 1.6 seconds and a pendulum length of 4 feet yields an oscillation time of 2.2 seconds, where the time is measured to the nearest tenth of a second. These results can be recorded as the ordered pairs (2, 1.6) and (4, 2.2).