SECTION 1.5

Inequalities

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A5.

PS1. Find: $\{x|x > 2\} \cap \{x|x > 5\}$ [P.1]

PS2. Evaluate $3x^2 - 2x + 5$ for $x = -3$. [P.1]

PS3. Evaluate $\dfrac{x+3}{x-2}$ for $x = 7$. [P.1/P.5]

PS4. Factor: $10x^2 + 9x - 9$ [P.4]

PS5. For what value of x is $\dfrac{x-3}{2x-7}$ undefined? [P.1/P.5]

PS6. Solve: $2x^2 - 11x + 15 = 0$ [1.3]

Properties of Inequalities

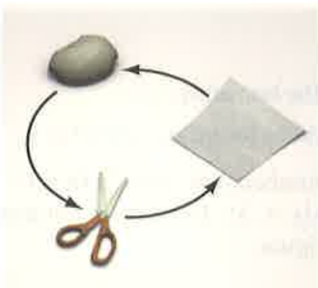
In Section P.1 we used inequalities to describe the order of real numbers and to represent subsets of real numbers. In this section we consider inequalities that involve a variable. In particular, we consider how to determine which real numbers make an inequality a true statement.

The **solution set** of an inequality is the set of all real numbers for which the inequality is a true statement. For instance, the solution set of $x + 1 > 4$ is the set of all real numbers greater than 3. Two inequalities are **equivalent inequalities** if they have the same solution set. We can solve many inequalities by producing *simpler* but equivalent inequalities until the solutions are readily apparent. To produce these simpler but equivalent inequalities, we often apply the following properties.

Math Matters

Another property of inequalities, called the *transitive property,* states that for real numbers a, b, and c, if $a > b$ and $b > c$, then $a > c$. We say that the relationship "is greater than" is a transitive relationship.

Not all relationships are transitive relationships. For instance, consider the game of scissors, paper, rock. In this game, scissors beats paper and paper beats rock, but scissors does not beat rock!



Properties of Inequalities

Let a, b, and c be real numbers.

1. ***Addition–Subtraction Property*** If the same real number is added to or subtracted from each side of an inequality, the resulting inequality is equivalent to the original inequality.

   $a < b$ and $a + c < b + c$ are equivalent inequalities.

2. ***Multiplication–Division Property***
   a. Multiplying or dividing each side of an inequality by the same *positive* real number produces an equivalent inequality.

   If $c > 0$, then $a < b$ and $ac < bc$ are equivalent inequalities.

   b. Multiplying or dividing each side of an inequality by the same *negative* real number produces an equivalent inequality provided the direction of the inequality symbol is *reversed.*

   If $c < 0$, then $a < b$ and $ac > bc$ are equivalent inequalities.

(continued)

EXAMPLE

Property 1 Adding or subtracting the same number to (from) each side of an inequality produces an equivalent inequality.

$$x - 4 < 7 \qquad\qquad x + 3 > 5$$
$$x - 4 + 4 < 7 + 4 \qquad\qquad x + 3 - 3 > 5 - 3$$
$$x < 11 \qquad\qquad x > 2$$

Property 2a Multiplying or dividing each side of an inequality by the same positive number produces an equivalent inequality.

$$3x < 12 \qquad\qquad \frac{2}{3}x > -4$$
$$\frac{3x}{3} < \frac{12}{3} \qquad\qquad \frac{3}{2} \cdot \frac{2}{3}x > \frac{3}{2}(-4)$$
$$x < 4 \qquad\qquad x > -6$$

Property 2b Multiplying or dividing each side of an inequality by the same negative number produces an equivalent inequality provided the direction of the inequality symbol is reversed.

$$-2x < 6 \qquad\qquad -\frac{3}{4}x > 3$$
$$\frac{-2x}{-2} > \frac{6}{-2} \qquad\qquad -\frac{4}{3}\left(-\frac{3}{4}x\right) < \left(-\frac{4}{3}\right)3$$
$$x > -3 \qquad\qquad x < -4$$

Note the difference between Property 2a and Property 2b. Property 2a states that an equivalent inequality is produced when each side of a given inequality is multiplied (divided) by the same *positive* real number and the inequality symbol is not changed. By contrast, Property 2b states that when each side of a given inequality is multiplied (divided) by a *negative* real number, we must *reverse* the direction of the inequality symbol to produce an equivalent inequality. For instance, multiplying both sides of $-b < 4$ by -1 produces the equivalent inequality $b > -4$. (We multiplied both sides of the first inequality by -1, and we changed the "less than" symbol to a "greater than" symbol.)

EXAMPLE 1 Solve Linear Inequalities

Solve each of the following inequalities.

a. $2x + 1 < 7$ **b.** $-3x - 2 \le 10$

Solution

a. $2x + 1 < 7$

$2x < 6$ • Subtract 1 from each side of the inequality.

$x < 3$ • Divide each side by 2 and keep the inequality symbol as is.

The inequality $2x + 1 < 7$ is true for all real numbers less than 3. In set-builder notation, the solution set is given by $\{x \mid x < 3\}$. In interval notation, the solution set is $(-\infty, 3)$. See the following figure.

Study tip

Solutions of inequalities can be stated using set-builder notation or interval notation. For instance, the solution set of $2x + 1 < 7$ can be written in set-builder notation as $\{x \mid x < 3\}$ or in interval notation as $(-\infty, 3)$.

TO REVIEW

Interval Notation
See page 6.

b. $-3x - 2 \leq 10$

$-3x \leq 12$ • Add 2 to each side and keep the inequality symbol as is.

$x \geq -4$ • Divide each side by -3 and reverse the direction of the inequality symbol.

The inequality $-3x - 2 \leq 10$ is true for all real numbers greater than or equal to -4. In set-builder notation, the solution set is given by $\{x \mid x \geq -4\}$. In interval notation, the solution set is $[-4, \infty)$. See the following figure.

▸ **Try Exercise 10, page 132**

Compound Inequalities

A **compound inequality** is formed by joining two inequalities with the connective word *and* or *or*. The inequalities shown below are compound inequalities.

$$x + 1 > 3 \quad \text{and} \quad 2x - 11 < 7$$
$$x + 3 > 5 \quad \text{or} \quad x - 1 < 9$$

The solution set of a compound inequality with the connective word *or* is the *union* of the solution sets of the two inequalities. The solution set of a compound inequality with the connective word *and* is the *intersection* of the solution sets of the two inequalities.

EXAMPLE 2 Solve Compound Inequalities

Solve each compound inequality. Write each solution in set-builder notation.

a. $2x < 10$ or $x + 1 > 9$ **b.** $x + 3 > 4$ and $2x + 1 > 15$

Solution

a. $2x < 10$ or $x + 1 > 9$

$x < 5$ $x > 8$ • Solve each inequality.

$\{x \mid x < 5\}$ $\{x \mid x > 8\}$ • Write each solution as a set.

$\{x \mid x < 5\} \cup \{x \mid x > 8\} = \{x \mid x < 5 \text{ or } x > 8\}$ • Write the union of the solution sets.

b. $x + 3 > 4$ and $2x + 1 > 15$

$x > 1$ $2x > 14$ • Solve each inequality.

$x > 7$

$\{x \mid x > 1\}$ $\{x \mid x > 7\}$ • Write each solution as a set.

$\{x \mid x > 1\} \cap \{x \mid x > 7\} = \{x \mid x > 7\}$ • Write the intersection of the solution sets.

▸ **Try Exercise 14, page 132**

Question • What is the solution set of the compound inequality $x > 1$ or $x < 3$?

Answer • The solution is the set of all real numbers. Using interval notation, the solution set is written as $(-\infty, \infty)$.

Note

We reserve the notation $a < b < c$ to mean $a < b$ and $b < c$. Thus the solution set of $2 > x > 5$ is the empty set, because there are no numbers less than 2 and greater than 5.

Note

The compound inequality $a < b$ and $b < c$ can be written in the compact form $a < b < c$. However, the compound inequality $a < b$ or $b > c$ cannot be expressed in a compact form.

The inequality given by

$$12 < x + 5 < 19$$

is equivalent to the compound inequality $12 < x + 5$ *and* $x + 5 < 19$. You can solve $12 < x + 5 < 19$ by either of the following methods.

Method 1 Find the intersection of the solution sets of the inequalities $12 < x + 5$ and $x + 5 < 19$.

$$12 < x + 5 \quad \text{and} \quad x + 5 < 19$$
$$7 < x \qquad\qquad x < 14$$

The solution set is $\{x \mid x > 7\} \cap \{x \mid x < 14\} = \{x \mid 7 < x < 14\}$.

Method 2 Subtract 5 from each of the three parts of the inequality.

$$12 < x + 5 < 19$$
$$12 - 5 < x + 5 - 5 < 19 - 5$$
$$7 < x < 14$$

The solution set is $\{x \mid 7 < x < 14\}$.

Absolute Value Inequalities

The solution set of the absolute value inequality $|x - 1| < 3$ is the set of all real numbers whose distance from 1 is *less than* 3. Therefore, the solution set consists of all numbers between -2 and 4. See Figure 1.6. In interval notation, the solution set is $(-2, 4)$.

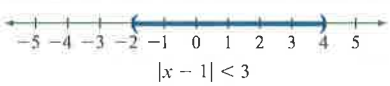


Figure 1.6

The solution set of the absolute value inequality $|x - 1| > 3$ is the set of all real numbers whose distance from 1 is *greater than* 3. Therefore, the solution set consists of all real numbers less than -2 *or* greater than 4. See Figure 1.7. In interval notation, the solution set is $(-\infty, -2) \cup (4, \infty)$.

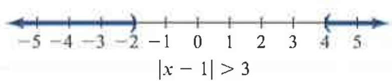


Figure 1.7

The following properties are used to solve absolute value inequalities.

Note

Some inequalities have a solution set that consists of all real numbers. For example, $|x + 9| \geq 0$ is true for all values of x. Because an absolute value is always nonnegative, the inequality is always true.

Properties of Absolute Value Inequalities

For any variable expression E and any nonnegative real number k,

$$|E| \leq k \quad \text{if and only if} \quad -k \leq E \leq k$$
$$|E| \geq k \quad \text{if and only if} \quad E \leq -k \quad \text{or} \quad E \geq k$$

These properties also hold true when the $<$ symbol is substituted for the $\leq$ symbol and when the $>$ symbol is substituted for the $\geq$ symbol.

EXAMPLE

If $|x| < 5$, then $-5 < x < 5$.

If $|x| > 7$, then $x < -7$ or $x > 7$.

In Example 3, we use the preceding properties to solve absolute value inequalities.

EXAMPLE 3 Solve Absolute Value Inequalities

Solve each of the following inequalities. Write each solution set in interval notation.

a. $|2 - 3x| < 7$ **b.** $|4x - 3| \geq 5$

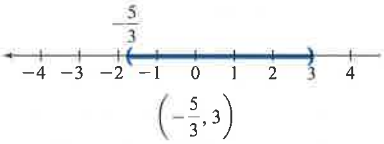


Figure 1.8

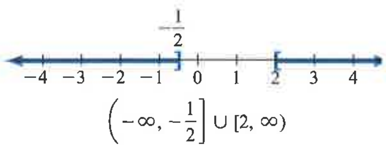


Figure 1.9

Solution

a. $|2 - 3x| < 7$ if and only if $-7 < 2 - 3x < 7$. Solve this compound inequality.

$-7 < 2 - 3x < 7$

$-9 < -3x < 5$ • Subtract 2 from each of the three parts of the inequality.

$3 > x > -\frac{5}{3}$ • Multiply each part of the inequality by $-\frac{1}{3}$ and reverse the inequality symbols.

In interval notation, the solution set is given by $\left(-\frac{5}{3}, 3\right)$. See Figure 1.8.

b. $|4x - 3| \geq 5$ implies $4x - 3 \leq -5$ or $4x - 3 \geq 5$. Solving each of these inequalities produces

$$4x - 3 \leq -5 \quad \text{or} \quad 4x - 3 \geq 5$$
$$4x \leq -2 \qquad\qquad 4x \geq 8$$
$$x \leq -\frac{1}{2} \qquad\qquad x \geq 2$$

The solution set is $\left(-\infty, -\frac{1}{2}\right] \cup [2, \infty)$. See Figure 1.9.

Try Exercise 22, page 132

Polynomial Inequalities

Any value of x that causes a polynomial in x to equal zero is called a **zero of the polynomial**. For example, -4 and 1 are both zeros of the polynomial $x^2 + 3x - 4$ because $(-4)^2 + 3(-4) - 4 = 0$ and $1^2 + 3 \cdot 1 - 4 = 0$.

Sign Property of Polynomials

Polynomials in x have the following property: For all values of x between two consecutive real zeros, all values of the polynomial are positive or all values of the polynomial are negative.

In our work with inequalities that involve polynomials, the real zeros of the polynomial are also referred to as **critical values of the inequality**. On a number line, the critical values of an inequality separate the real numbers that make the inequality true from those that make it false.

For instance, to solve the inequality $x^2 + 3x - 4 < 0$, we begin by solving the equation $x^2 + 3x - 4 = 0$ to find the real zeros of the polynomial.

$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -4 \qquad\qquad x = 1$$

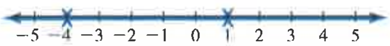


Figure 1.10

The real zeros are −4 and 1. They are the critical values of the inequality $x^2 + 3x - 4 < 0$, and they separate the real number line into three intervals, as shown in Figure 1.10.

To determine the intervals in which $x^2 + 3x - 4$ is less than 0, pick a number called a **test value** from each of the three intervals and then determine whether $x^2 + 3x - 4$ is less than 0 for each of these test values. For example, in the interval $(-\infty, -4)$, pick a test value of -5. Then

$$x^2 + 3x - 4 = (-5)^2 + 3(-5) - 4 = 6$$

Because 6 is not less than 0, by the sign property of polynomials, no number in the interval $(-\infty, -4)$ makes $x^2 + 3x - 4$ less than 0.

Now pick a test value from the interval $(-4, 1)$—say, 0. When $x = 0$,

$$x^2 + 3x - 4 = 0^2 + 3(0) - 4 = -4$$

Because −4 is less than 0, by the sign property of polynomials, all numbers in the interval $(-4, 1)$ make $x^2 + 3x - 4$ less than 0.

If we pick a test value of 2 from the interval $(1, \infty)$, then

$$x^2 + 3x - 4 = (2)^2 + 3(2) - 4 = 6$$

Because 6 is not less than 0, by the sign property of polynomials, no number in the interval $(1, \infty)$ makes $x^2 + 3x - 4$ less than 0.

The following table is a summary of our work.

Interval	Test Value x	$x^2 + 3x - 4 \overset{?}{<} 0$
$(-\infty, -4)$	-5	$(-5)^2 + 3(-5) - 4 < 0$ $6 < 0$ False
$(-4, 1)$	0	$(0)^2 + 3(0) - 4 < 0$ $-4 < 0$ True
$(1, \infty)$	2	$(2)^2 + 3(2) - 4 < 0$ $6 < 0$ False

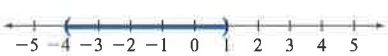
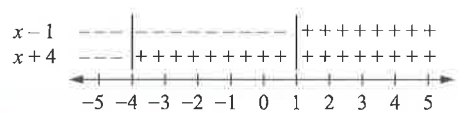


Figure 1.11

In interval notation, the solution set of $x^2 + 3x - 4 < 0$ is $(-4, 1)$. The solution set is graphed in Figure 1.11. Note that in this case the critical values −4 and 1 are not included in the solution set because they do not make $x^2 + 3x - 4$ less than 0.

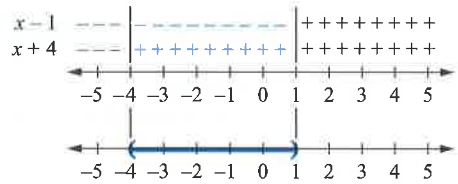
To avoid the extensive arithmetic, we often use a *sign diagram*. For example, note that the factor $(x + 4)$ is negative for all $x < -4$ and positive for all $x > -4$. The factor $(x - 1)$ is negative for all $x < 1$ and positive for all $x > 1$. These results are shown in Figure 1.12.



Sign diagram for $x^2 + 3x - 4 < 0$

Figure 1.12

Because we are trying to solve $x^2 + 3x - 4 < 0$, we want the interval for which the product of the factors is negative (the polynomial is less than zero). From the sign diagram, we can visually determine that this interval is where the factors have opposite signs. The solution set is the interval $(-4, 1)$. See Figure 1.13.



The solution set is the interval (−4, 1).

Figure 1.13

EXAMPLE 4 Solve a Polynomial Inequality

Find the solution set of $x^3 + 3x^2 - 4x - 12 \geq 0$. Write the answer in interval notation.

Solution

Find the zeros of the polynomial.

$$x^3 + 3x^2 - 4x - 12 = 0$$
$$(x + 3)(x + 2)(x - 2) = 0 \qquad \text{• Factor by grouping.}$$

The zeros are −3, −2, and 2. Draw a sign diagram using these values. See Figure 1.14.

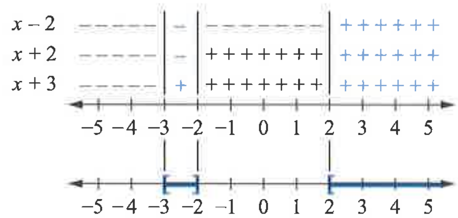


Figure 1.14

Because the inequality is ≥, find the intervals for which the product of the factors is positive or zero. From the diagram, the solution set is $[-3, 2] \cup [2, \infty)$. The inequality is ≥, so we use brackets, except after the infinity symbol.

▶ **Try Exercise 38, page 132**

Following is a summary of the steps used to solve polynomial inequalities by the critical value method.

Solving a Polynomial Inequality by the Critical Value Method

1. Write the inequality so that one side of the inequality is a nonzero polynomial and the other side is 0.
2. Find the real zeros of the polynomial. They are the critical values of the original inequality.
3. Use test values to determine which of the consecutive intervals formed by the critical values are to be included in the solution set.

Rational Inequalities

A rational expression is the quotient of two polynomials. **Rational inequalities** involve rational expressions, and they can be solved by an extension of the critical value method.

Definition of a Critical Value of a Rational Expression

A **critical value of a rational expression** is a number that causes the numerator of the rational expression to equal zero or the denominator of the rational expression to equal zero.

Rational expressions also have the property that they remain either positive for all values of the variable between consecutive critical values or negative for all values of the variable between consecutive critical values.

Following is a summary of the steps used to solve rational inequalities by the critical value method.

Solving a Rational Inequality Using the Critical Value Method

1. Write the inequality so that one side of the inequality is a rational expression and the other side is zero.
2. Find the real zeros of the numerator of the rational expression and the real zeros of its denominator. They are the critical values of the inequality.
3. Use test values to determine which of the consecutive intervals formed by the critical values are to be included in the solution set.

EXAMPLE 5 Solve a Rational Inequality

Solve: $\dfrac{3x+4}{x+1} \leq 2$

Solution

Write the inequality so that 0 appears on the right side of the inequality.

$$\frac{3x+4}{x+1} \leq 2$$

$$\frac{3x+4}{x+1} - 2 \leq 0$$

Write the left side as a rational expression.

$$\frac{3x+4}{x+1} - \frac{2(x+1)}{x+1} \leq 0$$

• The LCD is $x + 1$.

$$\frac{3x+4-2x-2}{x+1} \leq 0$$

• Simplify.

$$\frac{x+2}{x+1} \leq 0$$

The critical values of this inequality are -2 and -1 because the numerator $x + 2$ is equal to zero when $x = -2$ and the denominator $x + 1$ is equal to zero when $x = -1$. The critical values -2 and -1 separate the real number line into the three intervals $(-\infty, -2)$, $(-2, -1)$, and $(-1, \infty)$.

All values of x on the interval $(-2, -1)$ make $\frac{x+2}{x+1}$ negative, as desired. On the other intervals, the quotient $\frac{x+2}{x+1}$ is positive. See the sign diagram in Figure 1.15.

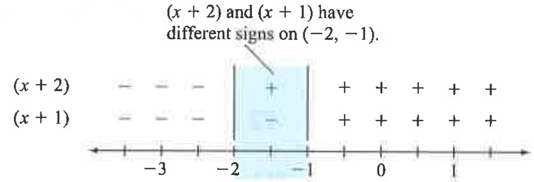


Figure 1.15

The solution set is $[-2, -1)$. The graph of the solution set is shown in Figure 1.16. Note that -2 is included in the solution set because $\frac{x+2}{x+1} = 0$ when $x = -2$. However, -1 is not included in the solution set because the denominator $(x + 1)$ is zero when $x = -1$.

Figure 1.16

Try Exercise 54, page 132

Applications of Inequalities

Many applied problems can be solved by using inequalities.

EXAMPLE 6 Solve an Application Concerning Leases

A real estate company needs a new copy machine. The company has decided to lease either the model ABC machine for \$75 a month plus 5 cents per copy or the model XYZ machine for \$210 a month and 2 cents per copy. Under what conditions is it less expensive to lease the XYZ machine?

Solution

Let x represent the number of copies the company produces per month. The dollar costs per month are $75 + 0.05x$ for model ABC and $210 + 0.02x$ for model XYZ. It will be less expensive to lease model XYZ provided

$$210 + 0.02x < 75 + 0.05x$$

$$210 - 0.03x < 75 \quad \text{• Subtract } 0.05x \text{ from each side.}$$

$$-0.03x < -135 \quad \text{• Subtract 210 from each side.}$$

$$x > 4500 \quad \text{• Divide each side by } -0.03. \text{ Reverse the inequality symbol.}$$

The company will find it less expensive to lease model XYZ if it produces more than 4500 copies per month.

Try Exercise 60, page 132

EXAMPLE 7 Solve an Application Concerning Test Scores

Tyra has test scores of 70 and 81 in her biology class. To receive a C grade, she must obtain an average greater than or equal to 72 but less than 82. What range of test scores on the one remaining test will enable Tyra to get a C for the course?

Solution

The average of three test scores is the sum of the scores divided by 3. Let x represent Tyra's next test score. The requirements for a C grade produce the following inequality.

$$72 \le \frac{70 + 81 + x}{3} < 82$$

$$216 \le 70 + 81 + x < 246$$ • Multiply each part of the inequality by 3.

$$216 \le 151 + x < 246$$ • Simplify.

$$65 \le x < 95$$ • Solve for x by subtracting 151 from each part of the inequality.

To get a C in the course, Tyra's remaining test score must be in the interval [65, 95).

▶ **Try Exercise 64, page 133**

In many business applications, companies are interested in the cost C of manufacturing x items, the revenue R generated by selling all the items, and the profit P made by selling the items.

In the next example, the cost, in dollars, of manufacturing x tennis racquets is given by $C = 32x + 120{,}000$. The 120,000 represents the fixed cost because it remains constant regardless of how many racquets are manufactured. The $32x$ represents the variable cost because this term varies depending on how many racquets are manufactured. Each additional racquet costs the company an additional \$32.

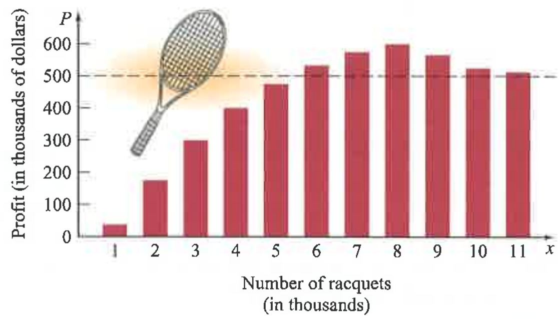
The revenue, in dollars, received from the sale of x tennis racquets is given by $R = x(200 - 0.01x)$. The quantity $(200 - 0.01x)$ is the price the company charges for each tennis racquet. The price varies depending on the number of racquets that are manufactured. For instance, if the number of racquets x that the company manufactures is small, the company will be able to demand almost \$200 for each racquet. As the number of racquets that the company manufactures increases (approaches 20,000), the company will be able to sell *all* the racquets only if it decreases the price of each racquet.

The following profit formula shows the relationship between profit P, revenue R, and cost C.

$$P = R - C$$

EXAMPLE 8 Solve a Business Application

A company determines that the cost C, in dollars, of producing x tennis racquets is $C = 32x + 120{,}000$. The revenue R, in dollars, from selling all of the tennis racquets is $R = x(200 - 0.01x)$.



How many racquets should the company manufacture and sell if the company wishes to earn a profit of at least \$500,000?

Solution

The profit is given by

$$\begin{aligned} P &= R - C \\ &= x(200 - 0.01x) - (32x + 120{,}000) \\ &= 200x - 0.01x^2 - 32x - 120{,}000 \\ &= -0.01x^2 + 168x - 120{,}000 \end{aligned}$$

The profit will be at least \$500,000 provided

$$-0.01x^2 + 168x - 120{,}000 \geq 500{,}000$$
$$-0.01x^2 + 168x - 620{,}000 \geq 0$$

Using the quadratic formula, we find that the approximate critical values of this last inequality are 5474.3 and 11,325.7. Test values show that the inequality is positive only on the interval (5474.3, 11,325.7). The company should manufacture at least 5475 tennis racquets but no more than 11,325 tennis racquets to produce the desired profit.

Try Exercise 62, page 133

EXERCISE SET 1.5

Concept Check

1. State whether each pair of inequalities are equivalent.

   a. $x > -3, x > 0$ b. $3x \leq -6, x \geq -2$

   c. $-2x < 0, x > 0$ d. $\frac{2}{3}x \geq -6, x \geq -9$

2. Write the inequality $|x + 2| > 3$ as a compound inequality without absolute value signs.

3. Write the inequality $|x - 5| \leq 8$ as a compound inequality without absolute value signs.

4. For each of the following, identify the critical values of the inequality.

   a. $x^2 + 8x < 0$

   b. $(2x + 5)(x - 4) \geq 0$

   c. $\dfrac{x - 4}{x + 2} \leq 0$

   d. $\dfrac{2x - 1}{x(x - 3)} > 0$

In Exercises 5 to 12, use the properties of inequalities to solve each inequality. Write the solution set using set-builder notation, and graph the solution set.

5. $2x + 3 < 11$ 6. $3x - 5 > 16$

Indicates Try It Exercises

7. $x + 4 > 3x + 16$ **8.** $5x + 6 < 2x + 1$

9. $-3(x + 2) \leq 5x + 7$ **10.** $-4(x - 5) \geq 2x + 15$

11. $-4(3x - 5) > 2(x - 4)$ **12.** $3(x + 7) \leq 5(2x - 8)$

In Exercises 13 to 20, solve each compound inequality. Write the solution set using set-builder notation, and graph the solution set.

13. $4x + 1 > -2$ and $4x + 1 \leq 17$

14. $2x + 5 > -16$ and $2x + 5 < 9$

15. $10 \geq 3x - 1 \geq 0$

16. $0 \leq 2x + 6 \leq 54$

17. $x + 2 < -1$ or $x + 3 \geq 2$

18. $x + 1 > 4$ or $x + 2 \leq 3$

19. $-4x + 5 > 9$ or $4x + 1 < 5$

20. $2x - 7 \leq 15$ or $3x - 1 \leq 5$

In Exercises 21 to 32, use interval notation to express the solution set of each inequality.

21. $|2x - 1| > 4$ **22.** $|2x - 9| < 7$

23. $|x + 3| \geq 5$ **24.** $|x - 10| \geq 2$

25. $|3x - 10| \leq 14$ **26.** $|2x - 5| \geq 1$

27. $|4 - 5x| \geq 24$ **28.** $|3 - 2x| \leq 5$

29. $|x - 5| \geq 0$ **30.** $|x - 7| \geq 0$

31. $|x - 4| \leq 0$ **32.** $|2x + 7| \leq 0$

In Exercises 33 to 44, use the critical value method to solve each polynomial inequality. Use interval notation to write each solution set.

33. $x^2 + 7x > 0$ **34.** $x^2 - 5x \leq 0$

35. $x^2 - 16 \leq 0$ **36.** $x^2 - 49 > 0$

37. $x^2 + 7x + 10 < 0$ **38.** $x^2 + 5x + 6 < 0$

39. $x^2 - 3x \geq 28$ **40.** $x^2 < -x + 30$

41. $x^3 - x^2 - 16x + 16 < 0$ **42.** $x^3 + x^2 - 9x - 9 > 0$

43. $x^4 - 20x^2 + 64 \geq 0$ **44.** $x^4 - 10x^2 + 9 \leq 0$

In Exercises 45 to 58, use the critical value method to solve each rational inequality. Write each solution set in interval notation.

45. $\dfrac{x + 4}{x - 1} < 0$ **46.** $\dfrac{x - 2}{x + 3} > 0$

47. $\dfrac{x - 5}{x + 8} \geq 3$ **48.** $\dfrac{x - 4}{x + 6} \leq 1$

49. $\dfrac{x}{2x + 7} \geq 4$ **50.** $\dfrac{x}{3x - 5} \leq -5$

51. $\dfrac{(x + 1)(x - 4)}{x - 2} < 0$ **52.** $\dfrac{x(x - 4)}{x + 5} > 0$

53. $\dfrac{x + 2}{x - 5} \leq 2$ **54.** $\dfrac{3x + 1}{x - 2} \geq 4$

55. $\dfrac{6x^2 - 11x - 10}{x} > 0$ **56.** $\dfrac{3x^2 - 2x - 8}{x - 1} \geq 0$

57. $\dfrac{x^2 - 6x + 9}{x - 5} \leq 0$ **58.** $\dfrac{x^2 + 10x + 25}{x + 1} \geq 0$

59. **Personal Finance** A bank offers two checking account plans. The monthly fee and charge per check for each plan are shown in the following diagram. Under what conditions is it less expensive to use the LowCharge plan?

Account Plan	Monthly Fee	Charge per Check
LowCharge	\$5.00	\$.01
FeeSaver	\$1.00	\$.08

60. **Personal Finance** You can rent a car for the day from Company A for \$29.00 plus \$0.12 a mile. Company B charges \$22.00 plus \$0.21 a mile. Find the number of miles m (to the nearest mile) per day for which it is cheaper to rent from Company A.

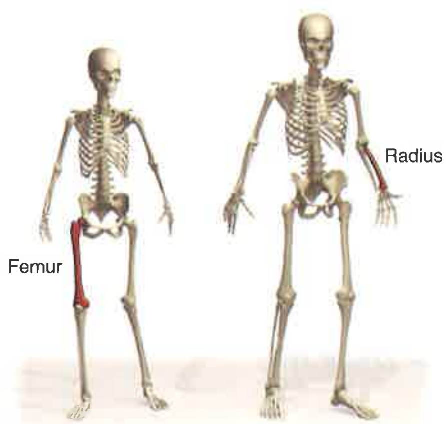
61. **Shipping Requirements** United Parcel Service (UPS) will ship only packages for which the length is less than or equal to 108 inches and the length plus the girth is less than or equal to 165 inches. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 47 inches and a width of 22 inches, determine the possible range of heights h for this package if you wish to ship it by UPS. (*Source:* http://www.ups.com.)

62. **Movie Theater Attendance** The attendance A, in billions of people, at movie theaters has been declining since the year 2000. A model of the decline is given by $A = -0.03x + 1.59$, where $x = 0$ corresponds to 2000. According to this model, in what year will movie attendance first be less than 1 billion people?

63. **Car Value** Based on data from the Kelley Blue Book website, the value V, in dollars, of a 2011 Corvette in excellent condition can be modeled by $V = -176.05m + 50{,}520$, where m is the number of miles, in thousands, on the odometer. Using this model, how many miles will a 2011 Corvette have if its value is \$41,000? Round to the nearest thousand miles. (*Source*: http://www.kbb.com)

64. **Average Temperatures** The average daily minimum-to-maximum temperature range for the city of Palm Springs during the month of September is 68° to 104° Fahrenheit. What is the corresponding temperature range measured on the Celsius temperature scale? (*Hint:* Let F be the average daily temperature in degrees Fahrenheit. Then $68° \le F \le 104°$. Now substitute $\frac{9}{5}C + 32$ for F and solve the resulting inequality for C.)

65. **Forensic Science** Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. These lengths are substituted into mathematical inequalities. For instance, an inequality that relates the height h, in centimeters, of an adult female and the length f, in centimeters, of her femur is $|h - (2.47f + 54.10)| \le 3.72$. Use this inequality to estimate the possible range of heights, rounded to the nearest tenth of a centimeter, for an adult female whose femur measures 32.24 centimeters.



66. **Forensic Science** An inequality that is used to calculate the height h of an adult male from the length r of his radius is

$$|h - (3.32r + 85.43)| \le 4.57$$

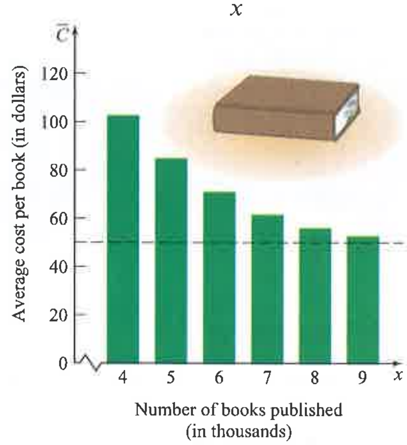
where h and r are both in centimeters. Use this inequality to estimate the possible range of heights, rounded to the nearest tenth of a centimeter, for an adult male whose radius measures 26.36 centimeters.

67. **Revenue** The monthly revenue R for a product is given by $R = 420x - 2x^2$, where x is the price in dollars of each unit produced. Find the interval, in terms of x, for which the monthly revenue is greater than \$0.

68. **Revenue** A shoe manufacturer finds that the monthly revenue R from a particular style of aerobics shoe is given by $R = 312x - 3x^2$, where x is the price in dollars of each pair of shoes sold. Find the interval, in terms of x, for which the monthly revenue is greater than or equal to \$5925.

69. **Publishing** A publisher has determined that if x books are published, the average cost per book is given by

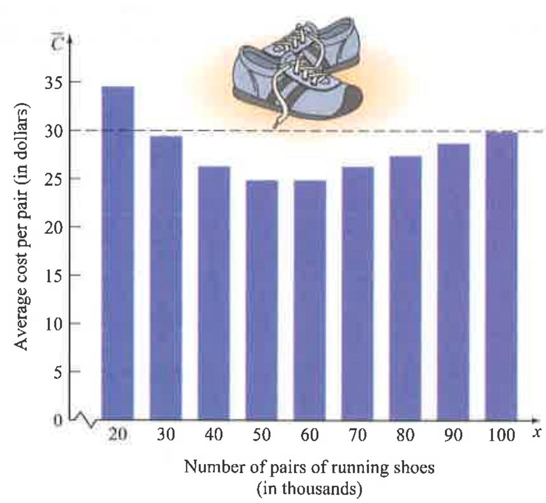
$$\overline{C} = \frac{14.25x + 350{,}000}{x}$$



How many books should be published if the company wants to bring the average cost per book below \$50?

70. **Manufacturing** A company manufactures running shoes. The company has determined that if it manufactures x pairs of shoes, the average cost, in dollars, per pair is

$$\overline{C} = \frac{0.00014x^2 + 12x + 400{,}000}{x}$$

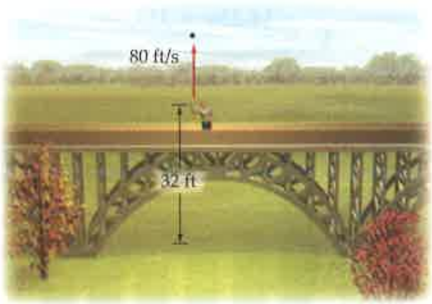


How many pairs of running shoes should the company manufacture if it wishes to bring the average cost below \$30 per pair?

71. **Height of a Projectile** The equation

$$s = -16t^2 + v_0 t + s_0$$

gives the height s, in feet above ground level, at the time t, in seconds, of an object thrown directly upward from a height s_0 feet above the ground and with an initial velocity of v_0 feet per second. A ball is thrown directly upward from a height of 32 feet above the ground with an initial velocity of 80 feet per second. Find the time interval during which the ball will be more than 96 feet above the ground.



Enrichment Exercises

72. **Mean Weight of Women** If a researcher wanted to know the mean weight (the *mean* is the sum of all the measurements divided by the number of measurements) of women in the United States, the weight of every woman would have to be measured and then the mean weight calculated—an impossible task. Instead, researchers find a representative sample of women and find the mean weight of the sample. Because the entire population of women is not used, there is a possibility that the calculated mean weight is not the true mean weight. For one study, researchers used the formula $\left|\frac{163 - \mu}{1.79}\right| < 2.33$, where μ is the true mean weight, in pounds, of all women, to be 98% sure of the range of values for the true mean weight. Using this inequality, what is the range of mean weights of women in the United States? Round to the nearest tenth of a pound. (*Source*: Based on data from the National Center for Health Statistics.)

73. **Mean Weight of Men** If a researcher wanted to know the mean weight (see Exercise 72 for a definition of mean) of men in the United States, the weight of every man would have to be measured and then the mean weight calculated—an impossible task. Instead, researchers find a representative sample of men and find the mean weight of the sample. Because the entire population of men is not used, there is a possibility that the calculated mean weight is not the true mean weight. For one study, researchers used the formula $\left|\frac{190 - \mu}{2.45}\right| < 2.575$, where μ is the true mean weight, in pounds, of all men, to be 99% sure of the range of values for the true mean weight. Using this inequality, what is the range of mean weights of men in the United States? Round to the nearest tenth of a pound. (*Source*: Based on data from the National Center for Health Statistics.)

SECTION 1.6 Variation and Applications

- Direct Variation
- Inverse Variation
- Joint and Combined Variations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A6.

PS1. Solve $1820 = k(28)$ for k. [1.1]

PS2. Solve $20 = \dfrac{k}{1.5^2}$ for k. [1.1]

PS3. Evaluate $k \cdot \dfrac{3}{5^2}$ given that $k = 225$. [P.1]

PS4. Evaluate $k \cdot \dfrac{4.5 \cdot 32}{8^2}$ given that $k = 12.5$. [P.1]

PS5. If the length of each side of a square is doubled, what effect does this have on its area? [P.1/P.2]

PS6. If the radius of a cylinder is tripled, does this triple the volume of the cylinder? [P.1/P.2]