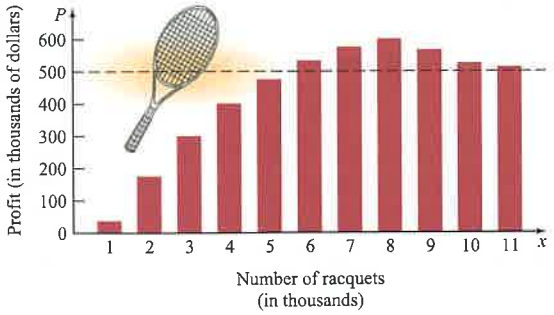


How many racquets should the company manufacture and sell if the company wishes to earn a profit of at least \$500,000?

Solution

The profit is given by

$$\begin{aligned} P &= R - C \\ &= x(200 - 0.01x) - (32x + 120{,}000) \\ &= 200x - 0.01x^2 - 32x - 120{,}000 \\ &= -0.01x^2 + 168x - 120{,}000 \end{aligned}$$

The profit will be at least \$500,000 provided

$$\begin{aligned} -0.01x^2 + 168x - 120{,}000 &\geq 500{,}000 \\ -0.01x^2 + 168x - 620{,}000 &\geq 0 \end{aligned}$$

Using the quadratic formula, we find that the approximate critical values of this last inequality are 5474.3 and 11,325.7. Test values show that the inequality is positive only on the interval (5474.3, 11,325.7). The company should manufacture at least 5475 tennis racquets but no more than 11,325 tennis racquets to produce the desired profit.

▸ Try Exercise 62, page 133

Answer graphs for Exercises 5 to 20 are on page AA1.

EXERCISE SET 1.5

Concept Check

1. State whether each pair of inequalities are equivalent.

   a. $x > -3, x > 0$ No **b.** $3x \leq -6, x \geq -2$ No

   c. $-2x < 0, x > 0$ Yes **d.** $\frac{2}{3}x \geq -6, x \geq -9$ Yes

2. Write the inequality $|x + 2| > 3$ as a compound inequality without absolute value signs. $x + 2 < -3$ or $x + 2 > 3$

3. Write the inequality $|x - 5| \leq 8$ as a compound inequality without absolute value signs. $x - 5 \geq -8$ and $x - 5 \leq 8$

4. For each of the following, identify the critical values of the inequality.

   a. $x^2 + 8x < 0$ $-8, 0$

   b. $(2x + 5)(x - 4) \geq 0$ $-\frac{5}{2}, 4$

   c. $\frac{x - 4}{x + 2} \leq 0$ $-2, 4$

   d. $\frac{2x - 1}{x(x - 3)} > 0$ $0, \frac{1}{2}, 3$

In Exercises 5 to 12, use the properties of inequalities to solve each inequality. Write the solution set using set-builder notation, and graph the solution set.

5. $2x + 3 < 11$ $\{x|x < 4\}$

6. $3x - 5 > 16$ $\{x|x > 7\}$

Indicates Try It Exercises

7. $x + 4 > 3x + 16$ $\{x|x < -6\}$

8. $5x + 6 < 2x + 1$ $\left\{x|x < -\frac{5}{3}\right\}$

9. $-3(x + 2) \le 5x + 7$ $\left\{x|x \ge -\frac{13}{8}\right\}$

10. $-4(x - 5) \ge 2x + 15$ $\left\{x|x \le \frac{5}{6}\right\}$

11. $-4(3x - 5) > 2(x - 4)$ $\{x|x < 2\}$

12. $3(x + 7) \le 5(2x - 8)$ $\left\{x|x \ge \frac{61}{7}\right\}$

In Exercises 13 to 20, solve each compound inequality. Write the solution set using set-builder notation, and graph the solution set.

13. $4x + 1 > -2$ and $4x + 1 \le 17$ $\left\{x|-\frac{3}{4} < x \le 4\right\}$

14. $2x + 5 > -16$ and $2x + 5 < 9$ $\left\{x|-\frac{21}{2} < x < 2\right\}$

15. $10 \ge 3x - 1 \ge 0$ $\left\{x|\frac{1}{3} \le x \le \frac{11}{3}\right\}$

16. $0 \le 2x + 6 \le 54$ $\{x|-3 \le x \le 24\}$

17. $x + 2 < -1$ or $x + 3 \ge 2$ $\{x|x < -3 \text{ or } x \ge -1\}$

18. $x + 1 > 4$ or $x + 2 \le 3$ $\{x|x \le 1 \text{ or } x > 3\}$

19. $-4x + 5 > 9$ or $4x + 1 < 5$ $\{x|x < 1\}$

20. $2x - 7 \le 15$ or $3x - 1 \le 5$ $\{x|x \le 11\}$

In Exercises 21 to 32, use interval notation to express the solution set of each inequality.

21. $|2x - 1| > 4$ $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

22. $|2x - 9| < 7$ $(1, 8)$

23. $|x + 3| \ge 5$ $(-\infty, -8] \cup [2, \infty)$

24. $|x - 10| \ge 2$ $(-\infty, 8] \cup [12, \infty)$

25. $|3x - 10| \le 14$ $\left[-\frac{4}{3}, 8\right]$

26. $|2x - 5| \ge 1$ $(-\infty, 2] \cup [3, \infty)$

27. $|4 - 5x| \ge 24$ $(-\infty, -4] \cup \left[\frac{28}{5}, \infty\right)$

28. $|3 - 2x| \le 5$ $[-1, 4]$

29. $|x - 5| \ge 0$ $(-\infty, \infty)$

30. $|x - 7| \ge 0$ $(-\infty, \infty)$

31. $|x - 4| \le 0$ $\{4\}$

32. $|2x + 7| \le 0$ $\left\{-\frac{7}{2}\right\}$

In Exercises 33 to 44, use the critical value method to solve each polynomial inequality. Use interval notation to write each solution set.

33. $x^2 + 7x > 0$ $(-\infty, -7) \cup (0, \infty)$

34. $x^2 - 5x \le 0$ $[0, 5]$

35. $x^2 - 16 \le 0$ $[-4, 4]$

36. $x^2 - 49 > 0$ $(-\infty, -7) \cup (7, \infty)$

37. $x^2 + 7x + 10 < 0$ $(-5, -2)$

38. $x^2 + 5x + 6 < 0$ $(-3, -2)$

39. $x^2 - 3x \ge 28$ $(-\infty, -4] \cup [7, \infty)$

40. $x^2 < -x + 30$ $(-6, 5)$

41. $x^3 - x^2 - 16x + 16 < 0$ $(-\infty, -4) \cup (1, 4)$

42. $x^3 + x^2 - 9x - 9 > 0$ $(-3, -1) \cup (3, \infty)$

43. $x^4 - 20x^2 + 64 \ge 0$ $(-\infty, -4] \cup [-2, 2] \cup [4, \infty)$

44. $x^4 - 10x^2 + 9 \le 0$ $[-3, -1] \cup [1, 3]$

In Exercises 45 to 58, use the critical value method to solve each rational inequality. Write each solution set in interval notation.

45. $\frac{x + 4}{x - 1} < 0$ $(-4, 1)$

46. $\frac{x - 2}{x + 3} > 0$ $(-\infty, -3) \cup (2, \infty)$

47. $\frac{x - 5}{x + 8} \ge 3$ $\left[-\frac{29}{2}, -8\right)$

48. $\frac{x - 4}{x + 6} \le 1$ $(-6, \infty)$

49. $\frac{x}{2x + 7} \ge 4$ $\left[-4, -\frac{7}{2}\right)$

50. $\frac{x}{3x - 5} \le -5$ $\left[\frac{25}{16}, \frac{5}{3}\right)$

51. $\frac{(x + 1)(x - 4)}{x - 2} < 0$ $(-\infty, -1) \cup (2, 4)$

52. $\frac{x(x - 4)}{x + 5} > 0$ $(-5, 0) \cup (4, \infty)$

53. $\frac{x + 2}{x - 5} \le 2$ $(-\infty, 5) \cup [12, \infty)$

54. $\frac{3x + 1}{x - 2} \ge 4$ $(2, 9]$

55. $\frac{6x^2 - 11x - 10}{x} > 0$ $\left(-\frac{2}{3}, 0\right) \cup \left(\frac{5}{2}, \infty\right)$

56. $\frac{3x^2 - 2x - 8}{x - 1} \ge 0$ $\left[-\frac{4}{3}, 1\right) \cup [2, \infty)$

57. $\frac{x^2 - 6x + 9}{x - 5} \le 0$ $(-\infty, 5)$

58. $\frac{x^2 + 10x + 25}{x + 1} \ge 0$ $\{-5\} \cup (-1, \infty)$

59. **Personal Finance** A bank offers two checking account plans. The monthly fee and charge per check for each plan are shown in the following diagram. Under what conditions is it less expensive to use the LowCharge plan? If you write more than 57 checks a month

Account Plan	Monthly Fee	Charge per Check
LowCharge	$5.00	$.01
FeeSaver	$1.00	$.08

60. **Personal Finance** You can rent a car for the day from Company A for $29.00 plus $0.12 a mile. Company B charges $22.00 plus $0.21 a mile. Find the number of miles m (to the nearest mile) per day for which it is cheaper to rent from Company A. At least 78 mi

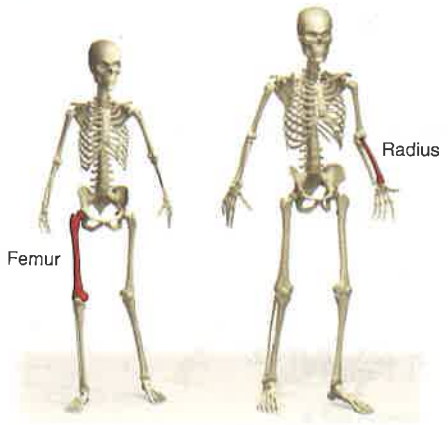
61. **Shipping Requirements** United Parcel Service (UPS) will ship only packages for which the length is less than or equal to 108 inches and the length plus the girth is less than or equal to 165 inches. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 47 inches and a width of 22 inches, determine the possible range of heights h for this package if you wish to ship it by UPS. (*Source:* http://www.ups.com.) $0 \text{ in.} < h \le 37 \text{ in.}$

62. **Movie Theater Attendance** The attendance A, in billions of people, at movie theaters has been declining since the year 2000. A model of the decline is given by $A = -0.03x + 1.59$, where $x = 0$ corresponds to 2000. According to this model, in what year will movie attendance first be less than 1 billion people? 2019

63. **Car Value** Based on data from the Kelley Blue Book website, the value V, in dollars, of a 2011 Corvette in excellent condition can be modeled by $V = -176.05m + 50{,}520$, where m is the number of miles, in thousands, on the odometer. Using this model, how many miles will a 2011 Corvette have if its value is \$41,000? Round to the nearest thousand miles. (*Source*: http://www.kbb.com) 54,000 mi

64. **Average Temperatures** The average daily minimum-to-maximum temperature range for the city of Palm Springs during the month of September is 68° to 104° Fahrenheit. What is the corresponding temperature range measured on the Celsius temperature scale? (*Hint:* Let F be the average daily temperature in degrees Fahrenheit. Then $68° \le F \le 104°$. Now substitute $\frac{9}{5}C + 32$ for F and solve the resulting inequality for C.) $20° \le C \le 40°$

65. **Forensic Science** Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. These lengths are substituted into mathematical inequalities. For instance, an inequality that relates the height h, in centimeters, of an adult female and the length f, in centimeters, of her femur is $|h - (2.47f + 54.10)| \le 3.72$. Use this inequality to estimate the possible range of heights, rounded to the nearest tenth of a centimeter, for an adult female whose femur measures 32.24 centimeters. 130.0 to 137.5 cm



66. **Forensic Science** An inequality that is used to calculate the height h of an adult male from the length r of his radius is

$$|h - (3.32r + 85.43)| \le 4.57$$

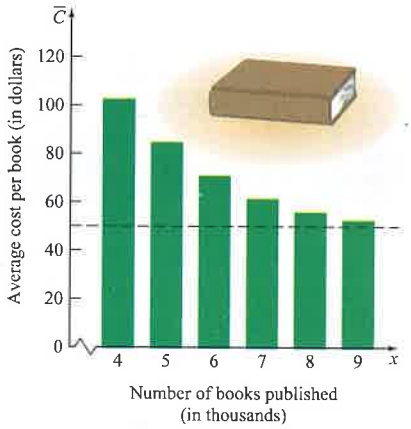
where h and r are both in centimeters. Use this inequality to estimate the possible range of heights, rounded to the nearest tenth of a centimeter, for an adult male whose radius measures 26.36 centimeters. 168.4 to 177.5 cm

67. **Revenue** The monthly revenue R for a product is given by $R = 420x - 2x^2$, where x is the price in dollars of each unit produced. Find the interval, in terms of x, for which the monthly revenue is greater than \$0. (\$0, \$210)

68. **Revenue** A shoe manufacturer finds that the monthly revenue R from a particular style of aerobics shoe is given by $R = 312x - 3x^2$, where x is the price in dollars of each pair of shoes sold. Find the interval, in terms of x, for which the monthly revenue is greater than or equal to \$5925. [\$25, \$79]

69. **Publishing** A publisher has determined that if x books are published, the average cost per book is given by

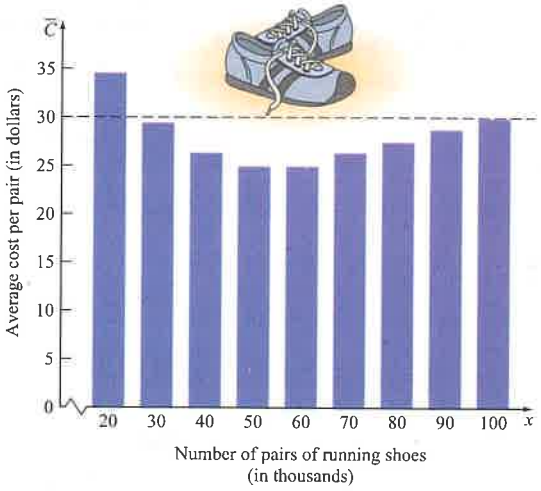
$$\overline{C} = \frac{14.25x + 350{,}000}{x}$$



How many books should be published if the company wants to bring the average cost per book below \$50? At least 9791 books

70. **Manufacturing** A company manufactures running shoes. The company has determined that if it manufactures x pairs of shoes, the average cost, in dollars, per pair is

$$\overline{C} = \frac{0.00014x^2 + 12x + 400{,}000}{x}$$

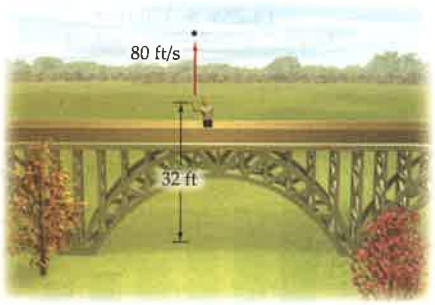


How many pairs of running shoes should the company manufacture if it wishes to bring the average cost below \$30 per pair? From 28,572 to 99,999 pairs of shoes

71. Height of a Projectile The equation

$$s = -16t^2 + v_0t + s_0$$

gives the height s, in feet above ground level, at the time t, in seconds, of an object thrown directly upward from a height s_0 feet above the ground and with an initial velocity of v_0 feet per second. A ball is thrown directly upward from a height of 32 feet above the ground with an initial velocity of 80 feet per second. Find the time interval during which the ball will be more than 96 feet above the ground. More than 1 s but less than 4 s



Enrichment Exercises

72. Mean Weight of Women If a researcher wanted to know the mean weight (the *mean* is the sum of all the measurements divided by the number of measurements) of women in the United States, the weight of every woman would have to be measured and then the mean weight calculated—an impossible task. Instead, researchers find a representative sample of women and find the mean weight of the sample. Because the entire population of women is not used, there is a possibility that the calculated mean weight is not the true mean weight. For one study, researchers used the formula $\left|\frac{163 - \mu}{1.79}\right| < 2.33$, where μ is the true mean weight, in pounds, of all women, to be 98% sure of the range of values for the true mean weight. Using this inequality, what is the range of mean weights of women in the United States? Round to the nearest tenth of a pound. (*Source*: Based on data from the National Center for Health Statistics.) 158.8 lb $< \mu <$ 167.2 lb

73. Mean Weight of Men If a researcher wanted to know the mean weight (see Exercise 72 for a definition of mean) of men in the United States, the weight of every man would have to be measured and then the mean weight calculated—an impossible task. Instead, researchers find a representative sample of men and find the mean weight of the sample. Because the entire population of men is not used, there is a possibility that the calculated mean weight is not the true mean weight. For one study, researchers used the formula $\left|\frac{190 - \mu}{2.45}\right| < 2.575$, where μ is the true mean weight, in pounds, of all men, to be 99% sure of the range of values for the true mean weight. Using this inequality, what is the range of mean weights of men in the United States? Round to the nearest tenth of a pound. (*Source*: Based on data from the National Center for Health Statistics.) 183.7 lb $< \mu <$ 196.3 lb

SECTION 1.6

Direct Variation
Inverse Variation
Joint and Combined Variations

Variation and Applications

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A6.

PS1. Solve $1820 = k(28)$ for k. [1.1] 65

PS2. Solve $20 = \frac{k}{1.5^2}$ for k. [1.1] 45

PS3. Evaluate $k \cdot \frac{3}{5^2}$ given that $k = 225$. [P.1] 27

PS4. Evaluate $k \cdot \frac{4.5 \cdot 32}{8^2}$ given that $k = 12.5$. [P.1] 28.125

PS5. If the length of each side of a square is doubled, what effect does this have on its area? [P.1/P.2] The area becomes 4 times as large.

PS6. If the radius of a cylinder is tripled, does this triple the volume of the cylinder? [P.1/P.2] No. The volume becomes 9 times as large.

Direct Variation

Many real-life situations involve variables that are related by a type of equation called a **variation**. For example, a stone thrown into a pond generates circular ripples whose circumferences and diameters increase in size. The equation $C = \pi d$ expresses the relationship between the circumference C of a circle and its diameter d. If d increases, then C increases. The circumference C is said to *vary directly* as the diameter d.

Definition of Direct Variation

The variable y **varies directly** as the variable x, or y is **directly proportional** to x, if and only if

$$y = kx$$

where k is a constant called the **constant of proportionality** or the **variation constant**.

Direct variation occurs in many daily applications. For example, suppose the cost of a newspaper is 50 cents. The cost C to purchase n newspapers is directly proportional to the number n. That is, $C = 50n$. In this example, the variation constant is 50.

To solve a problem that involves a variation, we typically write a general equation that relates the variables and then use given information to solve for the variation constant.

Alternative to Example 1
Some colleges are on the semester system, whereas other colleges use the quarter system. When a student transfers from one college to another, it is sometimes necessary to translate between the semester and the quarter systems. The number of semester hours completed is directly proportional to the number of quarter hours completed. If a student whose record shows 45 quarter hours is credited with completing 30 semester hours, how many semester hours credit will a student have after completing 240 quarter hours?

■ 160 semester hours

EXAMPLE 1 Solve a Direct Variation

The distance sound travels varies directly as the time it travels. If sound travels 1340 meters in 4 seconds, find the distance sound will travel in 5 seconds.

Solution

Write an equation that relates the distance d to the time t. Because d varies directly as t, our equation is $d = kt$. Because $d = 1340$ when $t = 4$, we obtain

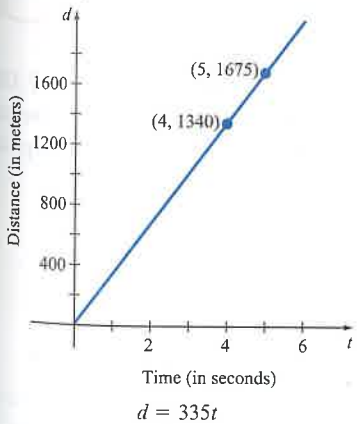
$$1340 = k \cdot 4 \qquad \text{which implies} \qquad k = \frac{1340}{4} = 335$$

Therefore, the specific equation that relates the d meters sound travels in t seconds is $d = 335t$. To find the distance sound travels in 5 seconds, replace t with 5 to produce

$$d = 335(5) = 1675$$

Under the same conditions, sound will travel 1675 meters in 5 seconds. See Figure 1.17.

▶ Try Exercise 22, page 140



$d = 335t$

Figure 1.17

Definition of Direct Variation as the *n*th Power

If y **varies directly as the *n*th power** of x, then

$$y = kx^n$$

where k is a constant.

Consequently, the specific formula for C is $C = \frac{5}{24}AT$. Now, when $A = 2400$ and $T = 6$, we have

$$C = \frac{5}{24}(2400)(6) = 3000$$

The cost of insulating the 2400-square-foot ceiling with 6-inch insulation is \$3000.

Try Exercise 34, page 141

Combined variations involve more than one type of variation.

Alternative to Example 6
The volume of a given mass of a gas varies directly as the temperature T and inversely as the pressure P. If the volume of the gas is 220 cubic centimeters when T is 40°C and P is 20 kilograms per square centimeter, what is the volume when T is 35°C and P is 10 kilograms per square centimeter?
■ 385 cm^3

EXAMPLE 6 Solve a Combined Variation

The weight that a horizontal beam with a rectangular cross section can safely support varies jointly as the width and the square of the depth of the cross section and inversely as the length of the beam. See Figure 1.21. If a 10-foot-long 4- by 4-inch beam safely supports a load of 256 pounds, what load L can be safely supported by a beam made of the same material and with a width w of 4 inches, a depth d of 6 inches, and a length l of 16 feet?

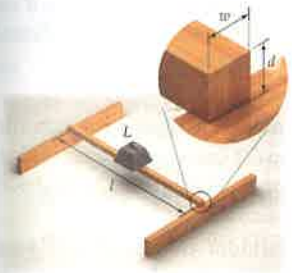


Figure 1.21

Solution

The general variation equation is $L = k \cdot \frac{wd^2}{l}$. Using the given data yields

$$256 = k \cdot \frac{4(4^2)}{10}$$

Solving for k produces $k = 40$, so the specific formula for L is

$$L = 40 \cdot \frac{wd^2}{l}$$

Substituting 4 for w, 6 for d, and 16 for l gives

$$L = 40 \cdot \frac{4(6^2)}{16} = 360 \text{ pounds}$$

Try Exercise 38, page 141

EXERCISE SET 1.6

Concept Check

In Exercises 1 to 12, write an equation that represents the relationship between the given variables. Use k as the variation constant.

1. d varies directly as t. $d = kt$
2. r varies directly as the square of s. $r = ks^2$
3. y varies inversely as x. $y = \frac{k}{x}$
4. p is inversely proportional to q. $p = \frac{k}{q}$
5. m varies jointly as n and p. $m = knp$
6. t varies jointly as r and the cube of s. $t = krs^3$
7. V varies jointly as l, w, and h. $V = klwh$
8. u varies directly as v and inversely as the square of w. $u = \frac{kv}{w^2}$
9. A is directly proportional to the square of s. $A = ks^2$
10. A varies jointly as h and the square of r. $A = khr^2$
11. F varies jointly as m_1 and m_2 and inversely as the square of d. $F = \frac{km_1m_2}{d^2}$
12. T varies jointly as t and r and the square of a. $T = ktra^2$

Indicates Try It Exercises

In Exercises 13 to 20, write the equation that expresses the relationship between the variables, and then use the given data to solve for the variation constant.

13. y varies directly as x, and $y = 64$ when $x = 48$. $y = kx, k = \frac{4}{3}$

14. m is directly proportional to n, and $m = 92$ when $n = 23$. $m = kn, k = 4$

15. r is directly proportional to the square of t, and $r = 144$ when $t = 108$. $r = kt^2, k = \frac{1}{81}$

16. C varies directly as r, and $C = 94.2$ when $r = 15$. $C = kr, k = 6.28$

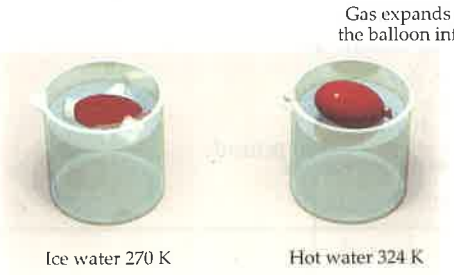
17. T varies jointly as r and the square of s, and $T = 210$ when $r = 30$ and $s = 5$. $T = krs^2, k = \frac{7}{25}$

18. u varies directly as v and inversely as the square root of w, and $u = 0.04$ when $v = 8$ and $w = 0.04$. $u = \frac{kv}{\sqrt{w}}, k = 0.001$

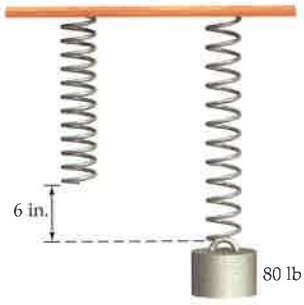
19. V varies jointly as l, w, and h, and $V = 240$ when $l = 8$, $w = 6$, and $h = 5$. $V = klwh, k = 1$

20. t varies directly as the cube of r and inversely as the square root of s, and $t = 10$ when $r = 5$ and $s = 0.09$. $t = \frac{kr^3}{\sqrt{s}}, k = 0.024$

21. Charles's Law Charles's Law states that the volume V occupied by a gas (at a constant pressure) is directly proportional to its absolute temperature T. An experiment with a balloon shows that the volume of the balloon is 0.85 liter at 270 K (absolute temperature).[1] What will the volume of the balloon be when its temperature is 324 K? 1.02 L



22. Hooke's Law Hooke's Law states that the distance a spring stretches varies directly as the weight on the spring. A weight of 80 pounds stretches a spring 6 inches. How far will a weight of 100 pounds stretch the spring? 7.5 in.



[1] Absolute temperature is measured on the Kelvin scale. A unit (called a kelvin) on the Kelvin scale is the same measure as a degree on the Celsius scale; however, 0 on the Kelvin scale corresponds to −273°C.

23. Semester Hours vs. Quarter Hours A student plans to transfer from a college that uses the quarter system to a college that uses the semester system. The number of semester hours a student receives credit for is directly proportional to the number of quarter hours the student has earned. A student with 51 quarter hours is given credit for 34 semester hours. How many semester hours credit should a student receive after completing 93 quarter hours? 62 semester h

24. Pressure and Depth The pressure a liquid exerts at a given point on a submarine is directly proportional to the depth of the point below the surface of the liquid. If the pressure at a depth of 3 feet is 187.5 pounds per square foot, find the pressure at a depth of 7 feet. 437.5 lb/ft^2

25. Amount of Juice Contained in a Grapefruit The amount of juice in a grapefruit is directly proportional to the cube of its diameter. A grapefruit with a 4-inch diameter contains 6 fluid ounces of juice. How much juice is contained in a grapefruit with a 5-inch diameter? Round to the nearest tenth of a fluid ounce. 11.7 fl oz

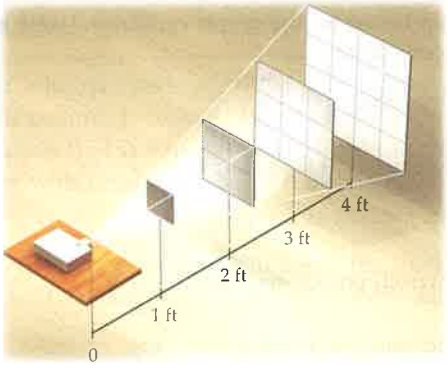
26. Motorcycle Jump The range of a projectile is directly proportional to the square of its velocity. If a motorcyclist can make a jump of 140 feet by coming off a ramp at 60 mph, find the distance the motorcyclist could expect to jump if the speed coming off the ramp were increased to 65 mph. Round to the nearest tenth of a foot. 164.3 ft

27. Period of a Pendulum The period T of a pendulum (the time it takes the pendulum to make one complete oscillation) varies directly as the square root of its length L. A pendulum 3 feet long has a period of 1.8 seconds.

a. Find the period of a pendulum that is 10 feet long. Round to the nearest tenth of a second. 3.3 s

b. What is the length of a pendulum that *beats seconds* (that is, has a 2-second period)? Round to the nearest tenth of a foot. 3.7 ft

28. Area of a Projected Picture The area of a projected picture on a movie screen varies directly as the square of the distance from the projector to the screen. If a distance of 20 feet produces a picture with an area of 64 square feet, what distance produces a picture with an area of 100 square feet? 25 ft



29. **Speed of a Bicycle Gear** The speed of a bicycle gear, in revolutions per minute, is inversely proportional to the number of teeth on the gear. If a gear with 64 teeth has a speed of 30 revolutions per minute, what will be the speed of a gear with 48 teeth? 40 revolutions/min

30. **Vibration of a Guitar String** The frequency of vibration of a guitar string under constant tension varies inversely as the length of the string. A guitar string with a length of 20 inches has a frequency of 144 vibrations per second. Find the frequency of a guitar string with a length of 18 inches. Assume the tension is the same for both strings. 160 vibrations/s

31. **Jet Engine Noise** The sound intensity of a jet engine, measured in watts per meter squared (W/m^2), is inversely proportional to the square of the distance between the engine and an airport ramp worker. For a certain jet, the sound intensity measures 0.5 W/m^2 at a distance of 7 meters from the ramp worker. What is the sound intensity for a ramp worker 10 meters from the jet? 0.245 W/m^2

32. **Illumination** The illumination a source of light provides is inversely proportional to the square of the distance from the source. If the illumination at a distance of 10 feet from the source is 50 footcandles, what is the illumination at a distance of 15 feet from the source? Round to the nearest tenth of a footcandle. 22.2 footcandles

33. **Volume Relationships** The volume V of a right circular cone varies jointly as the square of the radius r and the height h. Tell what happens to V when

a. r is tripled V is 9 times as large.

b. h is tripled V is 3 times as large.

c. both r and h are tripled V is 27 times as large.

34. **Safe Load** The load L that a horizontal beam can safely support varies jointly as the width w and the square of the depth d. If a beam with a width of 2 inches and a depth of 6 inches safely supports up to 200 pounds, how many pounds can a beam of the same length that has width 4 inches and depth 4 inches be expected to support? Round to the nearest pound. Assume the two beams are made of the same material. 178 lb

35. **Ideal Gas Law** The Ideal Gas Law states that the volume V of a gas varies jointly as the number of moles of gas n and the absolute temperature T and inversely as the pressure P. What happens to V when n is tripled and P is reduced by a factor of one-half? V is 6 times as large.

36. **Maximum Load** The maximum load a cylindrical column of circular cross section can support varies directly as the fourth power of the diameter and inversely as the square of the height. If a column 2 feet in diameter and 10 feet high supports up to 6 tons, how many tons can a column 3 feet in diameter and 14 feet high support? Round to the nearest tenth of a ton. Assume the two columns are made of the same material. 15.5 tons

37. **Earned Run Average** A pitcher's earned run average (ERA) is directly proportional to the number of earned runs the pitcher has allowed and is inversely proportional to the number of innings pitched. During the 2011 season, Clayton Kershaw of the Los Angeles Dodgers had an ERA of 2.28. He allowed 59 earned runs in 233.1 innings.

a. Determine the variation constant used to compute Kershaw's ERA. Round to the nearest tenth 9.0.

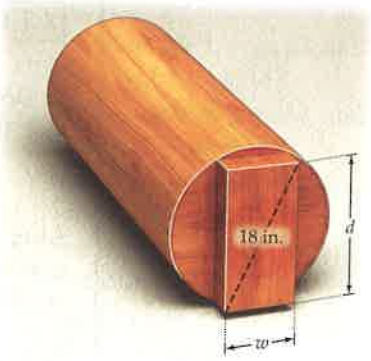
During the same season, C. J. Wilson of the Los Angeles Angels allowed 73 earned runs in 223.1 innings.

b. Use the variation constant from part a, to determine the ERA for C. J. Wilson. Round to the nearest hundredth. 2.94

38. **Safe Load** The load L a horizontal beam can safely support varies jointly as the width w and the square of the depth d and inversely as the length l. If a 12-foot beam with a width of 4 inches and a depth of 8 inches safely supports 800 pounds, how many pounds can a 16-foot beam that has a width of 3.5 inches and a depth of 6 inches be expected to support? Round to the nearest pound. Assume the two beams are made of the same material. 295 lb

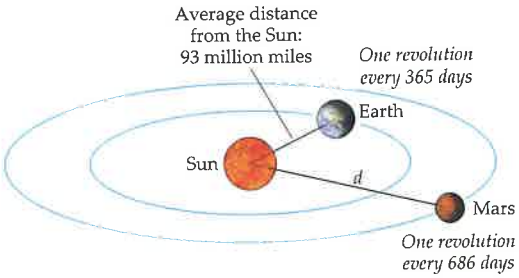
39. **Force, Speed, and Radius Relationships** The force needed to keep a car from skidding on a curve varies jointly as the weight of the car and the square of its speed and inversely as the radius of the curve. It takes 2800 pounds of force to keep an 1800-pound car from skidding on a curve with a radius of 425 feet at 45 mph. What force is needed to keep the same car from skidding when it takes a similar curve with a radius of 450 feet at 55 mph? Round to the nearest 10 pounds. 3950 lb

40. **Stiffness of a Beam** A cylindrical log is to be cut so that it will yield a beam that has a rectangular cross section of depth d and width w. The stiffness of a beam of given length is directly proportional to the width and the cube of the depth. The diameter of the log is 18 inches. What depth will yield the "stiffest" beam: $d = 10$ inches, $d = 12$ inches, $d = 14$ inches, or $d = 16$ inches? $d = 16$ in.



Enrichment Exercises

41. Kepler's Third Law Kepler's Third Law states that the square of the time t needed for a planet to make one complete revolution about the Sun is directly proportional to the cube of the average distance d between the planet and the Sun. The Earth, which averages 93 million miles from the Sun, completes one revolution in 365 days. Find the average distance from the Sun to Mars if Mars completes one revolution about the Sun in 686 days. Round to the nearest million miles. 142 million mi



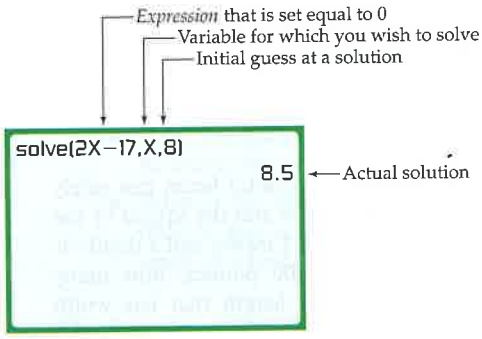
42. Newton's Law of Gravitation Newton's Law of Gravitation states that the force F, in newtons, between two masses m_1 and m_2 is directly proportional to the product of the masses and inversely proportional to the square of the distance r between the centers of the two masses. The force between Earth (mass $\approx 6 \times 10^{24}$ kg and radius ≈ 6400 km) and a 1-kg mass on the surface of Earth is 9.8 newtons. What is the force between Earth and a 1-kg mass that is 7200 km (the distance of an object in near-Earth orbit) from the center of Earth? Round to the nearest tenth. 7.7 newtons

Exploring Concepts with Technology

Use a Graphing Calculator to Solve Equations

Most graphing calculators can be used to solve equations. The following example shows how to solve an equation using the **solve(** feature that is available on a TI-83/TI-83 Plus/TI-84 Plus graphing calculator.

The calculator display that follows indicates that the solution of $2x - 17 = 0$ is 8.5.



The calculator display above was produced by the following keystrokes. Press [2nd] [catalog] S (scroll down to **solve(**) [ENTER]. Now enter 2 [X,T,Θ,n] [−] 17 [,] [X,T,Θ,n] [,] 8 [)] [ENTER]. In this example, the 8.5 represents the solution of $2x - 17 = 0$, which is close to our initial guess of 8. Because $2x - 17 = 0$ has only one solution, we are finished. Note that the **solve(** feature can be used only to solve equations of the form

$$\text{Expression} = 0$$

Also, you are required to indicate the variable you wish to solve for, and you must enter an initial guess. In the preceding display, we entered X as the variable and 8 as our initial guess.

The **solve(** feature can be used only to find *real* solutions. Also, the **solve(** feature finds only one solution each time the solution procedure is