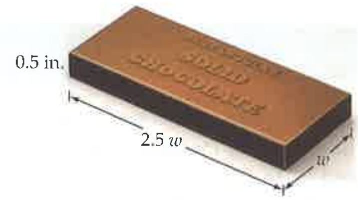
93. **Daredevil Motorcycle Jump** In March 2000, Doug Danger made a successful motorcycle jump over an L-1011 jumbo jet. The horizontal distance of his jump was 160 feet, and his height, in feet, during the jump was approximated by $h = -16t^2 + 25.3t + 20$, $t \geq 0$. He left the takeoff ramp at a height of 20 feet, and he landed on the landing ramp at a height of about 17 feet. How long, to the nearest tenth of a second, was he in the air?

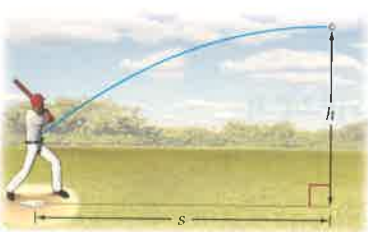
94. **Dimensions of a Candy Bar** A company makes rectangular solid candy bars that measure 5 inches by 2 inches by 0.5 inch. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should be the dimensions, to the nearest tenth of an inch, of the new candy bar if the company decides to keep the height at 0.5 inch and to make the length of the new candy bar 2.5 times as long as its width?



95. **Height of a Rocket** A model rocket is launched upward with an initial velocity of 220 feet per second. The height, in feet, of the rocket t seconds after the launch is given by the equation $h = -16t^2 + 220t$. How many seconds after the launch will the rocket be 350 feet above the ground? Round to the nearest tenth of a second.

96. **Baseball** The height h, in feet, of a baseball above the ground t seconds after it is hit is given by $h = -16t^2 + 52t + 4.5$. Use this equation to determine the number of seconds, to the nearest tenth of a second, from the time the ball is hit until the ball touches the ground.

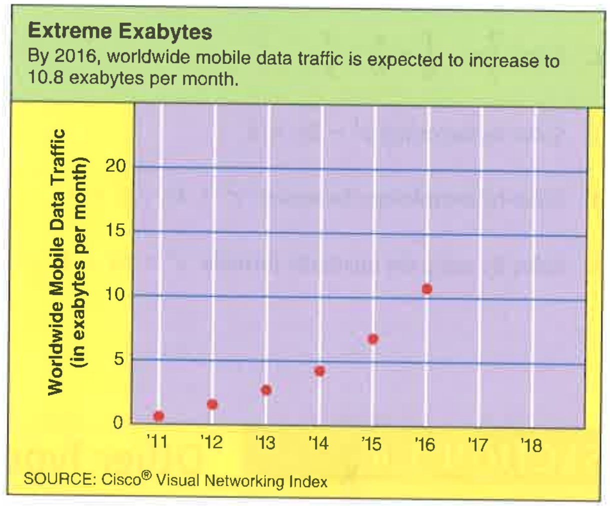
97. **Baseball** Two equations can be used to track the position of a baseball t seconds after it is hit. For instance, suppose $h = -16t^2 + 50t + 4.5$ gives the height, in feet, of a baseball t seconds after it is hit and $s = 103.9t$ gives the horizontal distance, in feet, of the ball from home plate t seconds after it is hit. (See the following figure.) Use these equations to determine whether this particular baseball will clear a 10-foot fence positioned 360 feet from home plate.



98. **Basketball** Michael Jordan was known for his hang time, which is the amount of time a player is in the air when making a jump toward the basket. An equation that approximates the height h, in inches, of one of Jordan's jumps is given by $h = -16t^2 + 26.6t$, where t is time in seconds. Use this equation to determine Michael Jordan's hang time, to the nearest tenth of a second, for this jump.

99. **Orbital Debris** The amount of space debris is increasing. The total mass M, in millions of kilograms, of objects in Earth orbit can be modeled by $M = 0.0001t^2 + 0.16t + 4.34$, where $t = 0$ corresponds to the year 2000. (*Source:* http://orbitaldebris.jsc.nasa.gov/) According to this model, in what year will the total mass of space debris objects first exceed 10 million kilograms?

100. **Mobile Data** The projected worldwide mobile data traffic D, in exabytes per month, can be modeled by $D = 0.40x^2 - 8.81x + 49.25, 11 \leq x \leq 16$, where $x = 11$ corresponds to the year 2011. (*Source:* www.cisco.com) Suppose this trend continues beyond the year 2016. Use the model to predict the first year in which worldwide mobile data traffic will always be greater than 25 exabytes per month. *Note:* 1 exabyte $= 10^{18}$ bytes, or 1 billion gigabytes.



101. **Centenarians** According to data provided by the U.S. Census Bureau, the number N, in thousands, of centenarians (a person whose age is 100 years or older) who will be living in the United States during a year from 2010 to 2050 can be approximated by $N = 0.3453x^2 - 9.417x + 164.1$, where x is the number of years after the beginning of 2000. Use this equation to determine in what year will there be 200,000 centenarians living in the United States.

102. **Automotive Engineering** The number N of feet that a car needs to stop on a certain road surface is given by the equation $N = -0.015v^2 + 3v$, $0 \leq v \leq 90$, where v is the speed of the car in miles per hour when the driver applies the brakes. What is the maximum speed, to the nearest mile per hour, that a motorist can be traveling and stop the car within 100 feet?

Enrichment Exercises

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ was given as $b^2 - 4ac$. We will call this the *coefficient* definition. There is, however, a more general definition of the discriminant that can be extended to polynomial equations of any degree. Let r_1 and r_2 be the solutions of the quadratic equation $ax^2 + bx + c = 0$. Then the discriminant of the equation is given by $a^2(r_1 - r_2)^2$. That is, the discriminant is the product of the square of the leading coefficient and the square of the difference between the roots. We will call this the *roots* definition of the discriminant.

103. Verify that the coefficient definition and the roots definition give the same value for the discriminant of each of the following equations.

a. $x^2 - x - 6 = 0$

b. $9x^2 - 6x - 1 = 0$

c. $x^2 + 4x + 4 = 0$

104. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $r_1 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $r_2 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$. Show that $a^2(r_1 - r_2)^2 = b^2 - 4ac$.

105. Let r_1, r_2, and r_3 be the solutions of the cubic equation $ax^3 + bx^2 + cx + d = 0$. Then the discriminant of the cubic equation is $a^4(r_1 - r_2)^2(r_1 - r_3)^2(r_2 - r_3)^2$. Find the discriminant of $x^3 - 4x^2 - 4x + 16 = 0$.

106. Suppose the discriminant of a cubic equation is 0. What can be said about the roots of that equation?

MID-CHAPTER 1 QUIZ

1. Solve: $6 - 4(2x + 1) = 5(3 - 2x)$

2. Solve: $\dfrac{2}{3}x - \dfrac{1}{4} = \dfrac{x}{6} + \dfrac{3}{2}$

3. Solve by factoring: $x^2 - 5x = 6$

4. Solve by completing the square: $x^2 + 4x - 2 = 0$

5. Solve by using the quadratic formula: $x^2 - 6x + 12 = 0$

6. A runner runs a course at a constant speed of 8 miles per hour. One hour later, a cyclist begins the same course at a constant speed of 16 miles per hour. How long after the runner starts does the cyclist overtake the runner?

7. A pharmacist mixes a 9% acetic acid solution with a 4% acetic solution. How many milliliters of each solution should the pharmacist use to make a 500-milliliter solution that is 6% acetic acid?

8. A mason can complete a wall in 10 hours, but an apprentice mason requires 15 hours to do the same job. How long will it take to build the wall with both people working?

SECTION 1.4

Polynomial Equations
Rational Equations
Radical Equations
Rational Exponent Equations
Equations That Are Quadratic in Form
Applications of Other Types of Equations

Other Types of Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A5.

PS1. Factor: $x^3 - 16x$ [P.4]

PS2. Factor: $x^4 - 36x^2$ [P.4]

PS3. Evaluate: $8^{2/3}$ [P.2]

PS4. Evaluate: $16^{3/2}$ [P.2]

PS5. Multiply: $(1 + \sqrt{x - 5})^2$, $x > 5$ [P.2/P.3]

PS6. Multiply: $(2 - \sqrt{x + 3})^2$, $x > -3$ [P.2/P.3]

Polynomial Equations

Some polynomial equations that are neither linear nor quadratic can be solved by the various techniques presented in this section. For instance, the **third-degree equation,** or **cubic equation,** in Example 1 can be solved by factoring the polynomial and using the zero product principle.

EXAMPLE 1 Solve a Polynomial Equation

Solve: $x^3 + 3x^2 - 4x - 12 = 0$

Solution

$$x^3 + 3x^2 - 4x - 12 = 0$$

$$(x^3 + 3x^2) - (4x + 12) = 0 \qquad \text{• Factor by grouping.}$$

$$x^2(x + 3) - 4(x + 3) = 0$$

$$(x + 3)(x^2 - 4) = 0$$

$$(x + 3)(x + 2)(x - 2) = 0 \qquad \text{• Use the zero product principle.}$$

$$x + 3 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -3 \qquad\qquad x = -2 \qquad\qquad x = 2$$

The solutions are −3, −2, and 2.

▶ **Try Exercise 14, page 118**

Rational Equations

TO REVIEW

Rational Expressions. See page 50.

A **rational equation** is one that involves rational expressions. The following two equations are rational equations.

$$\frac{2x}{x + 3} - 5 = \frac{x + 4}{x - 1} \qquad\qquad \frac{x + 1}{x^2 - 1} + \frac{x}{2x - 3} = \frac{4}{x - 1}$$

When solving a rational equation, be aware of the domain of the equation, which is the intersection of the domains of the rational expressions. For the first equation above, −3 and 1 are excluded as possible values of x and are not in the domain. For the second equation, −1, 1, and $\frac{3}{2}$ are excluded as possible values of x and are not in the domain.

The domain is important, as shown by trying to solve $\frac{9}{x - 3} + 2 = \frac{3x}{x - 3}$. We begin by noting that 3 is not in the domain of the rational expressions and then multiplying each side of the equation by $x - 3$.

$$\frac{9}{x - 3} + 2 = \frac{3x}{x - 3} \qquad \text{• 3 is not in the domain.}$$

$$(x - 3)\left(\frac{9}{x - 3} + 2\right) = (x - 3)\left(\frac{3x}{x - 3}\right) \qquad \text{• Multiply each side by } x - 3\text{, the LCD of the denominators.}$$

$$9 + 2(x - 3) = 3x \qquad \text{• Solve for } x.$$

$$9 + 2x - 6 = 3x$$

$$3 = x$$

However, the proposed solution, 3, is not in the domain, and replacing x with 3 in the original equation would require division by 0, which is not defined. Therefore, the equation has no solution.

Note

Just because an equation can be written does not mean that there is a solution, as the equation at the right illustrates. Recall from Section 1.1 that equations with no solution are called contradictions. A simple example of a contradiction is $x = x + 1$. This equation has no solution.

EXAMPLE 2 Solve a Rational Equation

Solve.

a. $\frac{2x + 1}{x + 4} + 3 = \frac{-2}{x + 4}$ **b.** $3x + \frac{4}{x - 2} = \frac{-4x + 12}{x - 2}$

c. $\frac{2x}{x - 3} + \frac{x + 1}{x + 4} = \frac{x - 1}{x - 3}$

(continued)

Solution

a.
$$\frac{2x+1}{x+4}+3=\frac{-2}{x+4}$$
• -4 is not in the domain.

$$(x+4)\left(\frac{2x+1}{x+4}+3\right)=(x+4)\left(\frac{-2}{x+4}\right)$$
• Multiply each side by $x+4$, the LCD of the denominators.

$$(2x+1)+3(x+4)=-2$$
• Solve for x.

$$5x+13=-2$$
$$5x=-15$$
$$x=-3$$

-3 checks as a solution. The solution is -3.

b.
$$3x+\frac{4}{x-2}=\frac{-4x+12}{x-2}$$
• 2 is not in the domain.

$$(x-2)\left(3x+\frac{4}{x-2}\right)=(x-2)\left(\frac{-4x+12}{x-2}\right)$$
• Multiply each side by $x-2$.

$$3x(x-2)+4=-4x+12$$
• Solve for x.

$$3x^2-6x+4=-4x+12$$
$$3x^2-2x-8=0$$
$$(3x+4)(x-2)=0$$
• Factor and use the zero product principle.

$$3x+4=0 \quad \text{or} \quad x-2=0$$
$$x=-\frac{4}{3} \qquad\qquad x=2$$

$-\frac{4}{3}$ checks as a solution; 2 is not in the domain and does not check as a solution. The solution is $-\frac{4}{3}$.

c.
$$\frac{2x}{x-3}+\frac{x+1}{x+4}=\frac{x-1}{x-3}$$
• 3 and -4 are not in the domain.

$$(x-3)(x+4)\left(\frac{2x}{x-3}+\frac{x+1}{x+4}\right)=(x-3)(x+4)\left(\frac{x-1}{x-3}\right)$$
• Multiply each side by the LCD of the denominators.

$$2x(x+4)+(x+1)(x-3)=(x+4)(x-1)$$
• Simplify.

$$2x^2+8x+x^2-2x-3=x^2+3x-4$$
$$2x^2+3x+1=0$$
$$(2x+1)(x+1)=0$$
• Factor and use the zero product principle.

$$2x+1=0 \quad \text{or} \quad x+1=0$$
$$x=-\frac{1}{2} \qquad\qquad x=-1$$

$-\frac{1}{2}$ and -1 check as solutions. The solutions are -1 and $-\frac{1}{2}$.

▸ **Try Exercise 28, page 118**

Radical Equations

Some equations that involve radical expressions can be solved by using the following principle.

The Power Principle

If P and Q are algebraic expressions and n is a positive integer, then every solution of $P = Q$ is a solution of $P^n = Q^n$.

EXAMPLE 3 Solve a Radical Equation

Solve: $2\sqrt{3x-2} = x + 1$

Solution

$$2\sqrt{3x-2} = x + 1$$

$$(2\sqrt{3x-2})^2 = (x+1)^2$$ • Square each side of the equation.

$$4(3x-2) = x^2 + 2x + 1$$ • Simplify.

$$12x - 8 = x^2 + 2x + 1$$

$$0 = x^2 - 10x + 9$$ • Write the quadratic equation in standard form.

$$0 = (x-1)(x-9)$$ • Factor and use the zero product principle.

$$x - 1 = 0 \qquad x - 9 = 0$$

$$x = 1 \qquad x = 9$$

Check:

$$2\sqrt{3x-2} = x + 1 \qquad 2\sqrt{3x-2} = x + 1$$

$$2\sqrt{3(1)-2} \stackrel{?}{=} 1 + 1 \qquad 2\sqrt{3(9)-2} \stackrel{?}{=} 9 + 1$$

$$2\sqrt{1} \stackrel{?}{=} 1 + 1 \qquad 2\sqrt{25} \stackrel{?}{=} 9 + 1$$

$$2 = 2 \qquad 10 = 10$$

The solutions check. The solutions are 1 and 9.

Try Exercise 34, page 118

Some care must be taken when using the power principle because the equation $P^n = Q^n$ may have more solutions than the original equation $P = Q$. As an example, consider $x = 3$. The only solution is the real number 3. Square each side of the equation to produce $x^2 = 9$, and you get both -3 and 3 as solutions. The -3 is called an *extraneous solution* because it is not a solution of the original equation $x = 3$.

Definition of an Extraneous Solution

Any solution of $P^n = Q^n$ that is not a solution of $P = Q$ is called an **extraneous solution**. Extraneous solutions *may* be introduced whenever each side of an equation is raised to an *even* power.

EXAMPLE 4 Solve Radical Equations

Solve: $\sqrt{x+1} - \sqrt{2x-5} = 1$

Solution

$$\sqrt{x+1} - \sqrt{2x-5} = 1$$

$$\sqrt{x+1} = 1 + \sqrt{2x-5}$$ • Isolate one of the radicals.

(*continued*)

$$(\sqrt{x+1})^2 = (1 + \sqrt{2x-5})^2$$ • Square each side.

$$x + 1 = 1 + 2\sqrt{2x-5} + (2x - 5)$$ • There is still a radical expression. Isolate the remaining radical.

$$-x + 5 = 2\sqrt{2x-5}$$

$$(-x+5)^2 = (2\sqrt{2x-5})^2$$ • Square each side.

$$x^2 - 10x + 25 = 4(2x - 5)$$

$$x^2 - 10x + 25 = 8x - 20$$

$$x^2 - 18x + 45 = 0$$ • Write the equation in standard form.

$$(x - 3)(x - 15) = 0$$ • Factor.

$$x - 3 = 0 \quad \text{or} \quad x - 15 = 0$$

$$x = 3 \qquad\qquad x = 15$$

Check:

$$\sqrt{x+1} - \sqrt{2x-5} = 1 \qquad\qquad \sqrt{x+1} - \sqrt{2x-5} = 1$$

$$\sqrt{3+1} - \sqrt{2(3)-5} \stackrel{?}{=} 1 \qquad\qquad \sqrt{15+1} - \sqrt{2(15)-5} \stackrel{?}{=} 1$$

$$\sqrt{4} - \sqrt{1} \stackrel{?}{=} 1 \qquad\qquad \sqrt{16} - \sqrt{25} \stackrel{?}{=} 1$$

$$2 - 1 = 1 \qquad\qquad 4 - 5 \neq 1$$

3 checks as a solution, but 15 does not. The solution is 3.

Try Exercise 38, page 118

Note

In the check at the right, 15 is an example of an *extraneous solution*. Squaring both sides of the equation created the extraneous solution.

Rational Exponent Equations

TO REVIEW

Definition of $\sqrt[n]{b^n}$
See page 24.

Recall that $\sqrt[n]{b^n} = |b|$ when n is a positive even integer and $\sqrt[n]{b^n} = b$ (the absolute value sign is not necessary) when n is a positive odd integer. These results can be restated using rational exponents.

$$(b^n)^{1/n} = |b|, \; n \text{ is a positive even integer}$$

$$(b^n)^{1/n} = b, \; n \text{ is a positive odd integer}$$

For instance, $(x^2)^{1/2} = |x|$ (n is an even integer) and $(x^3)^{1/3} = x$ (n is an odd integer). It is important to remember this when solving equations that involve a variable with a rational exponent. Here is an example that shows the details.

$$x^{2/3} = 16$$

$$(x^2)^{1/3} = 16$$ • Rewrite $x^{2/3}$ as $(x^2)^{1/3}$.

$$[(x^2)^{1/3}]^3 = 16^3$$ • Cube each side of the equation.

$$x^2 = 16^3$$

$$(x^2)^{1/2} = (16^3)^{1/2}$$ • To take the square root, raise each side of the equation to the 1/2 power.

$$|x| = (16^3)^{1/2}$$ • $(x^2)^{1/2} = |x|$

$$|x| = 4096^{1/2}$$

$$x = \pm 64$$ • Use the fact that if $|x| = a$ ($a > 0$), then $x = \pm a$.

Here is a check.

$$x^{2/3} = 16 \qquad\qquad x^{2/3} = 16$$

$$(-64)^{2/3} \stackrel{?}{=} 16 \quad \text{• Replace } x \text{ with } -64. \qquad (64)^{2/3} \stackrel{?}{=} 16 \quad \text{• Replace } x \text{ with } 64.$$

$$[(-64)^{1/3}]^2 \stackrel{?}{=} 16 \qquad\qquad [(64)^{1/3}]^2 \stackrel{?}{=} 16$$

$[-4]^2 \stackrel{?}{=} 16$ $\qquad$ $[4]^2 \stackrel{?}{=} 16$

$16 = 16$ • The solution checks. $\qquad$ $16 = 16$ • The solution checks.

The solutions are -64 and 64.

Although we could use this procedure every time we solve an equation containing a variable with a rational exponent, we will rely on a shortcut that recognizes the need for the absolute value symbol when the numerator of the rational exponent is an even integer. Here is the solution of $x^{2/3} = 16$, using this shortcut.

$x^{2/3} = 16$

$(x^{2/3})^{3/2} = 16^{3/2}$ • Raise each side of the equation to the 3/2 (the reciprocal of 2/3) power.

$|x| = 64$ • Because the numerator in the exponent of $x^{2/3}$ is an even number, the absolute value sign is necessary.

$x = \pm 64$

The solutions are -64 and 64.

Now consider $x^{3/4} = 8$. We solve this equation as

$x^{3/4} = 8$

$(x^{3/4})^{4/3} = 8^{4/3}$ • Raise each side of the equation to the 4/3 (the reciprocal of 3/4) power.

$x = 16$ • Because the numerator in the exponent of $x^{3/4}$ is an odd number, the absolute value sign is not necessary.

The solution is 16.

EXAMPLE 5 Solve an Equation That Involves a Variable with a Rational Exponent

Solve.

a. $2x^{4/5} - 47 = 115$ $\qquad$ **b.** $5x^{3/4} + 4 = 44$

Solution

a. $2x^{4/5} - 47 = 115$

$2x^{4/5} = 162$ • Add 47 to each side.

$x^{4/5} = 81$ • Divide each side by 2.

$(x^{4/5})^{5/4} = 81^{5/4}$ • Raise each side of the equation to the 5/4 (the reciprocal of 4/5) power.

$|x| = 243$ • Because the numerator in the exponent of $x^{4/5}$ is an even number, use absolute value.

$x = \pm 243$

The solutions are -243 and 243.

b. $5x^{3/4} + 4 = 44$

$5x^{3/4} = 40$ • Subtract 4 from each side.

$x^{3/4} = 8$ • Divide each side by 5.

$(x^{3/4})^{4/3} = 8^{4/3}$ • Raise each side of the equation to the 4/3 (the reciprocal of 3/4) power.

$x = 16$ • Because the numerator in the exponent of $x^{3/4}$ is an odd number, do not use absolute value.

Substituting 16 into $5x^{3/4} + 4 = 44$, we can verify that the solution is 16.

▸ Try Exercise 52, page 119

Equations That Are Quadratic in Form

The equation $4x^4 - 25x^2 + 36 = 0$ is said to be **quadratic in form,** which means that it can be written in the form

$$au^2 + bu + c = 0, \quad a \neq 0$$

where u is an algebraic expression involving x. For example, if we make the substitution $u = x^2$ (which implies that $u^2 = x^4$), then our original equation can be written as

$$4u^2 - 25u + 36 = 0$$

This quadratic equation can be solved for u, and then, using the relationship $u = x^2$, we can find the solutions of the original equation.

EXAMPLE 6 Solve an Equation That Is Quadratic in Form

Solve: $4x^4 - 25x^2 + 36 = 0$

Solution

Make the substitutions $u = x^2$ and $u^2 = x^4$ to produce the quadratic equation $4u^2 - 25u + 36 = 0$. Factor the quadratic polynomial on the left side of the equation.

$$(4u - 9)(u - 4) = 0$$

$$4u - 9 = 0 \quad \text{or} \quad u - 4 = 0$$

$$u = \frac{9}{4} \qquad\qquad u = 4$$

Substitute x^2 for u to produce

$$x^2 = \frac{9}{4} \quad \text{or} \quad x^2 = 4$$

$$x = \pm\sqrt{\frac{9}{4}} \qquad x = \pm\sqrt{4}$$

$$x = \pm\frac{3}{2} \qquad x = \pm 2$$

• Check in the original equation.

The solutions are $-2, -\frac{3}{2}, \frac{3}{2}$, and 2.

Try Exercise 56, page 119

The following table shows equations that are quadratic in form. Each equation is accompanied by an appropriate substitution that will enable it to be written in the form $au^2 + bu + c = 0$.

Equations That Are Quadratic in Form

Original Equation	Substitution	$au^2 + bu + c = 0$ Form
$x^4 - 8x^2 + 15 = 0$	$u = x^2$	$u^2 - 8u + 15 = 0$
$x^6 + x^3 - 12 = 0$	$u = x^3$	$u^2 + u - 12 = 0$
$x^{1/2} - 9x^{1/4} + 20 = 0$	$u = x^{1/4}$	$u^2 - 9u + 20 = 0$
$2x^{2/3} + 7x^{1/3} - 4 = 0$	$u = x^{1/3}$	$2u^2 + 7u - 4 = 0$
$15x^{-2} + 7x^{-1} - 2 = 0$	$u = x^{-1}$	$15u^2 + 7u - 2 = 0$

EXAMPLE 7 Solve an Equation That Is Quadratic in Form

Solve: $3x^{2/3} - 5x^{1/3} - 2 = 0$

Solution

Substituting u for $x^{1/3}$ gives

$$3u^2 - 5u - 2 = 0$$

$$(3u + 1)(u - 2) = 0 \qquad \text{• Factor.}$$

$$3u + 1 = 0 \quad \text{or} \quad u - 2 = 0$$

$$u = -\frac{1}{3} \qquad u = 2$$

$$x^{1/3} = -\frac{1}{3} \qquad x^{1/3} = 2 \qquad \text{• Replace } u \text{ with } x^{1/3}.$$

$$x = -\frac{1}{27} \qquad x = 8 \qquad \text{• Cube each side.}$$

A check will verify that both $-\frac{1}{27}$ and 8 are solutions.

Try Exercise 64, page 119

It is possible to solve equations that are quadratic in form without making a formal substitution. For example, to solve $x^4 + 5x^2 - 36 = 0$, factor the equation and apply the zero product principle.

$$x^4 + 5x^2 - 36 = 0$$

$$(x^2 + 9)(x^2 - 4) = 0$$

$$x^2 + 9 = 0 \quad \text{or} \quad x^2 - 4 = 0$$

$$x^2 = -9 \qquad x^2 = 4$$

$$x = \pm 3i \qquad x = \pm 2$$

The solutions are -2, 2, $-3i$, and $3i$.

Applications of Other Types of Equations

EXAMPLE 8 Solve a Uniform Motion Problem

Two buses are transporting a football team to a game that is 120 miles away. The second bus travels at an average speed that is 10 mph faster than the first bus and arrives 1 hour sooner than the first bus. Find the average speed of each bus.

Solution

Let r be the rate of the first bus. Then $r + 10$ is the rate of the second bus. Solving the uniform motion equation $d = rt$ for time gives $t = \frac{d}{r}$. Thus

$$\text{Time for first bus} = \frac{\text{Distance}}{\text{Rate of first bus}} = \frac{120}{r}$$

(continued)

$$\text{Time for second bus} = \frac{\text{Distance}}{\text{Rate of second bus}} = \frac{120}{r + 10}$$

$$\text{Time for second bus} = \text{Time for first bus} - 1$$

$$\frac{120}{r+10} = \frac{120}{r} - 1$$

$$r(r + 10)\left(\frac{120}{r + 10}\right) = r(r + 10)\left(\frac{120}{r} - 1\right)$$ • Multiply each side by the LCD $r(r + 10)$.

$$120r = (r + 10) \cdot 120 - r(r + 10)$$

$$120r = 120r + 1200 - r^2 - 10r$$

$$r^2 + 10r - 1200 = 0$$ • Write the quadratic equation in standard form.

$$(r + 40)(r - 30) = 0$$ • Factor.

Applying the zero product principle, $r = -40$ or $r = 30$. A negative average speed is not possible. The rate of the first bus is 30 miles per hour. The rate of the second bus is 40 miles per hour.

▶ **Try Exercise 72, page 119**

EXAMPLE 9 Solve a Work Problem

A small pipe takes 12 minutes longer than a larger pipe to empty a tank. Working together, they can empty the tank in 1.75 minutes. How long would it take the smaller pipe to empty the tank if the larger pipe is closed?

Solution

Let t be the time it takes the smaller pipe to empty the tank. Then $t - 12$ is the time for the larger pipe to empty the tank. Both pipes are open for 1.75 minutes. Therefore, $\frac{1.75}{t}$ is the portion of the tank emptied by the smaller pipe and $\frac{1.75}{t - 12}$ is the portion of the tank emptied by the larger pipe. Working together, they empty one tank. Thus $\frac{1.75}{t} + \frac{1.75}{t - 12} = 1$. Solve this equation for t.

$$\frac{1.75}{t} + \frac{1.75}{t - 12} = 1$$

$$t(t - 12)\left(\frac{1.75}{t} + \frac{1.75}{t - 12}\right) = t(t - 12) \cdot 1$$ • Multiply each side by the LCD $t(t - 12)$.

$$1.75(t - 12) + 1.75t = t^2 - 12t$$

$$1.75t - 21 + 1.75t = t^2 - 12t$$

$$0 = t^2 - 15.5t + 21$$ • Write the quadratic equation in standard form.

Using the quadratic formula, the solutions of the above equation are $t = 1.5$ and $t = 14$. Substituting $t = 1.5$ into the time for the larger pipe would give a negative time ($1.5 - 12 = -10.5$), so that answer is not possible. The time for the smaller pipe to empty the tank with the larger pipe closed is 14 minutes.

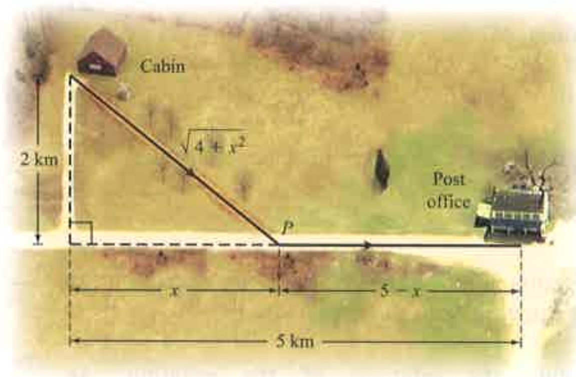
▶ **Try Exercise 74, page 119**

EXAMPLE 10 Solve an Application Involving Radicals

A cabin is located in a meadow at the end of a straight driveway 2 kilometers long. A post office is 5 kilometers from the driveway along a straight road. (See the diagram below.) A woman walks 2 kilometers per hour through the meadow to point P and then 5 kilometers per hour along the road to the post office. If it takes the woman 2.25 hours to reach the post office, what is the distance x of point P from the end of the driveway? Round to the nearest tenth of a kilometer.

Note

In the diagram, the expression $\sqrt{4 + x^2}$ for the distance from the cabin to point P is found by using the Pythagorean Theorem in the right triangle formed by the dashed lines and the woman's path through the meadow.



Solution

Recall that Distance = Rate × Time. Therefore, Time = $\dfrac{\text{Distance}}{\text{Rate}}$. Using this equation, we have

$$\text{Time to walk from cabin to } P = \frac{\text{Distance from cabin to } P}{\text{Rate of walking in meadow}} = \frac{\sqrt{4 + x^2}}{2}$$

$$\text{Time to walk from } P \text{ to post office} = \frac{\text{Distance from } P \text{ to post office}}{\text{Rate of walking on road}} = \frac{5 - x}{5}$$

The sum of these two times equals the total time (2.25 hours). Thus

$$\frac{\sqrt{4 + x^2}}{2} + \frac{5 - x}{5} = 2.25$$

Solve the equation.

$$\frac{\sqrt{4 + x^2}}{2} + \frac{5 - x}{5} = 2.25$$

$$10\left(\frac{\sqrt{4 + x^2}}{2} + \frac{5 - x}{5}\right) = 10(2.25) \qquad \text{• Clear the denominators.}$$

$$5\sqrt{4 + x^2} + 2(5 - x) = 22.5 \qquad \text{• Simplify.}$$

$$5\sqrt{4 + x^2} + 10 - 2x = 22.5$$

$$5\sqrt{4 + x^2} = 12.5 + 2x \qquad \text{• Isolate the radical.}$$

$$(5\sqrt{4 + x^2})^2 = (12.5 + 2x)^2 \qquad \text{• Square each side.}$$

$$25(4 + x^2) = 4x^2 + 50x + 156.25$$

$$100 + 25x^2 = 4x^2 + 50x + 156.25$$

$$21x^2 - 50x - 56.25 = 0 \qquad \text{• Write in standard form.}$$

(continued)

Using the quadratic formula to solve the last equation, we have $x \approx -0.8$ and $x \approx 3.2$. Because x cannot be negative, point P is 3.2 kilometers from the end of the driveway.

Try Exercise 84, page 120

EXERCISE SET 1.4

Concept Check

1. The factored form of the polynomial $x^3 - x^2 - 6x$ is $x(x-3)(x+2)$. What are the solutions of the equation $x^3 - x^2 - 6 = 0$?

2. What value of x is not in the domain of the rational equation $\frac{1}{x-5} + 3 = \frac{x+1}{x-5}$?

3. Refer to the equation in Exercise 2. What do you multiply each side of the equation by to clear the fractions?

4. A student found the solutions of the equation $\sqrt{x+7} - x = 5$ to be -6 and -3. Which of these solutions is an extraneous solution?

5. To what power do you raise each side of the equation $x^{2/3} = 9$ to solve for x?

6. What substitution do you use to rewrite the equation $2x^{1/2} + 7x^{1/4} - 4 = 0$ as the equation $2u^2 + 7u - 4 = 0$?

In Exercises 7 to 16, solve each polynomial equation by factoring and using the principle of zero products.

7. $x^3 - 25x = 0$
8. $x^3 - x = 0$
9. $x^3 - 2x^2 - x + 2 = 0$
10. $4x^3 + 4x^2 - 9x - 9 = 0$
11. $x^3 - 3x^2 - 5x + 15 = 0$
12. $x^3 - 4x^2 - 2x + 8 = 0$
13. $3x^3 + 2x^2 - 27x - 18 = 0$
14. $4x^3 + 5x^2 - 16x - 20 = 0$
15. $x^3 - 8 = 0$
16. $x^3 + 8 = 0$

In Exercises 17 to 30, solve the rational equation.

17. $\frac{5}{x+4} - 2 = \frac{7x+18}{x+4}$
18. $\frac{x+4}{x-2} + 3 = \frac{-2}{x-2}$
19. $2 + \frac{9}{r-3} = \frac{3r}{r-3}$
20. $\frac{t}{t-4} + 3 = \frac{4}{t-4}$
21. $\frac{3}{x+2} = \frac{5}{2x-7}$
22. $\frac{4}{y+2} = \frac{7}{y-4}$
23. $x - \frac{2x+3}{x+3} = \frac{2x+9}{x+3}$
24. $2x + \frac{3}{x-1} = \frac{-7x+10}{x-1}$
25. $\frac{5}{x-3} - \frac{3}{x-2} = \frac{4}{x-3}$
26. $\frac{4}{x-1} + \frac{7}{x+7} = \frac{5}{x-1}$
27. $\frac{x}{x+1} - \frac{x+2}{x-1} = \frac{x-12}{x+1}$
28. $\frac{2x+1}{x-3} - \frac{x-4}{x+5} = -\frac{10x+13}{x+5}$
29. $\frac{3-2x}{x+3} - \frac{2x+1}{x-4} = \frac{5x-29}{x-4}$
30. $\frac{5x+3}{3x-2} - \frac{x-1}{x-3} = \frac{2x+3}{x-3}$

In Exercises 31 to 44, solve the radical equation.

31. $\sqrt{x-4} - 6 = 0$
32. $\sqrt{10-x} = 4$
33. $\sqrt{9x-20} = x$
34. $x = \sqrt{12x-35}$
35. $\sqrt{-7x+2} + x = 2$
36. $\sqrt{-9x-9} + x = 1$
37. $\sqrt{3x-5} - \sqrt{x+2} = 1$
38. $\sqrt{x+7} - 2 = \sqrt{x-9}$
39. $\sqrt{2x+11} - \sqrt{2x-5} = 2$
40. $\sqrt{x+7} + \sqrt{x-5} = 6$
41. $\sqrt{x-4} + \sqrt{x+1} = 1$
42. $\sqrt{2x-9} + \sqrt{2x+6} = 3$
43. $\sqrt{2x-1} - \sqrt{x-1} = 1$
44. $\sqrt{6-x} + \sqrt{5x+6} = 6$

Indicates Try It Exercises

In Exercises 45 to 54, solve each equation containing a rational exponent on the variable.

45. $x^{1/3} = 2$
46. $x^{1/2} = 5$
47. $x^{2/5} = 9$
48. $x^{4/3} = 81$
49. $x^{3/2} = 27$
50. $x^{3/4} = 125$
51. $3x^{2/3} - 16 = 59$
52. $4x^{4/5} - 27 = 37$
53. $4x^{3/4} - 31 = 77$
54. $4x^{4/5} - 54 = 270$

In Exercises 55 to 70, solve each equation by first rewriting each equation as a quadratic equation.

55. $x^4 - 9x^2 + 14 = 0$
56. $x^4 - 10x^2 + 9 = 0$
57. $2x^4 - 11x^2 + 12 = 0$
58. $6x^4 - 7x^2 + 2 = 0$
59. $x^6 + x^3 - 6 = 0$
60. $6x^6 + x^3 - 15 = 0$
61. $x^{1/2} - 3x^{1/4} + 2 = 0$
62. $2x^{1/2} - 5x^{1/4} - 3 = 0$
63. $3x^{2/3} - 11x^{1/3} - 4 = 0$
64. $6x^{2/3} - 7x^{1/3} - 20 = 0$
65. $x^4 + 8x^2 - 9 = 0$
66. $4x^4 + 7x^2 - 36 = 0$
67. $x^{2/5} - x^{1/5} - 2 = 0$
68. $2x^{2/5} - x^{1/5} = 6$
69. $9x - 52\sqrt{x} + 64 = 0$
70. $8x - 38\sqrt{x} + 9 = 0$

71. **Boating** A small fishing boat heads to a point 24 miles downriver and then returns. The river's current moves at 3 miles per hour. If the trip up and back takes 6 hours and the boat keeps a constant speed relative to the water, what is the speed of the boat? [*Hint:* If v is the speed of the boat, then its speed downriver is $(v + 3)$ miles per hour and its speed upriver is $(v - 3)$ miles per hour.]

72. **Running** Maureen can run at a rate that is 2 miles per hour faster than her friend Hector's rate. While training for a mini marathon, Maureen gives Hector a half-hour head start and then begins chasing Hector on the same route. If Maureen passes Hector 12 miles from the starting point, how fast is each running?

73. **Fence Construction** A worker can build a fence in 8 hours. Working together, the worker and an assistant can build the fence in 5 hours. How long should it take the assistant, working alone, to build the fence?

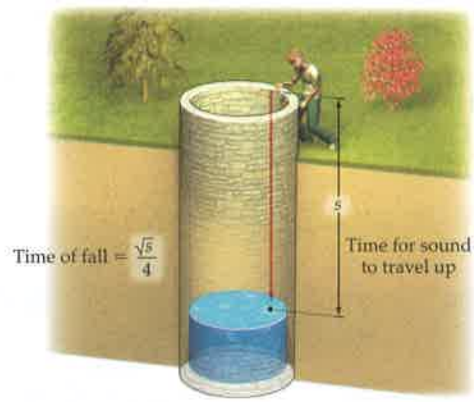
74. **Roof Repair** A roofer and an assistant can repair a roof together in 6 hours. Working alone, the assistant can repair the roof in 14 hours. If both the roofer and the assistant work together for 2 hours and then the assistant is left alone to finish the job, how much longer should the assistant need to finish the repairs?

75. **Painting a Room** An experienced painter and an apprentice can paint a room in 6 hours. Working alone, it takes the apprentice 5 hours less than twice the time needed by the experienced painter to paint the room. How long does it take the experienced painter to paint the room?

76. **Parallel Processing** Parallel processing uses two or more computers, working together, to solve a single problem. Using parallel processing, two computers can solve a problem in 12 minutes. If, working alone, one computer can solve a problem in 7 minutes less than the time needed by the second computer, how long would it take the faster computer working alone to solve the problem?

In Exercises 77 and 78, the depth s from the opening of a well to the water below can be determined by measuring the total time between the instant you drop a stone and the moment you hear it hit the water. The time, in seconds, it takes the stone to hit the water is given by $\sqrt{s}/4$, where s is measured in feet. The time, also in seconds, required for the sound of the impact to travel up to your ears is given by $s/1100$. Thus the total time T, in seconds, between the instant you drop the stone and the moment you hear its impact is

$$T = \frac{\sqrt{s}}{4} + \frac{s}{1100}$$



77. **Time of Fall** One of the world's deepest water wells is 7320 feet deep. Find the time between the instant you drop a stone and the time you hear it hit the water if the surface of the water is 7100 feet below the opening of the well. Round your answer to the nearest tenth of a second.

78. **Depth of a Well** Find the depth from the opening of a well to the water level if the time between the instant you drop a stone and the moment you hear its impact is 3 seconds. Round your answer to the nearest foot.

79. **Radius of a Cone** A conical funnel has a height h of 4 inches and a lateral surface area L of 15π square inches. Find the radius r of the cone. (*Hint:* Use the formula $L = \pi r\sqrt{r^2 + h^2}$.)

80. **Diameter of a Cone** As flour is poured onto a table, it forms a right circular cone whose height is one-third the diameter of the base. What is the diameter of the base when the cone has a volume of 192 cubic inches? Round to the nearest tenth of an inch.

81. **Precious Metals** A solid silver sphere has a diameter of 8 millimeters, and a second silver sphere has a diameter of 12 millimeters. The spheres are melted down and recast to form a single cube. What is the length s of each edge of the cube? Round your answer to the nearest tenth of a millimeter.

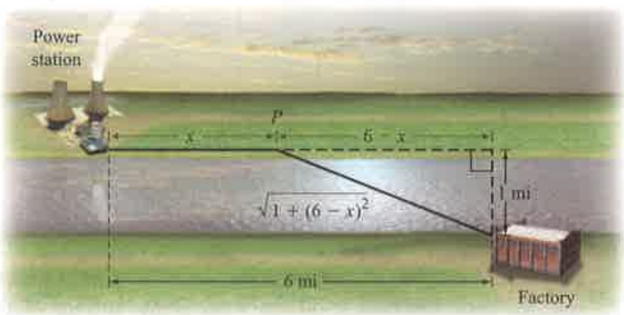
82. **Pendulum** The period T of a pendulum is the time it takes the pendulum to complete one swing from left to right and back. For a pendulum near the surface of Earth,

$$T = 2\pi\sqrt{\frac{L}{32}}$$

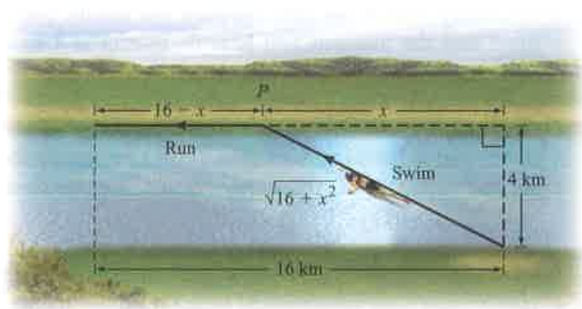
where T is measured in seconds and L is the length of the pendulum in feet. Find the length of a pendulum that has a period of 4 seconds. Round to the nearest tenth of a foot.

83. **Distance to the Horizon** On a ship, the distance d that you can see to the horizon is given by $d = \sqrt{1.5h}$, where h is the height of your eye measured in feet above sea level and d is measured in miles. How high is the eye level of a navigator who can see 14 miles to the horizon? Round to the nearest foot.

84. **Providing Power** A power station is on one side of a river that is 1 mile wide, and a factory is 6 miles downstream on the other side of the river. The cost is \$0.125 million per mile to run power lines over land and \$0.2 million per mile to run power lines under water. How far over land should the power line be run if the total cost of the project is to be \$1 million? Round to the nearest tenth of a mile. See the diagram below. (*Hint:* Cost for a segment equals cost per mile times the number of miles.)



85. **Triathlon Training** To prepare for a triathlon, a person swims across a river to point P and then runs along a path as shown in the diagram below.



The person swims at 7 kilometers per hour and runs at 22 kilometers per hour. For what distance x is the total time for swimming and running 2 hours? Round to the nearest tenth of a kilometer. (*Hint:* Time swimming + Time running = 2 hours, and $\frac{\text{Distance}}{\text{Rate}} = \text{Time}$.)

Enrichment Exercises

In Exercises 86 to 89, solve each equation.

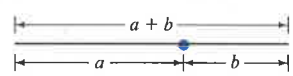
86. $x^4 + 3x^3 - 8x - 24 = 0$

87. $x^4 - 2x^3 + 27x - 54 = 0$

88. $\sqrt[3]{x^3 - 5x - 17} = x - 1$

89. $\sqrt[3]{x^3 - 2x - 13} = x - 1$

90. Two quantities, a and b, are in the **golden ratio** ϕ (also called the **golden mean**) if $\frac{a}{b} = \frac{a+b}{a} = \phi$. Geometrically, this can be interpreted as dividing a line segment into two parts so that the ratio of the length a of the longer part to the length b of the shorter part equals the ratio of the length of the whole segment to the length of the longer part. See the diagram below. Find the exact value of ϕ. (*Suggestion:* From the equation $\frac{a}{b} = \frac{a+b}{a} = \phi$, we have $\frac{a}{b} = \phi$ and $\frac{a+b}{a} = \phi$. Rewrite the first equation as $\frac{b}{a} = \frac{1}{\phi}$ and the second equation as $1 + \frac{b}{a} = \phi$. Use these two equations to write a single equation in the variable ϕ. The positive solution to this equation is the golden ratio.)



91. Refer to the definition of the golden ratio ϕ in Exercise 90. Here is a method of constructing a golden rectangle, a rectangle in which the ratio of the length to the width is ϕ. Begin with a square whose sides are 2 units. (You can use any length.) From the midpoint of one side, draw a line segment to an opposite vertex. Using a compass, create an arc that intersects an extension of the base of the square. Now complete the rectangle. (See the following diagram.) Show that $\frac{AD}{AB}$ is equal to the value of ϕ you found in Exercise 90.