



GO DIGITAL

1.3 Modeling with Linear Functions

Learning Target Use linear functions to model and analyze real-life situations.

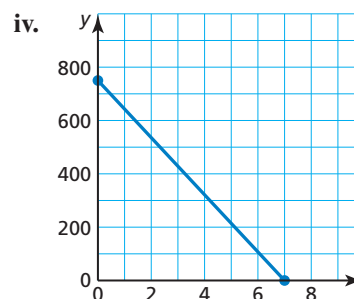
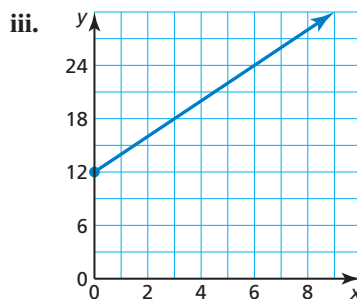
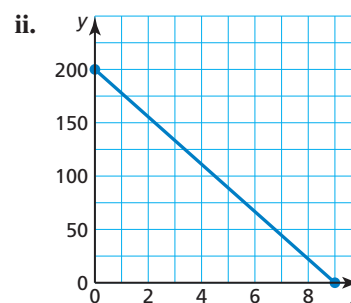
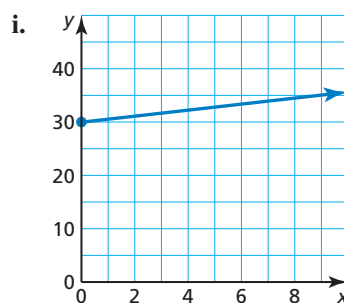
- Success Criteria**
- I can write equations of linear functions.
 - I can compare linear equations to solve real-life problems.
 - I can determine a line of best fit.

EXPLORE IT! Modeling with Linear Functions

Work with a partner. A company purchases a demolition robot for \$87,000. The spreadsheet shows how the robot depreciates over an 8-year period.

	A	B
1	Year, t	Value, V
2	0	\$87,000
3	1	\$79,750
4	2	\$72,500
5	3	\$65,250
6	4	\$58,000
7	5	\$50,750
8	6	\$43,500
9	7	\$36,250
10	8	\$29,000
11		

- Explain how you can write a linear function to represent the value V of the robot as a function of the number t of years.
- Sketch a graph of the function. Explain why this type of depreciation is called *straight line depreciation*.
- Interpret the slope and intercepts of the graph in the context of the problem.
- MP CHOOSE TOOLS** Use the Internet or another reference to find a real-life example of straight line depreciation. Then write a function that models the example you found and sketch its graph.
- Describe a real-life situation that can be modeled by each graph below. Exchange situations with your partner and match each of your partner's situations with its corresponding graph. Explain your reasoning.



Math Practice

Label Axes

After matching a situation with its graph, label the axes to clarify their correspondence with the quantities in the situation.

- How can you use a linear function to model and analyze a real-life situation?



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Writing Linear Equations

Vocabulary



line of fit, p. 22
line of best fit, p. 23
correlation coefficient, p. 23



KEY IDEA

Writing an Equation of a Line

Given slope m and y -intercept b

Use slope-intercept form:

$$y = mx + b$$

Given slope m and a point (x_1, y_1)

Use point-slope form:

$$y - y_1 = m(x - x_1)$$

Given points (x_1, y_1) and (x_2, y_2)

First use the slope formula to find m . Then use point-slope form with either given point.

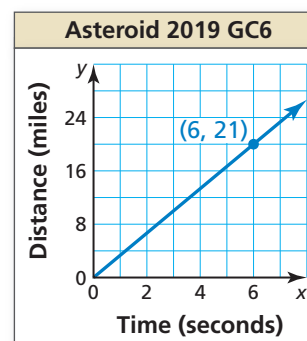
Asteroid 2019 GC6 is about as large as a house. NASA and other international organizations are constantly scanning our skies for threats from space rocks.



EXAMPLE 1 Writing a Linear Equation from a Graph



The graph shows the distance Asteroid 2019 GC6 travels in x seconds. Write an equation of the line and interpret the slope. The asteroid came within 136,000 miles of Earth in April, 2019. About how long does it take the asteroid to travel that distance?



SOLUTION

From the graph, you can see the slope is $m = \frac{21}{6} = 3.5$ and the y -intercept is $b = 0$. Use slope-intercept form to write an equation of the line.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$= 3.5x + 0 \quad \text{Substitute 3.5 for } m \text{ and 0 for } b.$$

The equation is $y = 3.5x$. The slope indicates that the asteroid travels 3.5 miles per second. Use the equation to find how long it takes the asteroid to travel 136,000 miles.

$$136,000 = 3.5x \quad \text{Substitute 136,000 for } y.$$

$$38,857 \approx x \quad \text{Divide each side by 3.5.}$$

▶ Because there are 3600 seconds in 1 hour and

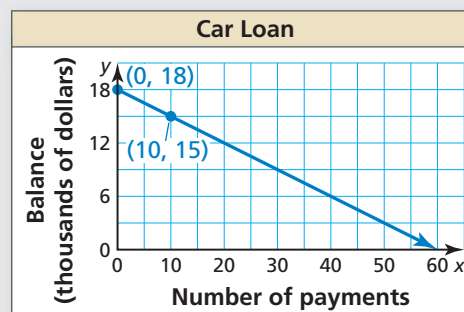
$$38,857 \text{ sec} \div \frac{3600 \text{ sec}}{1 \text{ h}} = 38,857 \text{ sec} \times \frac{1 \text{ h}}{3600 \text{ sec}} \approx 11 \text{ h,}$$

it takes the asteroid about 11 hours to travel 136,000 miles.

SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

- MP STRUCTURE** What does an equation of the form $y = mx$ indicate about x and y ?
- The graph shows the remaining balance y on a car loan after making x monthly payments.
 - Write an equation of the line and interpret the slope and y -intercept. What is the remaining balance after 36 payments?
 - Interpret the meaning of the x -intercept in this situation.
 - What happens to the equation you wrote in part (a) when the y -intercept remains the same and the x -intercept is changed to 72? What changed in the problem?





EXAMPLE 2 Comparing Linear Equations



Lakeside Inn	
Number of students, x	Total cost, y
100	\$1500
125	\$1800
150	\$2100
175	\$2400
200	\$2700

Two prom venues charge a rental fee plus a fee per student. The table shows the total costs (in dollars) for different numbers of students at Lakeside Inn. The total cost y (in dollars) for x students at Sunview Resort is represented by the equation

$$y = 10x + 600. \quad \text{Sunview Resort}$$

Which venue charges less per student? How many students must attend for the total costs to be the same?

SOLUTION

- Understand the Problem** You are given cost information for two venues. You need to compare the costs.
- Make a Plan** Compare the slopes to determine which venue charges less per student. Then write an equation that models the total cost at Lakeside Inn. Equate the cost expressions and solve to determine the number of students for which the total costs are equal.
- Solve and Check** The table shows a constant rate of change. Find the slope using any two points. Use $(x_1, y_1) = (100, 1500)$ and $(x_2, y_2) = (125, 1800)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 1500}{125 - 100} = \frac{300}{25} = 12$$

Comparing the slopes, Sunview Resort charges \$10 per student, which is less than the \$12 per student that Lakeside Inn charges.

Write an equation that represents the total cost at Lakeside Inn using the slope of 12 and a point from the table. Use $(x_1, y_1) = (100, 1500)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - 1500 &= 12(x - 100) && \text{Substitute for } m, x_1, \text{ and } y_1. \\ y - 1500 &= 12x - 1200 && \text{Distributive Property} \\ y &= 12x + 300 && \text{Add 1500 to each side.} \end{aligned}$$

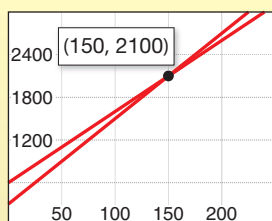
Equate the cost expressions and solve.

$$\begin{aligned} 10x + 600 &= 12x + 300 && \text{Set cost expressions equal.} \\ 300 &= 2x && \text{Combine like terms.} \\ 150 &= x && \text{Divide each side by 2.} \end{aligned}$$

▶ The total costs are the same when 150 students attend the prom.

Another Way

Another way to check your solution is to graph each cost equation and find the point of intersection. The x -value of the point of intersection is 150.



Check Notice that the table shows the total cost for 150 students at Lakeside Inn is \$2100. To check that your solution is correct, verify that the total cost at Sunview Resort is also \$2100 for 150 students.

$$\begin{aligned} y &= 10(150) + 600 && \text{Substitute 150 for } x. \\ &= 2100 \quad \checkmark && \text{Simplify.} \end{aligned}$$

SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

3. **WHAT IF?** Maple Ridge charges a rental fee plus a \$10 fee per student. The total cost is \$1900 for 140 students. Describe the number of students that must attend for the total cost at Maple Ridge to be less than the total costs at the other two venues. Use a graph to justify your answer.



Finding Lines of Fit and Lines of Best Fit

Data do not always show an *exact* linear relationship. When the data in a scatter plot show an approximately linear relationship, you can model the data with a **line of fit**.



KEY IDEA

Finding a Line of Fit

- Step 1** Make a scatter plot of the data.
- Step 2** Draw the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.
- Step 3** Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.
- Step 4** Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

EXAMPLE 3

Finding a Line of Fit

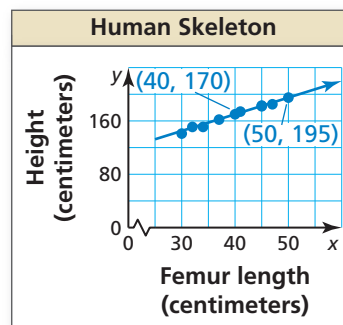


Femur length (cm), x	Height (cm), y
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182

The table shows the femur lengths (in centimeters) and heights (in centimeters) of several people. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 centimeters long.

SOLUTION

- Step 1** Make a scatter plot of the data.
The data show a linear relationship.
- Step 2** Draw the line that most closely appears to fit the data. One possibility is shown.
- Step 3** Choose two points on the line.
For the line shown, you might choose (40, 170) and (50, 195).
- Step 4** Write an equation of the line.



First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{195 - 170}{50 - 40} = \frac{25}{10} = 2.5$$

Use point-slope form to write an equation. Use $(x_1, y_1) = (40, 170)$.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 170 = 2.5(x - 40) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y - 170 = 2.5x - 100 \quad \text{Distributive Property}$$

$$y = 2.5x + 70 \quad \text{Add 170 to each side.}$$

Use the equation to estimate the height of a person whose femur is 35 centimeters long.

$$y = 2.5(35) + 70 \quad \text{Substitute 35 for } x.$$

$$= 157.5 \quad \text{Simplify.}$$

- ▶ The approximate height of a person whose femur is 35 centimeters long is 157.5 centimeters.



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The **line of best fit** is the line that best models a set of data and lies as close as possible to all of the data points. Many technology tools have a *linear regression* feature that you can use to find the line of best fit for a set of data.

The **correlation coefficient**, denoted by r , is a number that ranges from -1 to 1 . When r is near 1 , there is a strong positive correlation and the points lie close to a line with a positive slope. When r is near -1 , there is a strong negative correlation and the points lie close to a line with a negative slope. As r gets closer to 0 , the correlation becomes weaker.

EXAMPLE 4 Finding a Line of Best Fit Using Technology



Use technology to find an equation of the line of best fit for the data in Example 3. Estimate the height of a person whose femur is 35 centimeters long. Compare this height to your estimate in Example 3.

SOLUTION

Step 1 Enter the data into two lists.

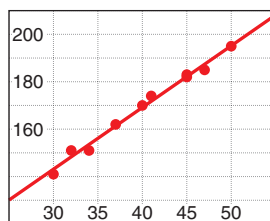
x	y
40	170
45	183
32	151
50	195
37	162
41	174
30	141

Step 2 Find an equation of the line of best fit. The values in the equation can be rounded to obtain $y = 2.6x + 65$.

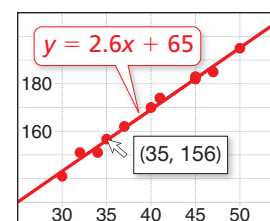
$y = mx + b$	
PARAMETERS	
$m = 2.6036$	$b = 64.9968$
STATISTICS	
$r^2 = 0.9891$	
$r = 0.9945$	

The value of r is close to 1.

Step 3 Graph the regression equation with the data.



Step 4 Find the value of y when $x = 35$.



Math Practice

Analyze Givens

How can you determine appropriate scales for your graph?

► The height of a person with a 35-centimeter femur is about 156 centimeters. This is less than the estimate of 157.5 centimeters found in Example 3.

SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

- VOCABULARY** A line of best fit has a correlation coefficient of -0.98 . What can you conclude about the slope of the line?
- The data pairs (x, y) represent the humerus lengths x (in centimeters) and heights y (in centimeters) of several females.

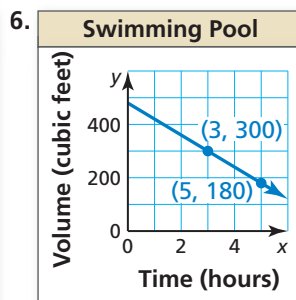
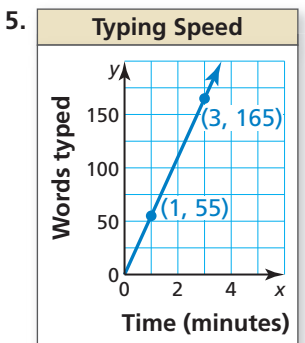
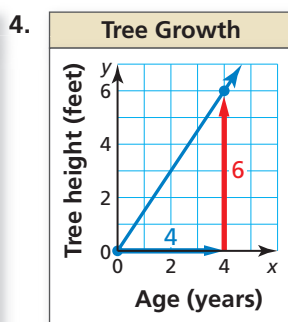
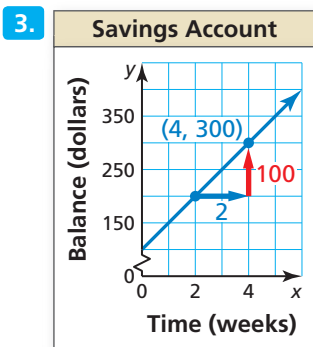
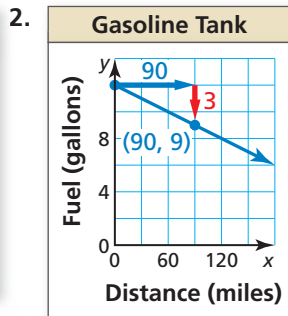
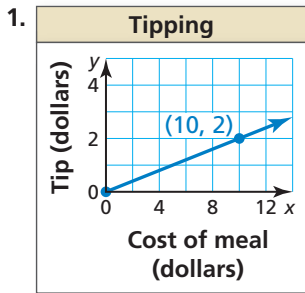
$(33, 166), (25, 142), (22, 130), (30, 154), (28, 152), (32, 159), (26, 141), (27, 145)$

 - Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a female whose humerus is 40 centimeters long.
 - Use technology to find an equation of the line of best fit for the data. Estimate the height of a female whose humerus is 40 centimeters long. Compare this height to your estimate in part (a).

1.3 Practice WITH CalcChat® AND CalcView®



In Exercises 1–6, use the graph to write an equation of the line and interpret the slope. ▶ *Example 1*



7. **MODELING REAL LIFE**

You consider buying a phone from one of two cell phone carriers. The table shows the total costs (in dollars) of the phone and service for different numbers of months at Carrier A. The total cost y (in dollars) of the phone and x months of service at Carrier B is represented by the equation $y = 55x + 300$. Which carrier charges less per month? After how many months of service are the total costs the same?

Carrier A	
Months, x	Total cost, y
3	\$500
6	\$650
9	\$800
12	\$950
15	\$1100

▶ *Example 2*

8. **MODELING REAL LIFE**

You and a friend sign up for a new volunteer project to increase your service hours. The table shows your total numbers of service hours after different numbers of weeks on the new project. Your friend initially has 8 service hours and spends 2 hours on the new project each week. Who initially has more service hours? Who volunteers more hours per week? After how many weeks do you and your friend have the same total number of service hours?



Your Service Hours	
Weeks, x	Total hours, y
4	15
6	20
8	25
10	30
12	35

9. **ERROR ANALYSIS** Describe and correct the error in writing a linear equation that models the data in the table.

x	y
12	17
20	23
28	29
36	35
42	41

X

$$m = \frac{20 - 12}{23 - 17} = \frac{8}{6} = \frac{4}{3}$$

$$y - 17 = \frac{4}{3}(x - 12)$$

$$y - 17 = \frac{4}{3}x - 16$$

$$y = \frac{4}{3}x + 1$$

10. **MP PROBLEM SOLVING** You notice that temperatures in Canada are reported in degrees Celsius. You know there is a linear relationship between degrees Fahrenheit and degrees Celsius, but you forget the formula. From science class, you remember the freezing point of water is 0°C or 32°F , and its boiling point is 100°C or 212°F .

- Write an equation that represents degrees Fahrenheit in terms of degrees Celsius.
- A forecast for Toronto shows a high of 15°C . What is this temperature in degrees Fahrenheit?
- Rewrite your equation in part (a) to represent degrees Celsius in terms of degrees Fahrenheit.
- A forecast for your city shows a high of 68°F . What is this temperature in degrees Celsius?



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In Exercises 11–14, determine whether the data show a linear relationship. If so, write an equation of a line of fit. Then estimate y when $x = 15$ and explain its meaning in the context of the situation. ▶ Example 3

11.

Minutes walking, x	1	6	11	13	16
Calories burned, y	6	27	50	56	70

12.

Months, x	9	13	18	22	23
Hair length (in.), y	3	5	7	10	11

13.

Hours, x	3	7	9	17	20
Battery life (%), y	86	61	50	26	0

14.

Shoe size, x	6	8	8.5	10	13
Heart rate (bpm), y	112	94	100	132	87

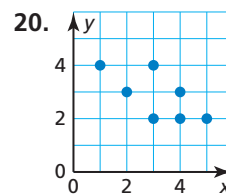
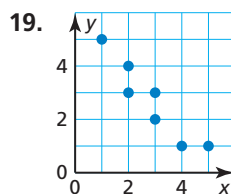
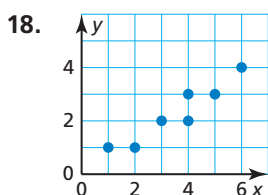
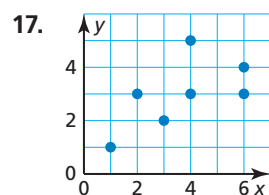
15. **MODELING REAL LIFE** The data pairs (x, y) represent the average annual tuition and fees y (in dollars) for private nonprofit colleges and universities in the United States x years after the 2012–2013 academic year. Use technology to find an equation of the line of best fit. Interpret the slope and y -intercept in this situation. Estimate the average annual tuition in the 2022–2023 academic year. ▶ Example 4

$(0, 31,890), (1, 32,500), (2, 33,090), (3, 34,150)$
 $(4, 35,080), (5, 35,720), (6, 35,830)$

16. **MODELING REAL LIFE** The table shows the numbers y (in billions) of active users on a social media site x years after 2009. Use technology to find an equation of the line of best fit. Interpret the slope and y -intercept in this situation. Estimate the number of active users on the site in 2022.

Years after 2009, x	Active users, y
0	0.36
1	0.61
3	1.06
5	1.39
7	1.86
8	2.13
9	2.32

MP USING TOOLS In Exercises 17–20, use technology to find an equation of the line of best fit for the data. Identify and interpret the correlation coefficient.



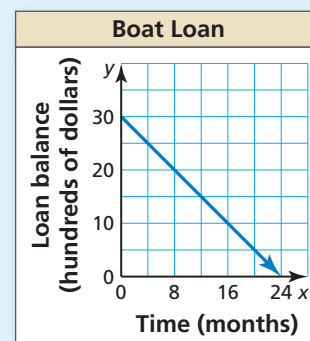
21. **MP REASONING** The table shows the numbers of tickets sold for a concert when different prices are charged. Write an equation of a line of fit for the data. Does it seem reasonable to use your model to predict the number of tickets sold when the ticket price is \$85? Explain.

Ticket price (dollars), x	17	20	22	26
Tickets sold, y	450	423	400	395

22. **HOW DO YOU SEE IT?**

You secure an interest-free loan to purchase a boat. You agree to make equal monthly payments for the next two years. The graph shows the amount of money you still owe.

- What is the slope of the line? What does the slope represent?
- What is the domain and range of the function? What does each represent?
- How much do you still owe after making payments for 12 months?



23. **OPEN-ENDED** Give two real-life quantities that have (a) a positive correlation, (b) a negative correlation, and (c) approximately no correlation. Explain.

24. **MAKING AN ARGUMENT** A set of data pairs has a correlation coefficient $r = 0.3$. Your friend says that because the correlation coefficient is positive, it is logical to use the line of best fit to make predictions. Is your friend correct? Explain your reasoning.

25. **COLLEGE PREP** Which equation has a graph that is a line passing through the point $(8, -5)$ and is perpendicular to the graph of $y = -4x + 1$?

- (A) $y = \frac{1}{4}x - 5$ (B) $y = -4x + 27$
(C) $y = -\frac{1}{4}x - 7$ (D) $y = \frac{1}{4}x - 7$

**26. THOUGHT PROVOKING**

Points A and B lie on the line $y = -x + 4$. Point C is the same distance from point A as it is from point B but does not lie on the same line. Choose coordinates for points A , B , and C . Write equations for the lines connecting points A and C and points B and C .

27. ABSTRACT REASONING If x and y have a strong positive correlation, and y and z have a strong negative correlation, then what can you conclude about the correlation between x and z ? Explain.

28. ANALYZING RELATIONSHIPS Data show a positive correlation between the attendance at a local beach and the attendance at a local amusement park.

- Does a positive correlation make sense in this situation? Explain.
- Is it reasonable to conclude that an increase in attendance at the beach will cause an increase in attendance at the amusement park? Explain.

29. PERFORMANCE TASK Your family wants to purchase a new vehicle that comes in either a gasoline model or an electric model.

- Using the information shown, the approximate number of miles your family drives per year, and gas and electricity prices in your area, determine which vehicle is a better buy. Use linear equations to support your answer.
- Research other factors that impact the cost of vehicle ownership. How might these factors support or change your answer?

Gasoline
Price: \$17,950
Fuel Economy:
26 mi/gal city
38 mi/gal highway

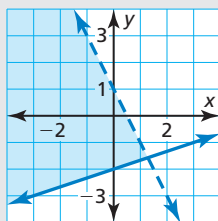
Electric
Price: \$29,120
Fuel Economy:
31 kWh/100 mi

REVIEW & REFRESH

In Exercises 30–33, solve the system using any method. Explain your choice of method.

- | | |
|---|---|
| <p>30. $3x + y = 7$
$-2x - y = 9$</p> <p>32. $2x + 2y = 3$
$x = 4y - 1$</p> | <p>31. $4x - 6y = 2$
$2x - 3y = 1$</p> <p>33. $y = x - 4$
$y = -4x + 6$</p> |
|---|---|

34. Write a system of inequalities represented by the graph.



- Solve the literal equation $z = 4y + 2x + 8$ for x .
- What percent of 25 is 14?

In Exercises 37–40, graph the function and its parent function. Then describe the transformation.

- | | |
|--|---|
| <p>37. $f(x) = \frac{3}{2}$</p> <p>39. $f(x) = 2(x - 1)^2$</p> | <p>38. $f(x) = 3x$</p> <p>40. $f(x) = - x + 2 - 7$</p> |
|--|---|

In Exercises 41 and 42, find the sum or difference.

- $(x^2 + 2x + 16) + (4x^2 - 7x - 18)$
- $(-5n^3 + n^2 - 12n) - (6n^2 + 4n - 13)$

43. MODELING REAL LIFE

Two internet service providers charge an installation fee plus a monthly service fee. The table shows the total costs (in dollars) for different numbers of months at Company A. The total

Company A	
Months, x	Total cost, y
2	\$180
3	\$220
4	\$260
5	\$300
6	\$340

cost y (in dollars) for x months of service at Company B is represented by $y = 45x + 50$. Which company charges less per month? After how many months of service are the total costs the same?

44. What number is 34% of 50?

In Exercises 45 and 46, write a function g whose graph represents the indicated transformation of the graph of f .

- $f(x) = 2x + 1$; translation 3 units up
- $f(x) = -3|x - 4|$; vertical shrink by a factor of $\frac{1}{2}$