SECTION 1.2

Formulas
Applications

Formulas and Applications

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A4.

PS1. The sum of two numbers is 32. If one of the numbers is represented by x, then the expression $32 - x$ represents the other number. Evaluate $32 - x$ for $x = 8\frac{1}{2}$. [P.1]

PS2. Evaluate $\frac{1}{2}bh$ for $b = \frac{2}{3}$ and $h = \frac{4}{5}$. [P.1]

PS3. What property has been applied to rewrite $2l + 2w$ as $2(l + w)$? [P.1]

PS4. What property has been applied to rewrite $\left(\frac{1}{2}b\right)h$ as $\frac{1}{2}(bh)$? [P.1]

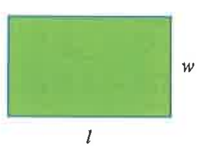
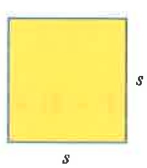
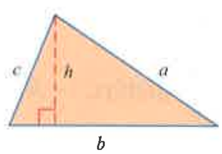
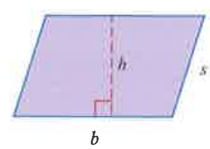
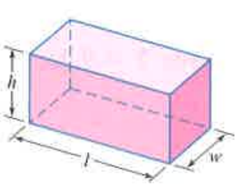
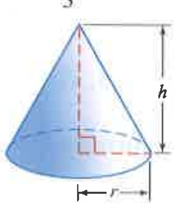
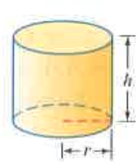
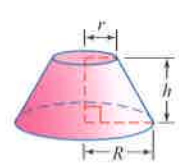
PS5. Add: $\frac{2}{5}x + \frac{1}{3}x$ [P.1]

PS6. Simplify: $\dfrac{1}{\frac{1}{a} + \frac{1}{b}}$ [P.5]

Formulas

A **formula** is an equation that expresses known relationships between two or more variables. Table 1.2 lists several formulas from geometry that are used in this text. The variable P represents perimeter, C represents circumference of a circle, A represents area, S represents surface area of an enclosed solid, and V represents volume.

Table 1.2 Formulas from Geometry

Rectangle	Square	Triangle	Circle	Parallelogram
$P = 2l + 2w$	$P = 4s$	$P = a + b + c$	$C = \pi d = 2\pi r$	$P = 2b + 2s$
$A = lw$	$A = s^2$	$A = \frac{1}{2}bh$	$A = \pi r^2$	$A = bh$
				
Rectangular Solid	**Right Circular Cone**	**Sphere**	**Right Circular Cylinder**	**Frustum of a Cone**
$S = 2(wh + lw + hl)$	$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$	$S = 4\pi r^2$	$S = 2\pi rh + 2\pi r^2$	$S = \pi(R + r)\sqrt{h^2 + (R - r)^2} + \pi r^2 + \pi R^2$
$V = lwh$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$
				

It is often necessary to solve a formula for a specified variable. Begin the process by isolating all terms that contain the specified variable on one side of the equation and all terms that do not contain the specified variable on the other side.

EXAMPLE 1 Solve a Formula for a Specified Variable

a. Solve $2l + 2w = P$ for l.

b. Solve $S = 2(wh + lw + hl)$ for h.

Solution

a. $2l + 2w = P$

$2l = P - 2w$ • Subtract $2w$ from each side to isolate the $2l$ term.

$l = \dfrac{P - 2w}{2}$ • Divide each side by 2.

b. $S = 2(wh + lw + hl)$

$S = 2wh + 2lw + 2hl$

$S - 2lw = 2wh + 2hl$ • Isolate the terms that involve the variable h on the right side.

$S - 2lw = 2h(w + l)$ • Factor $2h$ from the right side.

$\dfrac{S - 2lw}{2(w + l)} = h$ • Divide each side by $2(w + l)$.

▶ **Try Exercise 10, page 91**

Note

In Example 1a, the solution $l = \dfrac{P - 2w}{2}$ also can be written as $l = \dfrac{P}{2} - w$.

Question • If $ax + b = c$, does $x = \dfrac{c}{a} - b$?

Formulas are often used to compare the performances of athletes. Here is an example of a formula that is used in professional football.

EXAMPLE 2 Calculate a Quarterback Rating

The National Football League uses the following formula to rate quarterbacks.

$$\text{QB rating} = \frac{100}{6}[0.05(C - 30) + 0.25(Y - 3) + 0.2T + (2.375 - 0.25I)]$$

In this formula, C is the percentage of pass completions, Y is the average number of yards gained per pass attempt, T is the percentage of touchdown passes, and I is the percentage of interceptions.

During the 2011 season, Aaron Rodgers, the quarterback of the Green Bay Packers, completed 68.3% of his passes. He averaged 9.25 yards per pass attempt, 9.0% of his passes were for touchdowns, and 1.2% of his passes were intercepted. Determine Rodgers's quarterback rating for the 2011 season.

Solution

Because C is defined as a percentage, $C = 68.3$. We are also given $Y = 9.25$, $T = 9.0$, and $I = 1.2$.

Substitute these values into the rating formula.

Answer • No. $x = \dfrac{c - b}{a}$, provided $a \neq 0$.

QB rating

$$= \frac{100}{6}[0.05(68.3 - 30) + 0.25(9.25 - 3) + 0.2(9.0) + (2.375 - 0.25(1.2))]$$

$$\approx 122.5$$

Aaron Rodgers's quarterback rating for the 2011 season was 122.5.

Try Exercise 20, page 92

Applications

Linear equations emerge in a variety of application problems. In solving such problems, it generally helps to apply specific techniques in a series of small steps. The following general strategies should prove helpful in the remaining portion of this section.

Strategies for Solving Application Problems

1. Read the problem carefully. If necessary, reread the problem several times.
2. When appropriate, draw a sketch and label parts of the drawing with the specific information given in the problem.
3. Determine the unknown quantities, and label them with variables. Write down any equation that relates the variables.
4. Use the information from step 3, along with a known formula or some additional information given in the problem, to write an equation.
5. Solve the equation obtained in step 4, and check to see whether the results satisfy all the conditions of the original problem.

EXAMPLE 3 Dimensions of a Painting

A dog agility course must be contained inside a rectangle where the ratio of the length to the width is 1.2. If there are 440 feet of fencing available to enclose the course, what are the dimensions of the rectangle?

Photo Courtesy of Lois Aufmann

Solution

1. Read the problem carefully.
2. Draw a rectangle and label it. We have used w for its width and l for its length. See Figure 1.3.
3. The problem states that the ratio of length to width is 1.2. Write this as an equation.

   $\frac{l}{w} = 1.2$ or, multiplying each side of the equation by w, $l = 1.2w$.

   The problem also states that the perimeter is 440 feet. Write this as an equation. The formula for perimeter is $P = 2l + 2w$.
4. Combine the information from the preceding steps to write an equation.

$$P = 2l + 2w$$

$$440 = 2(1.2w) + 2w \quad \bullet\ P = 440,\ l = 1.2w$$

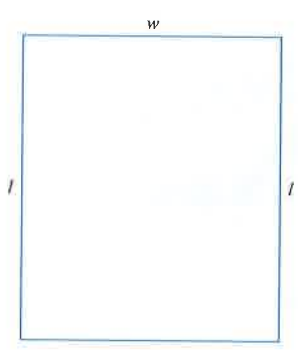


Figure 1.3

(continued)

5. Solve for w.

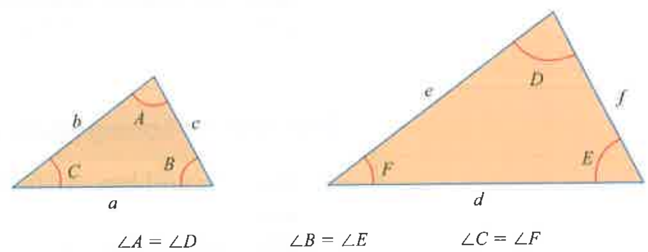
$$440 = 2(1.2w) + 2w$$
$$440 = 2.4w + 2w$$
$$440 = 4.4w$$
$$100 = w$$

The width is 100 feet. The length is $l = 1.2(100) = 120$.

The width of the rectangle is 100 feet, and the length is 120 feet.

Try Exercise 24, page 92

Similar triangles are ones for which the measures of corresponding angles are equal. The triangles below are similar.



$\angle A = \angle D$ $\angle B = \angle E$ $\angle C = \angle F$

An important relationship among the sides of similar triangles is that the ratios of corresponding sides are equal. Thus, for the triangles above,

$$\frac{a}{b} = \frac{d}{e} \qquad \frac{a}{c} = \frac{d}{f} \qquad \frac{b}{c} = \frac{e}{f}$$

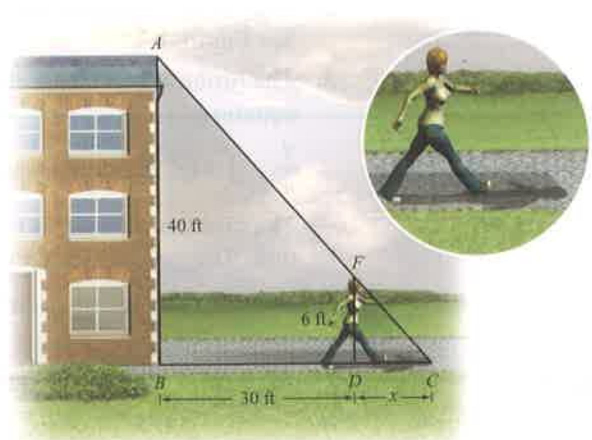
This fact is used in many applications.

EXAMPLE 4 A Problem Involving Similar Triangles

A person 6 feet tall is in the shadow of a building 40 feet tall and is walking directly away from the building. When the person is 30 feet from the building, the tip of the person's shadow is at the same point as the tip of the shadow of the building. How much farther must the person walk to be just out of the shadow of the building? Round to the nearest tenth of a foot.

Solution

Let x be the distance the person has to walk. Draw a picture of the situation using similar triangles.



Triangles ABC and FDC are similar triangles. Therefore, the ratios of the lengths of the corresponding sides are equal. Using this fact, we can write an equation.

$$\frac{30 + x}{40} = \frac{x}{6}$$

Now solve the equation.

$$\frac{30 + x}{40} = \frac{x}{6}$$

$$120\left(\frac{30 + x}{40}\right) = 120\left(\frac{x}{6}\right)$$ • Multiply each side by 120, the LCD of 40 and 6.

$$3(30 + x) = 20x$$ • Solve for x.

$$90 + 3x = 20x$$

$$90 = 17x$$

$$5.3 \approx x$$

The person must walk an additional 5.3 feet.

▶ **Try Exercise 30, page 93**

Many business applications can be solved by using the equation

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

EXAMPLE 5 A Business Application

It costs a tennis shoe manufacturer \$37.15 to produce a pair of tennis shoes that sells for \$69.95. How many pairs of tennis shoes must the manufacturer sell to make a profit of \$20,172.00?

Solution

The *profit* is equal to the *revenue* minus the *cost*. If x equals the number of pairs of tennis shoes to be sold, then the revenue will be \$69.95x and the cost will be \$37.15x. Therefore,

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$20{,}172.00 = 69.95x - 37.15x$$

$$20{,}172.00 = 32.80x$$

$$615 = x$$

The manufacturer must sell 615 pairs of tennis shoes to make the desired profit.

▶ **Try Exercise 32, page 93**

Simple interest problems can be solved by using the formula $I = Prt$, where I is the interest, P is the principal, r is the simple interest rate per period, and t is the number of periods.

EXAMPLE 6 An Investment Problem

An accountant invests part of a \$6000 bonus in a 5% simple interest account and invests the remainder of the money at 8.5% simple interest. Together the investments earn \$370 per year. Find the amount invested at each rate.

(continued)

Solution

Let x be the amount invested at 5%. The remainder of the money is $\$6000 - x$, which is the amount invested at 8.5%. Using the simple interest formula $I = Prt$ with $t = 1$ year yields

$$\text{Interest at } 5\% = x \cdot 0.05 = 0.05x$$
$$\text{Interest at } 8.5\% = (6000 - x) \cdot (0.085) = 510 - 0.085x$$

The interest earned on the two accounts equals $370.

$$\begin{aligned} 0.05x + (510 - 0.085x) &= 370 \\ -0.035x + 510 &= 370 \\ -0.035x &= -140 \\ x &= 4000 \end{aligned}$$

The accountant invested $4000 at 5% and the remaining $2000 at 8.5%.

Try Exercise 38, page 93

Many *uniform motion problems* can be solved by using the formula $d = rt$, where d is the distance traveled, r is the rate of speed, and t is the time.

EXAMPLE 7 A Uniform Motion Problem

A runner runs a course at a constant speed of 6 mph. One hour after the runner begins, a cyclist starts on the same course at a constant speed of 15 mph. How long after the runner starts does the cyclist overtake the runner?

Solution

If we represent the time the runner has spent on the course by t, then the time the cyclist takes to overtake the runner is $t - 1$. The following table organizes the information and helps us determine how to write the distance each person travels.

	Rate r	·	Time t	=	Distance d
Runner	6	·	t	=	$6t$
Cyclist	15	·	$t - 1$	=	$15(t - 1)$

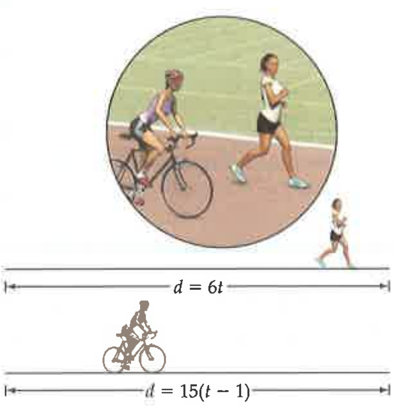


Figure 1.4

Figure 1.4 indicates that the runner and the cyclist cover the same distance. Thus

$$\begin{aligned} 6t &= 15(t - 1) \\ 6t &= 15t - 15 \\ -9t &= -15 \\ t &= 1\frac{2}{3} \end{aligned}$$

The cyclist overtakes the runner $1\frac{2}{3}$ hours after the runner starts.

Try Exercise 42, page 93

Percent mixture problems involve combining solutions or alloys that have different concentrations of a common substance. Percent mixture problems can be solved by using the formula $pA = Q$, where p is the percent of concentration (in decimal form), A is the amount of the solution or alloy, and Q is the quantity of a substance in the solution or alloy. For example, in 4 liters of a 25% acid solution, p is the percent of acid (0.25 as a decimal), A is the amount of solution (4 liters), and Q is the amount of acid in the solution, which equals (0.25)(4) liters = 1 liter.

EXAMPLE 8 A Percent Mixture Problem

A chemist mixes an 11% hydrochloric acid solution with a 6% hydrochloric acid solution. How many milliliters of each solution should the chemist use to make a 600-milliliter solution that is 8% hydrochloric acid?

Solution

Let x be the number of milliliters of the 11% solution. Because the solution after mixing will have a total of 600 milliliters of fluid, $600 - x$ is the number of milliliters of the 6% solution. See Figure 1.5.

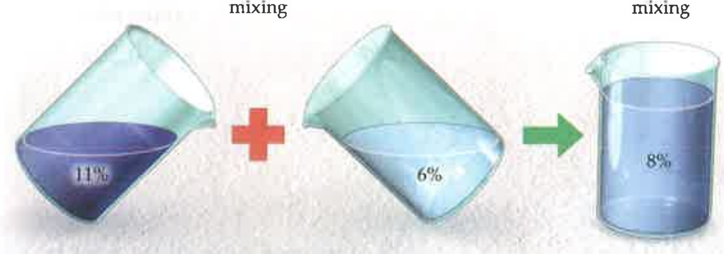


Figure 1.5

Because all the hydrochloric acid in the solution after mixing comes from either the 11% solution or the 6% solution, the number of milliliters of hydrochloric acid in the 11% solution added to the number of milliliters of hydrochloric acid in the 6% solution must equal the number of milliliters of hydrochloric acid in the 8% solution.

$$\begin{pmatrix}\text{ml of acid in}\\ \text{11\% solution}\end{pmatrix} + \begin{pmatrix}\text{ml of acid in}\\ \text{6\% solution}\end{pmatrix} = \begin{pmatrix}\text{ml of acid in}\\ \text{8\% solution}\end{pmatrix}$$

$$0.11x + 0.06(600 - x) = 0.08(600)$$

$$\begin{aligned} 0.11x + 36 - 0.06x &= 48 \\ 0.05x + 36 &= 48 \\ 0.05x &= 12 \\ x &= 240 \end{aligned}$$

The chemist should use 240 milliliters of the 11% solution and 360 milliliters of the 6% solution to make a 600-milliliter solution that is 8% hydrochloric acid.

▶ **Try Exercise 50, page 94**

Value mixture problems involve combining two or more ingredients that have different prices into a single blend. The solution of a value mixture problem is based on the equation $V = CA$, where V is the value of the ingredient, C is the unit cost of the ingredient, and A is the amount of the ingredient.

For instance, if the cost C of tea is \$4.30 per pound, then 5 pounds (the amount A) of tea has a value $V = (4.30)(5) = 21.50$, or \$21.50. The solution of a value mixture problem is based on the sum of the values of all ingredients taken separately equaling the value of the mixture.

EXAMPLE 9 A Value Mixture Problem

How many ounces of pure silver that costs \$30.50 per ounce must be mixed with 60 ounces of a silver alloy that costs \$24.35 per ounce to produce a silver alloy that costs \$26.00 per ounce?

Solution

Let x be the number of ounces of pure silver being added. The value of the silver added is $30.50x$. The value of the 60 ounces of the existing alloy is 24.35(60). Mixing the x ounces of the pure silver to the 60 ounces of the existing alloy yields an alloy that contains $(x + 60)$ ounces. The value of the new alloy is $26.00(x + 60)$.

$$\begin{aligned} \begin{pmatrix}\text{Value of}\\ \text{pure silver}\end{pmatrix} + \begin{pmatrix}\text{Value of}\\ \text{existing alloy}\end{pmatrix} &= \begin{pmatrix}\text{Value of}\\ \text{new alloy}\end{pmatrix} \\ 30.50x \quad + \quad 24.35(60) &= 26.00(x + 60) \\ 30.50x + 1461 &= 26x + 1560 \\ 4.5x + 1461 &= 1560 \\ 4.5x &= 99 \\ x &= 22 \end{aligned}$$

22 ounces of pure silver must be added.

▶ **Try Exercise 60, page 94**

To solve a *work problem*, use the equation

$$\text{Rate of work} \times \text{Time worked} = \text{Part of task completed}$$

For example, if a painter can paint a wall in 15 minutes, then the painter can paint $\frac{1}{15}$ of the wall in 1 minute. The painter's *rate of work* is $\frac{1}{15}$ of the wall each minute. In general, if a task can be completed in x minutes, then the rate of work is $\frac{1}{x}$ of the task each minute.

EXAMPLE 10 A Work Problem

Pump A can fill a pool in 6 hours, and pump B can fill the same pool in 3 hours. How long will it take to fill the pool if both pumps are used?

Solution

Because pump A fills the pool in 6 hours, $\frac{1}{6}$ represents the part of the pool filled by pump A in 1 hour. Because pump B fills the pool in 3 hours, $\frac{1}{3}$ represents the part of the pool filled by pump B in 1 hour.

Let t equal the number of hours to fill the pool using both pumps. Then

$$t \cdot \frac{1}{6} = \frac{t}{6}$$ • Part of the pool filled by pump A

$$t \cdot \frac{1}{3} = \frac{t}{3}$$ • Part of the pool filled by pump B

$$\begin{pmatrix}\text{Part filled}\\ \text{by pump A}\end{pmatrix} + \begin{pmatrix}\text{Part filled}\\ \text{by pump B}\end{pmatrix} = \begin{pmatrix}\text{1 filled}\\ \text{pool}\end{pmatrix}$$

$$\frac{t}{6} + \frac{t}{3} = 1$$

Multiplying each side of the equation by 6 produces

$$t + 2t = 6$$
$$3t = 6$$
$$t = 2$$

Check: Pump A fills $\frac{2}{6}$, or $\frac{1}{3}$, of the pool in 2 hours and pump B fills $\frac{2}{3}$ of the pool in 2 hours, so 2 hours is the time required to fill the pool if both pumps are used.

Try Exercise 62, page 94

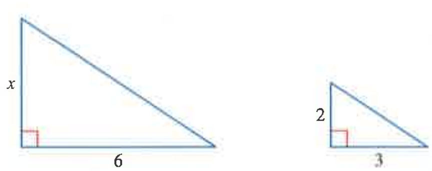
EXERCISE SET 1.2

Concept Check

1. What is the first step of solving the formula $A = P + Prt$ for r?

2. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$. Find the value of A if $h = 6$, $b_1 = 5$, and $b_2 = 7$.

3. The two right triangles shown below are similar. What must $\frac{x}{6}$ equal?



4. Part of \$4000 is invested in a 3% simple interest account and the remainder in a 5% simple interest account. If x represents the amount invested at 3%, what expression represents the amount invested at 5%? Write an expression for the amount of interest earned on the two accounts together in 1 year.

5. Suppose x pounds of almonds that cost \$7 per pound are mixed with 10 pounds of walnuts that cost \$9.00 per pound. Write an expression that represents the total value, in dollars, of the nut mixture.

6. Marya can mow her lawn in 45 minutes. Write an expression that represents the fraction of the lawn that she can mow in m minutes.

In Exercises 7 to 16, solve the formula for the specified variable.

7. $V = \frac{1}{3}\pi r^2 h$; h (geometry)

8. $P = S - Sdt$; t (business)

9. $I = Prt$; t (business)

10. $A = P + Prt$; P (business)

11. $F = \frac{Gm_1m_2}{d^2}$; m_1 (physics)

12. $A = \frac{1}{2}h(b_1 + b_2)$; b_1 (geometry)

13. $a_n = a_1 + (n - 1)d$; d (mathematics)

14. $y - y_1 = m(x - x_1)$; x (mathematics)

15. $S = \frac{a_1}{1 - r}$; r (mathematics)

Indicates Try It Exercises

16. $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$; V_2 (chemistry)

17. **Quarterback Rating** During the 2011 season, Matt Schaub, the quarterback of the Houston Texans, completed 61% of his passes. He averaged 8.49 yards per pass attempt, 5.1% of his passes were for touchdowns, and 2.05% of his passes were intercepted. Determine Schaub's quarterback rating for the 2011 season. Round to the nearest tenth. (*Hint*: See Example 2, page 84.)

18. **Quarterback Rating** During the 2011 season, Alex Smith, the quarterback of the San Francisco 49ers, completed 61.3% of his passes. He averaged 7.07 yards per pass attempt, 3.8% of his passes were for touchdowns, and 1.1% of his passes were intercepted. Determine Smith's quarterback rating for the 2011 season. Round to the nearest tenth. (*Hint*: See Example 2, page 84.)

The simplified measure of gobbledygook (SMOG) readability formula is often used to estimate the reading grade level required if a person is to fully understand the written material being assessed. The formula is given by

$$\text{SMOG reading grade level} = \sqrt{w} + 3$$

where *w* is the number of words that have three or more syllables in a sample of 30 sentences. Use this information in Exercises 19 and 20.

19. A sample of 30 sentences from *Alice's Adventures in Wonderland*, by Lewis Carroll, shows a total of 42 words that have three or more syllables. Use the SMOG reading grade level formula to estimate the reading grade level required to fully understand this novel. Round the reading grade level to the nearest tenth.

Mary Evan Picture Library/Image Works

20. A sample of 30 sentences from *A Tale of Two Cities*, by Charles Dickens, shows a total of 105 words that have three or more syllables. Use the SMOG reading grade level formula to estimate the reading grade level required to fully understand this novel. Round the reading grade level to the nearest tenth.

Another reading level formula is the Gunning-Fog Index. Here is the formula.

$$\text{Gunning-Fog Index} = 0.4\,(A + P)$$

where *A* is the average number of words per sentence and *P* is the percentage of words that have three or more syllables. The Gunning-Fog Index is defined as the minimum grade level required if a person is to easily understand the text on the first reading. Use this information in Exercises 21 and 22.

21. In a large sample of sentences from the novel *The Red Badge of Courage*, by Stephen Crane, the average number of words per sentence is 14.8 and the percentage of words with three or more syllables is 15.1. Use the Gunning-Fog Index formula to estimate the reading grade level required to easily understand this novel. Round the grade level to the nearest tenth.

22. In a large sample of sentences from the novel *Emma*, by Jane Austen, the average number of words per sentence is 18.8 and the percentage of words with three or more syllables is 14.2. Use the Gunning-Fog Index formula to estimate the reading grade level required to easily understand this novel. Round the grade level to the nearest tenth.

In Exercises 23 to 66, solve by using the strategies for solving application problems (see page 85).

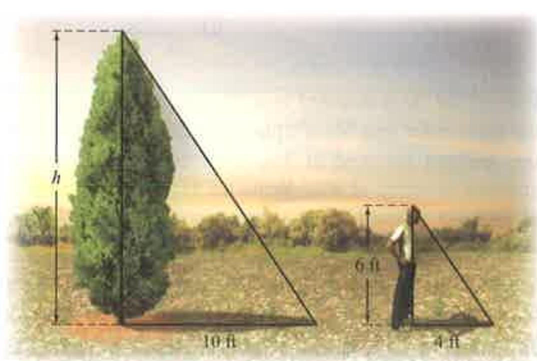
23. **Geometry** The length of a rectangle is 3 feet less than twice the width of the rectangle. If the perimeter of the rectangle is 174 feet, find the width and the length.

24. **Sports** The ratio of the length (including the endzone) to the width of an NFL football field is 2.25. If the perimeter of the football field is 1040 feet, what are the dimensions of the field?

25. **Geometry** A triangle has a perimeter of 84 centimeters. Each of the two longer sides of the triangle is three times as long as the shortest side. Find the length of each side of the triangle.

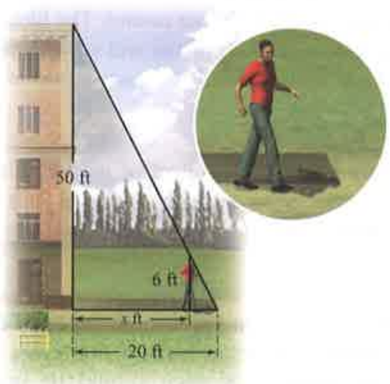
26. **Geometry** A triangle has a perimeter of 161 miles. The length of each of the two shorter sides of the triangle is two-thirds the length of the longest side. Find the length of each side of the triangle.

27. **Height of a Tree** One way to approximate the height of a tree is to measure its shadow and then measure the shadow of a known height. Use similar triangles and the diagram below to estimate the height of the tree.

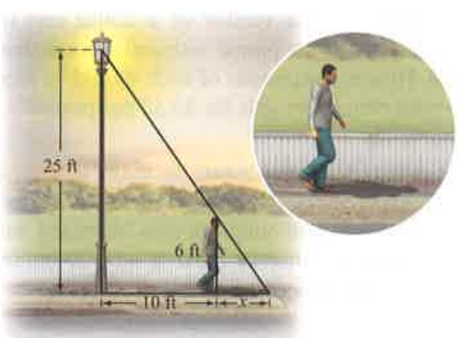


28. **Height of a Building** A building casts a shadow 50 feet long. A rod 4 feet tall placed near the building casts a shadow 3 inches long. Use similar triangles to determine the height of the building.

29. **Shadow Length** A building 50 feet tall casts a shadow 20 feet long. A person 6 feet tall is walking directly away from the building toward the edge of the building's shadow. How far from the building will the person be when the person's shadow just begins to emerge from that of the building?



30. **Shadow Length** A person 6 feet tall is standing at the base of a lamppost that is 25 feet tall and then begins to walk away from the lamppost. When the person is 10 feet from the lamppost, what is the length of the person's shadow? Round to the nearest tenth of a foot.



31. **Business** It costs a manufacturer $8.95 to produce sunglasses that sell for $29.99. How many pairs of sunglasses must the manufacturer sell to make a profit of $17,884?

32. **Business** It costs a restaurant owner $0.28 per glass for orange juice, which sells for $1.75 per glass. How many glasses of orange juice must the restaurant owner sell to make a profit of $2058?

33. **Determine Individual Prices** A book and a bookmark together sell for $10.10. If the price of the book is $10.00 more than the price of the bookmark, find the price of the book and the price of the bookmark.

34. **Share an Expense** Three people decide to share the cost of a yacht. By bringing in an additional partner, they can reduce the cost to each person by $4000. What is the total cost of the yacht?

35. **Business** The price of a computer fell 20% this year. If the computer now costs $750, how much did it cost last year?

36. **Business** The price of a magazine subscription rose 4% this year. If the subscription now costs $26, how much did it cost last year?

37. **Investment** An investment adviser invested $14,000 in two accounts. One investment earned 8% annual simple interest, and the other investment earned 6.5% annual simple interest. The amount of interest earned for 1 year was $1024. How much was invested in each account?

38. **Investment** A total of $7500 is deposited into two simple interest accounts. In one account, the annual simple interest rate is 5%, and in the second account, the annual simple interest rate is 7%. The amount of interest earned for 1 year was $405. How much was invested in each account?

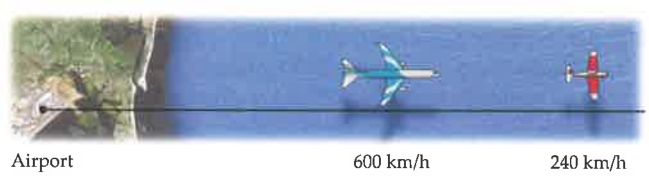
39. **Investment** An investment of $2500 is made at an annual simple interest rate of 5.5%. How much additional money must be invested at an annual simple interest rate of 8% so that the total interest earned is 7% of the total investment?

40. **Investment** An investment of $4600 is made at an annual simple interest rate of 6.8%. How much additional money must be invested at an annual simple interest rate of 9% so that the total interest earned is 8% of the total investment?

41. **Uniform Motion** Running at an average rate of 6 meters per second, a sprinter ran to the end of a track. The sprinter then jogged back to the starting point at an average rate of 2 meters per second. The total time for the sprint and the jog back was 2 minutes 40 seconds. Find the length of the track.

42. **Uniform Motion** A motorboat left a harbor and traveled to an island at an average rate of 15 knots. The average speed on the return trip was 10 knots. If the total trip took 7.5 hours, how many nautical miles is the harbor from the island?

43. **Uniform Motion** A plane leaves an airport traveling at an average speed of 240 kilometers per hour. How long will it take a second plane traveling the same route at an average speed of 600 kilometers per hour to catch up with the first plane if it leaves 3 hours later?



44. Uniform Motion A plane leaves Chicago headed for Los Angeles at 540 mph. One hour later, a second plane leaves Los Angeles headed for Chicago at 660 mph. If the air route from Chicago to Los Angeles is 1800 miles, how long will it take for the first plane to pass the second plane? How far from Chicago will they be at that time?

45. Speed of Sound in Air Two seconds after firing a rifle at a target, the shooter hears the impact of the bullet. Sound travels at 1100 feet per second and the bullet at 1865 feet per second. Determine the distance to the target (to the nearest foot).

46. Speed of Sound in Water Sound travels through seawater 4.62 times as fast as through air. The sound of an exploding mine on the surface of the water and partially submerged reaches a ship through the water 4 seconds before it reaches the ship through the air. How far is the ship from the explosion (to the nearest foot)? Use 1100 feet per second as the speed of sound through air.

47. Uniform Motion A car traveling at 80 kilometers per hour is passed by a second car going in the same direction at a constant speed. After 30 seconds, the two cars are 500 meters apart. Find the speed of the second car.

48. Uniform Motion Two planes leave Chicago at the same time, one traveling east and the other traveling west. The two planes fly at the same speed, r miles per hour. Sometime between 2 and 4 hours after they leave, they are 1500 miles apart. What are all the possible values of r?

49. Metallurgy How many grams of pure silver must a silversmith mix with a 45% silver alloy to produce 200 grams of a 50% alloy?

50. Chemistry How many liters of a 40% sulfuric acid solution should be mixed with 4 liters of a 24% sulfuric acid solution to produce a 30% solution?

51. Chemistry How many liters of water should be evaporated from 160 liters of a 12% saline solution so that the solution that remains is a 20% saline solution?

52. Automotive A radiator contains 6 liters of a 25% antifreeze solution. How much should be drained and replaced with pure antifreeze to produce a 33% antifreeze solution?

53. Metallurgy How much pure gold should be melted with 15 grams of 14-karat gold to produce 18-karat gold? (*Hint*: A karat is a measure of the purity of gold in an alloy. Pure gold measures 24 karats. An alloy that measures x karats is $\frac{x}{24}$ gold. For example, 18-karat gold is $\frac{18}{24} = \frac{3}{4}$ gold.)

54. Metallurgy How much 14-karat gold should be melted with 4 ounces of pure gold to produce 18-karat gold? (*Hint*: See Exercise 53.)

55. Tea Mixture A tea merchant wants to make 20 pounds of a blended tea costing $5.60 per pound. The blend is made using a $6.50-per-pound grade of tea and a $4.25-per-pound grade of tea. How many pounds of each grade of tea should be used?

56. Gold Alloy How many ounces of pure gold that costs $850 per ounce must be mixed with 25 ounces of a gold alloy that costs $500 per ounce to make a new alloy that costs $725 per ounce?

57. Trail Mix A grocery mixes some dried cranberries that cost $6 per pound with some granola that costs $3 per pound. How many pounds of each should be used to make a 25-pound mixture that costs $3.84 per pound?

58. Coffee Mixture A coffee shop decides to blend a coffee that sells for $12 per pound with a coffee that sells for $9 per pound to produce a blend that will sell for $10 per pound. How much of each should be used to yield 20 pounds of the new blend?

59. Coffee Mixture The vendor of a coffee cart mixes coffee beans that cost $8 per pound with coffee beans that cost $4 per pound. How many pounds of each should be used to make a 50-pound blend that sells for $5.50 per pound?

60. Silver Alloy A jeweler wants to make a silver alloy to be used to make necklaces. How many ounces of a silver alloy that costs $6.50 per ounce should be mixed with one that costs $8.00 per ounce to make a new 20-ounce alloy that costs $7.40 per ounce?

61. Install Electrical Wires An electrician can install the electric wires in a house in 14 hours. A second electrician requires 18 hours. How long would it take both electricians, working together, to install the wires?

62. Print a Report Printer A can print a report in 3 hours. Printer B can print the same report in 4 hours. How long would it take both printers, working together, to print the report?

63. Painting A painter can paint a kitchen in 10 hours. An apprentice can paint the same kitchen in 15 hours. If they worked together, how long would it take them to paint the kitchen?

64. **Sports** A snowmaking machine at a ski resort can produce enough snow for a beginner's ski trail in 16 hours. With a typical natural snowfall, it takes 24 hours to deposit enough snow to open the beginner's ski trail. If the snowmaking machine is run during a typical natural snowfall, how long will it take to deposit enough snow to open the beginner's trail?

65. **Road Construction** A new machine that deposits cement for a road requires 12 hours to complete a one-half mile section of road. An older machine requires 16 hours to pave the same amount of road. After depositing cement for 4 hours, the new machine develops a mechanical problem and quits working. The older machine is brought into place and continues the job. How long does it take the older machine to complete the job?

66. **Masonry** A mason can lay the bricks in a sidewalk in 12 hours. The mason's apprentice requires 16 hours to do the same job. After working together for 4 hours, the mason leaves for another job, and the apprentice continues working. How long will it take the apprentice to complete the job?

Enrichment Exercises

67. **Chemistry** A large vat contains a solution that is 30% salt and 70% water. Eight kilograms of the solution are removed from the vat and placed in an open container. After 2 kg of water evaporate from the open container, a lab technician takes an additional 2 kilograms of solution from the vat and places it in the open container. Now what is the percent salt concentration in the open container?

68. **Chemistry** A 10-pound solution is 99% water and 1% sugar. After some of the water evaporates, the solution is 98% water. Now what is the weight of the solution?

69. **Uniform Motion** Marlene rides her bicycle to her friend Jon's house and returns home by the same route. Marlene rides her bike at constant speeds of 6 mph on level ground, 4 mph when going uphill, and 12 mph when going downhill. If her total time riding was 1 hour, how far is it to Jon's house? (*Hint*: Let d_1 be the distance traveled on level ground and let d_2 be the distance traveled on the hill. Then the distance between the two houses is $d_1 + d_2$. Write an equation for the total time. For instance, the time spent in traveling to Jon's house on level ground is $\frac{d_1}{6}$.)

SECTION 1.3

- Solving Quadratic Equations by Factoring
- Solving Quadratic Equations by Taking Square Roots
- Solving Quadratic Equations by Completing the Square
- Solving Quadratic Equations by Using the Quadratic Formula
- The Discriminant of a Quadratic Equation
- Applications of Quadratic Equations

Quadratic Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A5.

PS1. Factor: $x^2 - x - 42$ [P.4]

PS2. Factor: $6x^2 - x - 15$ [P.4]

PS3. Write $3 + \sqrt{-16}$ in $a + bi$ form. [P.6]

PS4. If $a = -3$, $b = -2$, and $c = 5$, evaluate $\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ [P.1/P.2]

PS5. If $a = 2$, $b = -3$, and $c = 1$, evaluate $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ [P.1/P.2]

PS6. If $x = 3 - i$, evaluate $x^2 - 6x + 10$. [P.6]

Solving Quadratic Equations by Factoring

In Section 1.1 you solved linear equations. In this section you will learn to solve a type of equation that is referred to as a *quadratic equation.*

Math Matters

The term *quadratic* is derived from the Latin word *quadrare*, which means "to make square." Because the area of a square that measures x units on each side is x^2, we refer to equations that can be written in the form $ax^2 + bx + c = 0$ as equations that are "quadratic in x."

Definition of a Quadratic Equation

A **quadratic equation** in x is an equation that can be written in the **standard quadratic form**

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$.

Several methods can be used to solve a quadratic equation. For instance, if you can factor $ax^2 + bx + c$ into linear factors, then $ax^2 + bx + c = 0$ can be solved by applying the following property.

The Zero Product Principle

If A and B are algebraic expressions such that $AB = 0$, then $A = 0$ or $B = 0$.

The zero product principle states that if the product of two factors is 0, then at least one of the factors must be 0. In Example 1, the zero product principle is used to solve a quadratic equation.

EXAMPLE 1 Solve by Factoring

Solve each quadratic equation by factoring.

a. $x^2 + 2x - 15 = 0$ **b.** $2x^2 - 5x = 12$

Solution

a. $x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$ • Factor.

$x - 3 = 0$ or $x + 5 = 0$ • Set each factor equal to 0.

$x = 3$ $x = -5$ • Solve each linear equation.

A check shows that 3 and -5 are both solutions of $x^2 + 2x - 15 = 0$.

b. $2x^2 - 5x = 12$

$2x^2 - 5x - 12 = 0$ • Write in standard quadratic form.

$(x - 4)(2x + 3) = 0$ • Factor.

$x - 4 = 0$ or $2x + 3 = 0$ • Set each factor equal to 0.

$x = 4$ $2x = -3$ • Solve each linear equation.

$x = -\frac{3}{2}$

A check shows that 4 and $-\frac{3}{2}$ are both solutions of $2x^2 - 5x = 12$.

▸ Try Exercise 12, page 105

Some quadratic equations have a solution that is called a *double root*. For instance, consider $x^2 - 8x + 16 = 0$. Solving this equation by factoring, we have

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$ • Factor.

$x - 4 = 0$ or $x - 4 = 0$ • Set each factor equal to 0.

$x = 4$ $x = 4$ • Solve each linear equation.

The only solution of $x^2 - 8x + 16 = 0$ is 4. In this situation, the single solution 4 is called a **double solution** or **double root** because it was produced by solving the two identical equations $x - 4 = 0$, both of which have 4 as a solution.