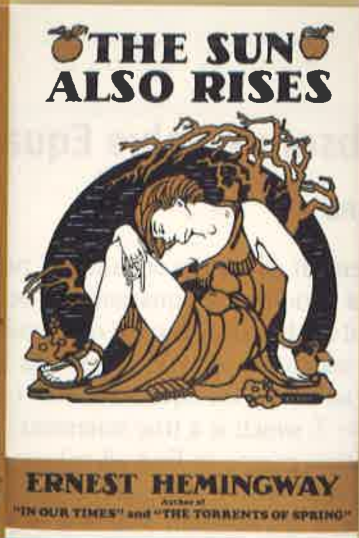
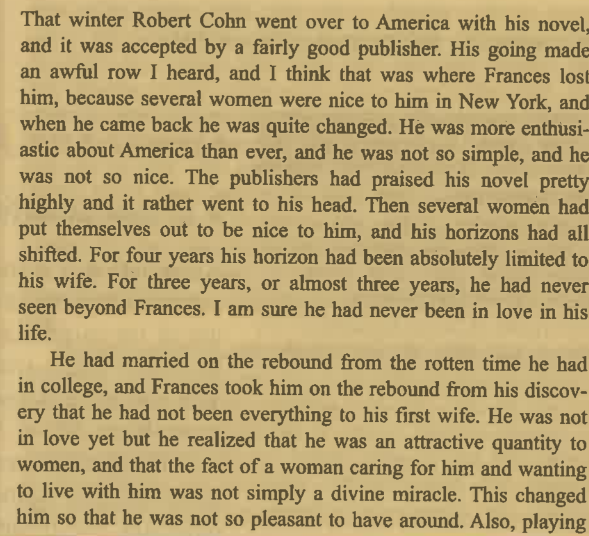



That winter Robert Cohn went over to America with his novel, and it was accepted by a fairly good publisher. His going made an awful row I heard, and I think that was where Frances lost him, because several women were nice to him in New York, and when he came back he was quite changed. He was more enthusiastic about America than ever, and he was not so simple, and he was not so nice. The publishers had praised his novel pretty highly and it rather went to his head. Then several women had put themselves out to be nice to him, and his horizons had all shifted. For four years his horizon had been absolutely limited to his wife. For three years, or almost three years, he had never seen beyond Frances. I am sure he had never been in love in his life.

He had married on the rebound from the rotten time he had in college, and Frances took him on the rebound from his discovery that he had not been everything to his first wife. He was not in love yet but he realized that he was an attractive quantity to women, and that the fact of a woman caring for him and wanting to live with him was not simply a divine miracle. This changed him so that he was not so pleasant to have around. Also, playing

CHAPTER 1 Equations and Inequalities

Gobbledygook

Gobbledygook is usually considered language that is unintelligible jargon or purposely difficult to understand. However, there is another context in which the word *gobbledygook* occurs. Simplified Measure of Gobbledygook (SMOG) is a readability formula created by G. Harry McLaughlin. It can be used by educators to determine the grade level of reading material given to students. The SMOG formula is a radical equation, one of the topics in this chapter. See Exercises 19 and 20 on page 92. The book *The Sun Also Rises* by Ernest Hemingway has a SMOG readability grade level of approximately 12.

SECTION 1.1

Linear and Absolute Value Equations

Linear Equations

An **equation** is a statement about the equality of two expressions. If either of the expressions contains a variable, the equation may be a true statement for some values of the variable and a false statement for other values. For example, the equation $2x + 1 = 7$ is a true statement for $x = 3$, but it is false for any number except 3. The number 3 is said to **satisfy** the equation $2x + 1 = 7$ because substituting 3 for x produces $2(3) + 1 = 7$, which is a true statement.

To **solve** an equation means to find all values of the variable that satisfy the equation. The values that satisfy an equation are called **solutions** or **roots** of the equation. For instance, 2 is a solution of $x + 3 = 5$.

Equivalent equations are equations that have exactly the same solution or solutions. The process of solving an equation is often accomplished by producing a sequence of equivalent equations until we arrive at an equation or equations of the form

$$\text{Variable} = \text{Constant}$$

To produce these equivalent equations, apply the properties of real numbers and the following two properties of equality.

Addition and Subtraction Property of Equality

Adding the same expression to each side of an equation or subtracting the same expression from each side of an equation produces an equivalent equation.

EXAMPLE

Begin with the equation $2x - 7 = 11$. Replacing x with 9 shows that 9 is a solution of the equation. Now add 7 to each side of the equation. The resulting equation is $2x = 18$, and the solution of the new equation is still 9.

Multiplication and Division Property of Equality

Multiplying or dividing each side of an equation by the same nonzero expression produces an equivalent equation.

EXAMPLE

Begin with the equation $\frac{2}{3}x = 8$. Replacing x with 12 shows that 12 is a solution of the equation. Now multiply each side of the equation by $\frac{3}{2}$. The resulting equation is $x = 12$, and the solution of the new equation is still 12.

Many applications can be modeled by *linear equations* in one variable.

Definition of a Linear Equation

A **linear equation,** or first-degree equation, in the single variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real numbers, with $a \neq 0$.

Linear equations are solved by applying the properties of real numbers and the properties of equality.

EXAMPLE 1 Solve a Linear Equation in One Variable

Solve: $3x - 5 = 7x - 11$

Solution

$$3x - 5 = 7x - 11$$

$$3x - 7x - 5 = 7x - 7x - 11$$ • Subtract $7x$ from each side of the equation.

$$-4x - 5 = -11$$

$$-4x - 5 + 5 = -11 + 5$$ • Add 5 to each side of the equation.

$$-4x = -6$$

$$\frac{-4x}{-4} = \frac{-6}{-4}$$ • Divide each side of the equation by -4.

$$x = \frac{3}{2}$$ • The equation is now in the form Variable = Constant.

As shown to the left, $\frac{3}{2}$ satisfies the original equation. The solution is $\frac{3}{2}$.

▸ Try Exercise 8, page 81

Study tip

You should check a proposed solution by substituting it back into the original equation.

$$3x - 5 = 7x - 11$$

$$3\left(\frac{3}{2}\right) - 5 \stackrel{?}{=} 7\left(\frac{3}{2}\right) - 11$$

$$\frac{9}{2} - 5 \stackrel{?}{=} \frac{21}{2} - 11$$

$$-\frac{1}{2} = -\frac{1}{2}$$

When an equation contains parentheses, use the distributive property to remove the parentheses.

EXAMPLE 2 Solve a Linear Equation in One Variable

Solve: $8 - 5(2x - 7) = 3(16 - 5x) + 5$

Solution

$$8 - 5(2x - 7) = 3(16 - 5x) + 5$$

$$8 - 10x + 35 = 48 - 15x + 5$$ • Use the distributive property.

$$-10x + 43 = -15x + 53$$ • Simplify.

$$-10x + 15x + 43 = -15x + 15x + 53$$ • Add $15x$ to each side of the equation.

$$5x + 43 = 53$$

$$5x + 43 - 43 = 53 - 43$$ • Subtract 43 from each side of the equation.

$$5x = 10$$

$$\frac{5x}{5} = \frac{10}{5}$$ • Divide each side of the equation by 5.

$$x = 2$$ • Check in the original equation.

The solution is 2.

▸ Try Exercise 12, page 81

If an equation involves fractions, it is helpful to multiply each side of the equation by the least common denominator (LCD) of all denominators to produce an equivalent equation that does not contain fractions.

EXAMPLE 3 Solve by Clearing Fractions

Solve: $\frac{2}{3}x + \frac{3}{5} = \frac{7}{3} - \frac{3}{10}x$

Solution

$$\frac{2}{3}x + \frac{3}{5} = \frac{7}{3} - \frac{3}{10}x$$

$$30\left(\frac{2}{3}x + \frac{3}{5}\right) = 30\left(\frac{7}{3} - \frac{3}{10}x\right)$$ • Multiply each side by 30, the LCD of the denominators.

$$30\left(\frac{2}{3}x\right) + 30\left(\frac{3}{5}\right) = 30\left(\frac{7}{3}\right) - 30\left(\frac{3}{10}x\right)$$ • Use the distributive property.

$$20x + 18 = 70 - 9x$$ • Simplify.

$$29x + 18 = 70$$ • Add $9x$ to each side of the equation.

$$29x = 52$$ • Subtract 18 from each side of the equation.

$$x = \frac{52}{29}$$ • Divide each side of the equation by 29.

The solution is $\frac{52}{29}$.

Try Exercise 18, page 81

Contradictions, Conditional Equations, and Identities

An equation that has no solutions is called a **contradiction.** The equation $x = x + 1$ is a contradiction. No number is equal to itself increased by 1.

An equation that is true for some values of the variable but not true for other values of the variable is called a **conditional equation.** For example, $x + 2 = 8$ is a conditional equation because it is true for $x = 6$ and false for any number not equal to 6.

An **identity** is an equation that is true for all values of the variable for which all terms of the equation are defined. It has an infinite number of solutions. Examples of identities include the equations $x + x = 2x$ and $4(x + 3) - 1 = 4x + 11$.

EXAMPLE 4 Classify Equations

Classify each equation as a contradiction, a conditional equation, or an identity.

a. $x + 1 = x + 4$

b. $4x + 3 = x - 9$

c. $5(3x - 2) - 7(x - 4) = 8x + 18$

Solution

a. Subtract x from both sides of $x + 1 = x + 4$ to produce the equivalent equation $1 = 4$. Because $1 = 4$ is a false statement, the original equation $x + 1 = x + 4$ has no solutions. It is a contradiction.

b. Solve using the procedures that produce equivalent equations.

$$4x + 3 = x - 9$$

$$3x + 3 = -9$$ • Subtract x from each side.

$$3x = -12$$ • Subtract 3 from each side.

$$x = -4$$ • Divide each side by 3.

Check to confirm that -4 is a solution. The equation $4x + 3 = x - 9$ is true for $x = -4$, but it is not true for any other values of x. Thus $4x + 3 = x - 9$ is a conditional equation.

c. Simplify the left side of the equation to show that it is *identical* to the right side.

$$5(3x - 2) - 7(x - 4) = 8x + 18$$
$$15x - 10 - 7x + 28 = 8x + 18$$
$$8x + 18 = 8x + 18$$

The original equation $5(3x - 2) - 7(x - 4) = 8x + 18$ is true for all real numbers x. The equation is an identity.

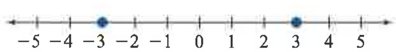
Try Exercise 28, page 81

Question • Dividing each side of $x = 4x$ by x produces $1 = 4$. Are the equations $x = 4x$ and $1 = 4$ equivalent equations?

Absolute Value Equations

The absolute value of a real number x is the distance between the number x and the number 0 on the real number line. Thus the solutions of $|x| = 3$ are all real numbers that are 3 units from 0. Therefore, the solutions of $|x| = 3$ are $x = 3$ or $x = -3$. See Figure 1.1.

The following property is used to solve absolute value equations.



$|x| = 3$

Figure 1.1

A Property of Absolute Value Equations

For any variable expression E and any nonnegative real number k,

$$|E| = k \quad \text{if and only if} \quad E = k \quad \text{or} \quad E = -k$$

EXAMPLE

If $|x| = 5$, then $x = 5$ or $x = -5$.

If $|x| = \frac{3}{2}$, then $x = \frac{3}{2}$ or $x = -\frac{3}{2}$.

If $|x| = 0$, then $x = 0$.

Note

Some absolute value equations have no solutions. For example, $|x + 2| = -5$ is false for all values of x. Because an absolute value is always nonnegative, the equation is never true.

EXAMPLE 5 Solve an Absolute Value Equation

Solve: $|2x - 5| = 21$

Solution

$|2x - 5| = 21$ implies $2x - 5 = 21$ or $2x - 5 = -21$. Solving each of these linear equations produces

$$2x - 5 = 21 \quad \text{or} \quad 2x - 5 = -21$$
$$2x = 26 \qquad\qquad 2x = -16$$
$$x = 13 \qquad\qquad x = -8$$

The solutions are -8 and 13.

Try Exercise 42, page 81

Answer • No. The real number 0 is a solution of $x = 4x$, but 0 is not a solution of $1 = 4$.

Applications of Linear Equations

Linear equations often can be used to model real-world data.

EXAMPLE 6 Movie Theater Ticket Prices

Movie theater ticket prices have been increasing steadily in recent years (see Table 1.1). An equation that models the average U.S. movie theater ticket price p, in dollars, is given by

$$p = 0.293t + 6.590$$

where t is the number of years after 2006. (This means that $t = 0$ corresponds to 2006.) Use this equation to predict the year in which the average U.S. movie theater ticket price will reach \$9.00.

Table 1.1 Average U.S. Movie Theater Ticket Price

Year	Price (in dollars)
2006	6.55
2007	6.88
2008	7.18
2009	7.50
2010	7.89
2011	7.93

Source: Motion Picture Association of America, Inc., www.mpaa.org.

Solution

$$p = 0.293t + 6.590$$

$$9.00 = 0.293t + 6.590 \quad \text{• Substitute 9.00 for } p.$$

$$2.41 = 0.293t \quad \text{• Solve for } t.$$

$$t \approx 8.2$$

Our equation predicts that the average U.S. movie theater ticket price will reach \$9.00 about 8.2 years after 2006, which is 2014.

▶ Try Exercise 54, page 82

EXAMPLE 7 Driving Time

Alicia is driving along a highway that passes through Centerville (see Figure 1.2). Her distance d, in miles, from Centerville is given by the equation

$$d = |135 - 60t|$$

where t is the time in hours since the start of her trip and $0 \le t \le 5$. Determine when Alicia will be exactly 15 miles from Centerville.

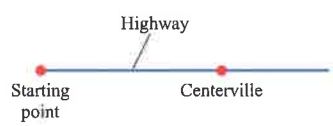


Figure 1.2

Solution

Substitute 15 for d.

$$d = |135 - 60t|$$

$$15 = |135 - 60t|$$

$$15 = 135 - 60t \quad \text{or} \quad -15 = 135 - 60t \quad \text{• Solve for } t.$$

$$-120 = -60t \qquad -150 = -60t$$

$$2 = t \qquad \frac{5}{2} = t$$

Alicia will be exactly 15 miles from Centerville after she has driven for 2 hours and after she has driven for $2\frac{1}{2}$ hours.

▶ Try Exercise 56, page 82

EXERCISE SET 1.1

Concept Check

1. Is 2 a solution of the equation $3(x - 4) + 5 = 2x - 5$?

2. By what number can you multiply each side of the equation $\frac{1}{5}x - 15 = \frac{3}{10}x$ in order to clear the equation of fractions?

3. What is the difference between a conditional equation and an identity?

4. If $|x + 9| = 5$, what are the possible values of $x + 9$? Rewrite the equation without absolute value signs.

In Exercises 5 to 26, solve each equation and check your solution.

5. $2x + 10 = 40$

6. $-3y + 20 = 2$

7. $5x + 2 = 2x - 10$

8. $4x - 11 = 7x + 20$

9. $2(x - 3) - 5 = 4(x - 5)$

10. $5(x - 4) - 7 = -2(x - 3)$

11. $3x + 5(1 - 2x) = 4 - 3(x + 1)$

12. $6 - 2(4x + 1) = 3x - 2(2x + 5)$

13. $4(2r - 17) + 5(3r - 8) = 0$

14. $6(5s - 11) - 12(2s + 5) = 0$

15. $\frac{3}{4}x + \frac{1}{2} = \frac{2}{3}$

16. $\frac{x}{4} - 5 = \frac{1}{2}$

17. $\frac{2}{3}x - 5 = \frac{1}{2}x - 3$

18. $\frac{1}{2}x + 7 - \frac{1}{4}x = \frac{19}{2}$

19. $0.2x + 0.4 = 3.6$

20. $0.04x - 0.2 = 0.07$

21. $x + 0.08(60) = 0.20(60 + x)$

22. $6(t + 1.5) = 12t$

23. $5[x - (4x - 5)] = 3 - 2x$

24. $6[3y - 2(y - 1)] - 2 + 7y = 0$

25. $\frac{40 - 3x}{5} = \frac{6x + 7}{8}$

26. $\frac{12 + x}{-4} = \frac{5x - 7}{3} + 2$

In Exercises 27 to 36, classify each equation as a contradiction, a conditional equation, or an identity.

27. $-3(x - 5) = -3x + 15$

28. $2x + \frac{1}{3} = \frac{6x + 1}{3}$

29. $2x + 7 = 3(x - 1)$

30. $4[2x - 5(x - 3)] = 6$

31. $\frac{4x + 8}{4} = x + 8$

32. $3[x - (4x - 1)] = -3(2x - 5)$

33. $3[x - 2(x - 5)] - 1 = -3x + 29$

34. $4[3(x - 5) + 7] = 12x - 32$

35. $2x - 8 = -x + 9$

36. $|3(x - 4) + 7| = |3x - 5|$

In Exercises 37 to 52, solve each absolute value equation for *x*.

37. $|x| = 4$

38. $|x| = 7$

39. $|x - 5| = 2$

40. $|x - 8| = 3$

41. $|2x - 5| = 11$

42. $|2x - 3| = 21$

43. $|2x + 6| = 10$

44. $|2x + 14| = 60$

45. $\left|\frac{x - 4}{2}\right| = 8$

46. $\left|\frac{x + 3}{4}\right| = 6$

47. $|2x + 5| = -8$

Indicates Try It Exercises

48. $|4x - 1| = -17$

49. $2|x + 3| + 4 = 34$

50. $3|x - 5| - 16 = 2$

51. $3|2x - 5| + 2 = 11$

52. $5 - 4|2 - 5x| = -7$

53. **Biology** The male magnificent frigatebird inflates a red pouch under his neck to attract females. Along with the inflated pouch, the bird makes a drumming-like sound whose frequency F, in hertz, is related to the volume V, in cubic centimeters, of the pouch by the equation $F = -5.5V + 5400$.

Martinmark/Dreamstime.com

Use the equation to estimate the volume of the pouch when the frequency of the sound is 550 hertz. Round to the nearest cubic centimeter.

54. **Health** According to one formula for lean body mass (LBM, in kilograms) given by R. Hume, the mass of the body minus fat is

$$\text{LBM} = 0.3281W + 0.3393H - 29.5336$$

where W is a person's weight in kilograms and H is the person's height in centimeters. If a person is 175 centimeters tall, what should that person weigh to have an LBM of 55 kilograms? Round to the nearest kilogram.

55. **Travel** Ruben is driving along a highway that passes through Barstow. His distance d, in miles, from Barstow is given by the equation $d = |210 - 50t|$, where t is the time, in hours, since the start of his trip and $0 \le t \le 6$. When will Ruben be exactly 60 miles from Barstow?

56. **Automobile Gas Mileage** The gas mileage m, in miles per gallon, obtained during a long trip is given by

$$m = -\frac{1}{2}|s - 55| + 25$$

where s is the speed of Kate's automobile in miles per hour and $40 \le s \le 70$. At what constant speed can Kate drive to obtain a gas mileage of exactly 22 miles per gallon?

57. **Fuel Consumption** An engine burns fuel at a constant rate of $\frac{1}{36}$ gallon per minute. If the engine originally contains 18 gallons of fuel, then the equation $g = 18 - \frac{1}{36}t$ gives the amount of fuel g, in gallons, t minutes after the engine is started. In how many minutes will the engine run out of gas?

58. **Diving Pressure** The pressure p, in pounds per square inch (psi), on a diver at a depth of d feet below the surface of the ocean can be approximated by the equation $p = 0.445d + 14.7$. If the pressure on a diver is 24 psi, find the depth of the diver. Round to the nearest tenth of a foot.

59. **Computer Science** If p% of a file remains to be downloaded using a cable modem, then

$$p = 100 - \frac{30}{N}t$$

where N is the size of the file in megabytes and t is the number of seconds since the download began. In how many minutes will 25% of a 110-megabyte file remain to be downloaded? Round to the nearest tenth of a minute.

60. **Aviation** The number of miles that remain to be flown by a commercial jet traveling from Boston to Los Angeles can be approximated by the equation

$$\text{Miles remaining} = 2650 - 475t$$

where t is the number of hours since leaving Boston. In how many hours will the plane be 1000 miles from Los Angeles? Round to the nearest tenth of an hour.

61. **Exercise Heart Rate** Various formulas are used to calculate the maximum heart rate (MHR), in beats per minute (bpm), a person should attain during exercise. One set of formulas by G. P. Whyte and colleagues is $\text{MHR(men)} = 202 - 0.55a$ and $\text{MHR(women)} = 216 - 1.09a$ where a is the age of the person exercising. According to these equations, what is the MHR for a male and for a female both of whom are 25 years old? Round to the nearest beat per minute.

62. **Exercise Heart Rate** Using the formulas in Exercise 61, what is the age of a woman whose recommended maximum heart rate is 150? Round to the nearest year.

Enrichment Exercises

In Exercises 63 to 66, solve for *x*.

63. $\dfrac{\frac{1}{2}}{1 + x} + \dfrac{1}{x} = \dfrac{x + 3}{2}$

64. $\dfrac{2x^2 - x + 2}{x} - 3 = 2(x - 4)$

65. $|x| + |x - 1| = 3$ (*Hint:* Consider three cases: $x < 0$, $0 \le x \le 1$, and $x > 1$.)

66. Explain why we suggest that you consider three cases in Exercise 65.