EXERCISE SET 1.1

Concept Check

1. Is 2 a solution of the equation $3(x - 4) + 5 = 2x - 5$? Yes
2. By what number can you multiply each side of the equation $\frac{1}{5}x - 15 = \frac{3}{10}x$ in order to clear the equation of fractions? 10 (or a multiple of 10)
3. What is the difference between a conditional equation and an identity? A conditional equation is true for some values of the variable but not true for other values of the variable. An identity has an infinite number of solutions.
4. If $|x + 9| = 5$, what are the possible values of $x + 9$? Rewrite the equation without absolute value signs. 5 and -5; $x + 9 = 5$ or $x + 9 = -5$

In Exercises 5 to 26, solve each equation and check your solution.

5. $2x + 10 = 40$ 15
6. $-3y + 20 = 2$ 6
7. $5x + 2 = 2x - 10$ -4
8. $4x - 11 = 7x + 20$ $-\frac{31}{3}$
9. $2(x - 3) - 5 = 4(x - 5)$ $\frac{9}{2}$
10. $5(x - 4) - 7 = -2(x - 3)$ $\frac{33}{7}$
11. $3x + 5(1 - 2x) = 4 - 3(x + 1)$ 1
12. $6 - 2(4x + 1) = 3x - 2(2x + 5)$ 2
13. $4(2r - 17) + 5(3r - 8) = 0$ $\frac{108}{23}$
14. $6(5s - 11) - 12(2s + 5) = 0$ 21
15. $\frac{3}{4}x + \frac{1}{2} = \frac{2}{3}$ $\frac{2}{9}$
16. $\frac{x}{4} - 5 = \frac{1}{2}$ 22
17. $\frac{2}{3}x - 5 = \frac{1}{2}x - 3$ 12
18. $\frac{1}{2}x + 7 - \frac{1}{4}x = \frac{19}{2}$ 10
19. $0.2x + 0.4 = 3.6$ 16
20. $0.04x - 0.2 = 0.07$ 6.75
21. $x + 0.08(60) = 0.20(60 + x)$ 9
22. $6(t + 1.5) = 12t$ $\frac{3}{2}$
23. $5[x - (4x - 5)] = 3 - 2x$ $\frac{22}{13}$
24. $6[3y - 2(y - 1)] - 2 + 7y = 0$ $-\frac{10}{13}$
25. $\frac{40 - 3x}{5} = \frac{6x + 7}{8}$ $\frac{95}{18}$
26. $\frac{12 + x}{-4} = \frac{5x - 7}{3} + 2$ $-\frac{32}{23}$

In Exercises 27 to 36, classify each equation as a contradiction, a conditional equation, or an identity.

27. $-3(x - 5) = -3x + 15$ Identity
28. $2x + \frac{1}{3} = \frac{6x + 1}{3}$ Identity
29. $2x + 7 = 3(x - 1)$ Conditional equation
30. $4[2x - 5(x - 3)] = 6$ Conditional equation
31. $\frac{4x + 8}{4} = x + 8$ Contradiction
32. $3[x - (4x - 1)] = -3(2x - 5)$ Conditional equation
33. $3[x - 2(x - 5)] - 1 = -3x + 29$ Identity
34. $4[3(x - 5) + 7] = 12x - 32$ Identity
35. $2x - 8 = -x + 9$ Conditional equation
36. $|3(x - 4) + 7| = |3x - 5|$ Identity

In Exercises 37 to 52, solve each absolute value equation for *x*.

37. $|x| = 4$ $-4, 4$
38. $|x| = 7$ $-7, 7$
39. $|x - 5| = 2$ 3, 7
40. $|x - 8| = 3$ 5, 11
41. $|2x - 5| = 11$ $-3, 8$
42. $|2x - 3| = 21$ $-9, 12$
43. $|2x + 6| = 10$ $-8, 2$
44. $|2x + 14| = 60$ $-37, 23$
45. $\left|\frac{x - 4}{2}\right| = 8$ $-12, 20$
46. $\left|\frac{x + 3}{4}\right| = 6$ $-27, 21$
47. $|2x + 5| = -8$ No solution

Indicates Try It Exercises

48. $|4x - 1| = -17$ No solution

49. $2|x + 3| + 4 = 34$ $-18, 12$

50. $3|x - 5| - 16 = 2$ $-1, 11$

51. $3|2x - 5| + 2 = 11$ $1, 4$

52. $5 - 4|2 - 5x| = -7$ $-\frac{1}{5}, 1$

53. **Biology** The male magnificent frigatebird inflates a red pouch under his neck to attract females. Along with the inflated pouch, the bird makes a drumming-like sound whose frequency F, in hertz, is related to the volume V, in cubic centimeters, of the pouch by the equation $F = -5.5V + 5400$.

Use the equation to estimate the volume of the pouch when the frequency of the sound is 550 hertz. Round to the nearest cubic centimeter. 882 cm^3

54. **Health** According to one formula for lean body mass (LBM, in kilograms) given by R. Hume, the mass of the body minus fat is

$$\text{LBM} = 0.3281W + 0.3393H - 29.5336$$

where W is a person's weight in kilograms and H is the person's height in centimeters. If a person is 175 centimeters tall, what should that person weigh to have an LBM of 55 kilograms? Round to the nearest kilogram. 77 kg

55. **Travel** Ruben is driving along a highway that passes through Barstow. His distance d, in miles, from Barstow is given by the equation $d = |210 - 50t|$, where t is the time, in hours, since the start of his trip and $0 \le t \le 6$. When will Ruben be exactly 60 miles from Barstow?
After 3 h and after 5 h 24 min

56. **Automobile Gas Mileage** The gas mileage m, in miles per gallon, obtained during a long trip is given by

$$m = -\frac{1}{2}|s - 55| + 25$$

where s is the speed of Kate's automobile in miles per hour and $40 \le s \le 70$. At what constant speed can Kate drive to obtain a gas mileage of exactly 22 miles per gallon?
49 mph or 61 mph

57. **Fuel Consumption** An engine burns fuel at a constant rate of $\frac{1}{36}$ gallon per minute. If the engine originally contains 18 gallons of fuel, then the equation $g = 18 - \frac{1}{36}t$ gives the amount of fuel g, in gallons, t minutes after the engine is started. In how many minutes will the engine run out of gas?
648 min

58. **Diving Pressure** The pressure p, in pounds per square inch (psi), on a diver at a depth of d feet below the surface of the ocean can be approximated by the equation $p = 0.445d + 14.7$. If the pressure on a diver is 24 psi, find the depth of the diver. Round to the nearest tenth of a foot. 20.9 ft

59. **Computer Science** If $p\%$ of a file remains to be downloaded using a cable modem, then

$$p = 100 - \frac{30}{N}t$$

where N is the size of the file in megabytes and t is the number of seconds since the download began. In how many minutes will 25% of a 110-megabyte file remain to be downloaded? Round to the nearest tenth of a minute. 4.6 min

60. **Aviation** The number of miles that remain to be flown by a commercial jet traveling from Boston to Los Angeles can be approximated by the equation

$$\text{Miles remaining} = 2650 - 475t$$

where t is the number of hours since leaving Boston. In how many hours will the plane be 1000 miles from Los Angeles? Round to the nearest tenth of an hour. 3.5 h

61. **Exercise Heart Rate** Various formulas are used to calculate the maximum heart rate (MHR), in beats per minute (bpm), a person should attain during exercise. One set of formulas by G. P. Whyte and colleagues is $\text{MHR(men)} = 202 - 0.55a$ and $\text{MHR(women)} = 216 - 1.09a$ where a is the age of the person exercising. According to these equations, what is the MHR for a male and for a female both of whom are 25 years old? Round to the nearest beat per minute. Men: 188 bmp, women: 189 bmp

62. **Exercise Heart Rate** Using the formulas in Exercise 61, what is the age of a woman whose recommended maximum heart rate is 150? Round to the nearest year. 61 years

Enrichment Exercises

In Exercises 63 to 66, solve for x.

63. $\dfrac{1}{\dfrac{2}{1+x}} + \dfrac{1}{x} = \dfrac{x+3}{2}$ 1

64. $\dfrac{2x^2 - x + 2}{x} - 3 = 2(x - 4)$ $-\frac{1}{2}$

65. $|x| + |x - 1| = 3$ (*Hint:* Consider three cases: $x < 0$, $0 \le x \le 1$, and $x > 1$.) $-1, 2$

66. Explain why we suggest that you consider three cases in Exercise 65. To remove the absolute value signs, we must know whether the quantities inside the absolute value are positive or negative. Both quantities are negative when $x < 0$; one is positive (or 0) and one is negative (or 0) when $0 \le x \le 1$; and both are positive when $x > 1$.

SECTION 1.2

Formulas
Applications

Formulas and Applications

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A4.

PS1. The sum of two numbers is 32. If one of the numbers is represented by x, then the expression $32 - x$ represents the other number. Evaluate $32 - x$ for $x = 8\frac{1}{2}$. [P.1] $23\frac{1}{2}$

PS2. Evaluate $\frac{1}{2}bh$ for $b = \frac{2}{3}$ and $h = \frac{4}{5}$. [P.1] $\frac{4}{15}$

PS3. What property has been applied to rewrite $2l + 2w$ as $2(l + w)$? [P.1] Distributive property

PS4. What property has been applied to rewrite $\left(\frac{1}{2}b\right)h$ as $\frac{1}{2}(bh)$? [P.1] Associative property of multiplication

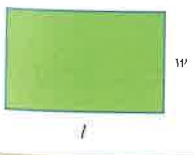
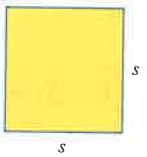
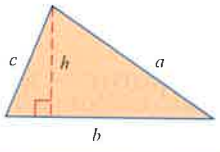
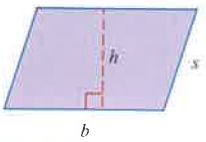
PS5. Add: $\frac{2}{5}x + \frac{1}{3}x$ [P.1] $\frac{11}{15}x$

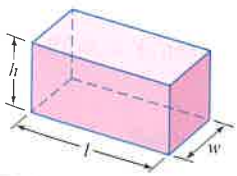
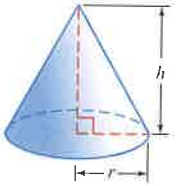
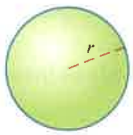
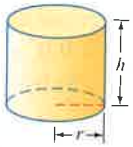
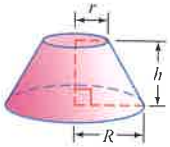
PS6. Simplify: $\dfrac{1}{\frac{1}{a} + \frac{1}{b}}$ [P.5] $\frac{ab}{a + b}$

Formulas

A **formula** is an equation that expresses known relationships between two or more variables. Table 1.2 lists several formulas from geometry that are used in this text. The variable P represents perimeter, C represents circumference of a circle, A represents area, S represents surface area of an enclosed solid, and V represents volume.

Table 1.2 Formulas from Geometry

Rectangle	Square	Triangle	Circle	Parallelogram
$P = 2l + 2w$ $A = lw$	$P = 4s$ $A = s^2$	$P = a + b + c$ $A = \frac{1}{2}bh$	$C = \pi d = 2\pi r$ $A = \pi r^2$	$P = 2b + 2s$ $A = bh$
				

Rectangular Solid	Right Circular Cone	Sphere	Right Circular Cylinder	Frustum of a Cone
$S = 2(wh + lw + hl)$ $V = lwh$	$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$ $V = \frac{1}{3}\pi r^2 h$	$S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	$S = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	$S = \pi(R + r)\sqrt{h^2 + (R - r)^2} + \pi r^2 + \pi R^2$ $V = \frac{1}{3}\pi h(r^2 + rR + R^2)$
				

5. Solve for w.

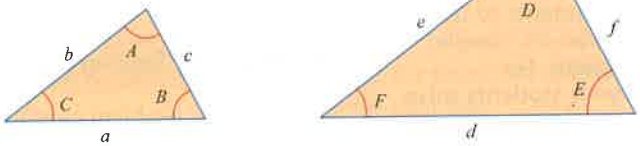
$$440 = 2(1.2w) + 2w$$
$$440 = 2.4w + 2w$$
$$440 = 4.4w$$
$$100 = w$$

The width is 100 feet. The length is $l = 1.2(100) = 120$.

The width of the rectangle is 100 feet, and the length is 120 feet.

▶ Try Exercise 24, page 92

Similar triangles are ones for which the measures of corresponding angles are equal. The triangles below are similar.



$\angle A = \angle D$ $\angle B = \angle E$ $\angle C = \angle F$

An important relationship among the sides of similar triangles is that the ratios of corresponding sides are equal. Thus, for the triangles above,

$$\frac{a}{b} = \frac{d}{e} \qquad \frac{a}{c} = \frac{d}{f} \qquad \frac{b}{c} = \frac{e}{f}$$

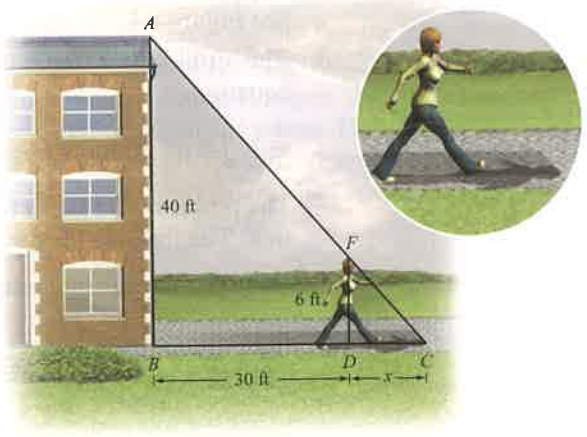
This fact is used in many applications.

EXAMPLE 4 A Problem Involving Similar Triangles

A person 6 feet tall is in the shadow of a building 40 feet tall and is walking directly away from the building. When the person is 30 feet from the building, the tip of the person's shadow is at the same point as the tip of the shadow of the building. How much farther must the person walk to be just out of the shadow of the building? Round to the nearest tenth of a foot.

Solution

Let x be the distance the person has to walk. Draw a picture of the situation using similar triangles.



Alternative to Example 4
A person 5 feet, 6 inches tall is in the shadow of a telephone pole that is 50 feet tall. The person is walking away from the pole along the line of the shadow. When the person is 26 feet from the pole, the tip of the person's shadow is at the same point as the tip of the shadow of the pole. How much farther must the person walk to be just out of the shadow of the pole? Round to the nearest tenth of a foot.
■ 3.2 ft

Triangles ABC and FDC are similar triangles. Therefore, the ratios of the lengths of the corresponding sides are equal. Using this fact, we can write an equation.

$$\frac{30 + x}{40} = \frac{x}{6}$$

Now solve the equation.

$$\frac{30 + x}{40} = \frac{x}{6}$$

$$120\left(\frac{30 + x}{40}\right) = 120\left(\frac{x}{6}\right)$$ • Multiply each side by 120, the LCD of 40 and 6.

$$3(30 + x) = 20x$$ • Solve for x.

$$90 + 3x = 20x$$

$$90 = 17x$$

$$5.3 \approx x$$

The person must walk an additional 5.3 feet.

Try Exercise 30, page 93

Many business applications can be solved by using the equation

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Alternative to Example 5
A lamp manufacturer spends $12.45 to produce a lamp that sells for $39.95. How many lamps must the manufacturer sell to make a profit of $11,000?
■ 400 lamps

EXAMPLE 5 A Business Application

It costs a tennis shoe manufacturer $37.15 to produce a pair of tennis shoes that sells for $69.95. How many pairs of tennis shoes must the manufacturer sell to make a profit of $20,172.00?

Solution

The *profit* is equal to the *revenue* minus the *cost*. If x equals the number of pairs of tennis shoes to be sold, then the revenue will be 69.95x$ and the cost will be 37.15x$. Therefore,

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$20{,}172.00 = 69.95x - 37.15x$$

$$20{,}172.00 = 32.80x$$

$$615 = x$$

The manufacturer must sell 615 pairs of tennis shoes to make the desired profit.

Try Exercise 32, page 93

Simple interest problems can be solved by using the formula $I = Prt$, where I is the interest, P is the principal, r is the simple interest rate per period, and t is the number of periods.

Alternative to Example 6
A technician invests part of $5000 in a 4% simple interest account and the remainder of the money at 6% simple interest. Together the investments earn $270 per year. Find the amount invested at each rate.
■ $1500 at 4%, $3500 at 6%

EXAMPLE 6 An Investment Problem

An accountant invests part of a $6000 bonus in a 5% simple interest account and invests the remainder of the money at 8.5% simple interest. Together the investments earn $370 per year. Find the amount invested at each rate.

(continued)

INSTRUCTOR NOTE
Encourage your students to write down what their variables represent. It will be difficult for them to establish an equation if they do not know precisely what each variable represents.

Let t equal the number of hours to fill the pool using both pumps. Then

$$t \cdot \frac{1}{6} = \frac{t}{6} \quad \text{• Part of the pool filled by pump A}$$

$$t \cdot \frac{1}{3} = \frac{t}{3} \quad \text{• Part of the pool filled by pump B}$$

$$\begin{pmatrix}\text{Part filled}\\ \text{by pump A}\end{pmatrix} + \begin{pmatrix}\text{Part filled}\\ \text{by pump B}\end{pmatrix} = \begin{pmatrix}\text{1 filled}\\ \text{pool}\end{pmatrix}$$

$$\frac{t}{6} + \frac{t}{3} = 1$$

Multiplying each side of the equation by 6 produces

$$\begin{aligned} t + 2t &= 6 \\ 3t &= 6 \\ t &= 2 \end{aligned}$$

Check: Pump A fills $\frac{2}{6}$, or $\frac{1}{3}$, of the pool in 2 hours and pump B fills $\frac{2}{3}$ of the pool in 2 hours, so 2 hours is the time required to fill the pool if both pumps are used.

Try Exercise 62, page 94

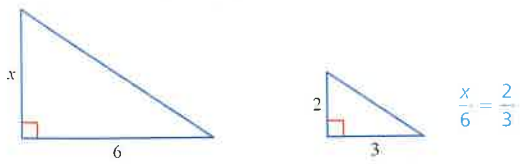
EXERCISE SET 1.2

Concept Check

1. What is the first step of solving the formula $A = P + Prt$ for r? Subtract P from each side.

2. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$. Find the value of A if $h = 6$, $b_1 = 5$, and $b_2 = 7$. 36

3. The two right triangles shown below are similar. What must $\frac{x}{6}$ equal? $\frac{2}{3}$



$\frac{x}{6} = \frac{2}{3}$

4. Part of \$4000 is invested in a 3% simple interest account and the remainder in a 5% simple interest account. If x represents the amount invested at 3%, what expression represents the amount invested at 5%? Write an expression for the amount of interest earned on the two accounts together in 1 year. $4000 - x$; $0.03x + 0.05(4000 - x)$

5. Suppose x pounds of almonds that cost \$7 per pound are mixed with 10 pounds of walnuts that cost \$9.00 per pound. Write an expression that represents the total value, in dollars, of the nut mixture. $7x + 90$

6. Marya can mow her lawn in 45 minutes. Write an expression that represents the fraction of the lawn that she can mow in m minutes. $\frac{m}{45}$

In Exercises 7 to 16, solve the formula for the specified variable.

7. $V = \frac{1}{3}\pi r^2 h$; h (geometry) $h = \frac{3V}{\pi r^2}$

8. $P = S - Sdt$; t (business) $t = \frac{S - P}{Sd}$

9. $I = Prt$; t (business) $t = \frac{I}{Pr}$

10. $A = P + Prt$; P (business) $\frac{A}{1 + rt} = P$

11. $F = \frac{Gm_1m_2}{d^2}$; m_1 (physics) $m_1 = \frac{Fd^2}{Gm_2}$

12. $A = \frac{1}{2}h(b_1 + b_2)$; b_1 (geometry) $\frac{2A - hb_2}{h} = b_1$

13. $a_n = a_1 + (n - 1)d$; d (mathematics) $d = \frac{a_n - a_1}{n - 1}$

14. $y - y_1 = m(x - x_1)$; x (mathematics) $\frac{y - y_1 + mx_1}{m} = x$

15. $S = \frac{a_1}{1 - r}$; r (mathematics) $r = \frac{S - a_1}{S}$

Indicates Try It Exercises